NG SCHOOL MATHEMATICS

TEACHERS' EDITION

LET'S DO

LET'S TALK

LET'S USE

Topic	Kinc		Book Two
	Units P and R	Units a, b, c, and d	Units e, f, g, and h
A Sets, Logical Reasoning, and Patterns	Concept of sets: **P**13–14 Comparisons of sizes: **P**1–8, **P**11–12, **P**15 Similarities and differences: **P**17–32 Patterns: **P**9–10, **P**16 Maze: **P**32	Comparing sets: **a**1–9 Equivalent sets: **a**5–8 Cardinal number of a set: **a**13–14 Empty set: **a**19 Union of sets and addition: **b**1–13, **b**25–32, **b**41 Sets and subtraction: **b**15–23, **b**33–37, **b**45 Sets of ten: **c**1–4 Skip counting: **d**54, **d**62 Logic: **a**59–60, **d**42 Patterns: **c**14	Cardinal number of a set: **e**1–7 Maze: **e**40, **g**22 Sets and addition: **e**43 Sets and subtraction: **e**47 Counting sequences: **e**15, 62; **f**36; **g**6, 11, 13; **h**38 Informal logic: **e**52, 57; **f**55; **g**12, 34, 37; **h**20, 50 Patterns: **f**10 Attribute pieces: **g**23 Puzzle problems: **g**34, **g**62, **h**8 Multiplication and sets: **g**50–52, **g**55, **g**57
B Numeration and Place Value	More than and less than concept: **P**33–37, **P**43–47 Matching, one-to-one correspondence: **P**37–42 Numerals and numbers 0 to 10: **R**1–7, **R**11–12, **R**17–32 Recognition of number of a set: **R**8–10, **R**13–16, **R**37–38	Introducing ten: **c**1–4 Grouping by tens: **c**5–13 Counting to 99: **c**15–26, **d**44 One more than: **a**9, **a**27, 29, 31, 33, 35 Greater than and less than: **a**41–49, **d**55–61 Numeral writing: **a**15–23, **a**27–36	Numeral writing: **e**3 2- and 3-digit numerals: **e**7–11, **f**13–18, **g**1–13 Counting to 99: **e**13–17, **h**62 Greater than and less than: **e**17–23, **g**12 Roman numerals: **e**32–39 Expanded notation: **f**49, **g**16–17 Regrouping: **h**33, **h**41–44, **h**52–56 Fraction numerals: **h**21–29
C Addition and Subtraction of Whole Numbers	Addition combinations 1 to 10 (intuitive): **R**32–36, **R**39–44 Subtraction related to sums of 10 or less (intuitive): **R**45–46 Order of numbers 1 to 10: **R**48	Addition and subtraction facts, sums through 9: **b**1–51, **c**35–45 Zero principle for addition: **b**9 Inverse relation (+ and −): **c**57–60, **d**49–52 Missing addends: **c**53–56, **d**45–53 Sums of ten: **d**1–11 Addition and subtraction facts, sums greater than 10: **d**13–23, **d**35–43 Commutative (order) principle: **d**41, **d**51	Addition and subtraction facts, sums of 10 or less: **e**41–51, **e**59 Missing addends: **e**53–61 Inverse relation (+ and −): **e**57–58, **h**13–14 Basic principles: **f**1–9 Addition and subtraction facts, sums to 18: **f**11–35, **g**35–47, **h**1–19 Addition and subtraction without regrouping: **f**47–61, **g**15–21 Addition and subtraction with regrouping: **h**31–61
D Multiplication and Division of Whole Numbers			Multiplication concepts: **g**49–61
E Fractional Numbers		Introduction to fractions, $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$: **d**25–33	Halves, thirds, and fourths: **h**21–29
F Problem Solving and Applications		Solving money picture stories: **c**47–51 Discussions: **a**2, 14, 26, 42; **b**2, 16, 26, 40; **c**16, 36, 46, 54; **d**2, 14, 26, 46, 56 Picture story for zero: **a**20	Money problems: **e**25–31, **g**14 Addition and subtraction problems: **e**60; **f**22, 32, 34, 54, 60; **g**20, 46, 60; **h**16, 18, 45, 58, 60 Multiplication problems: **g**59–60 Addition and subtraction problems with regrouping: **h**45, **h**58, **h**60 Discussions: **e**2, 26, 42; **f**48
G Estimation			Introductory concepts leading to regrouping: **h**31, **h**39
H Number Theory		Skip counting sequences: **d**54 Skip counting even numbers: **d**62	Odd and even numbers: **e**12 Skip counting: **e**62, **f**36, **h**38 Nomograph addition: **f**24 Square numbers: **f**62
I Measurement		Concept of length: **b**53–57 Centimetres: **b**57–61 Length of paths: **b**62, **d**34 Readiness for area concepts: **c**62 Readiness for volume concepts: **d**24	Time: **e**33–39 Centimetres: **f**37–40 Perimeter: **f**41–42 Area: **f**43–44 Litres: **f**46 Volume: **g**48
J Geometry, Number Line, and Graphs	Recognition of simple shapes: **P**17, **P**27, **P**28, **P**30, **P**48	Number line: **a**47; **b**11, 21, 29, 35, 43, 47; **d**5, 19–22, 48, 54 Recognition of basic shapes: **a**12, **a**50–54 Open and closed curves: **a**55 Segments and paths: **a**57 Same size figures: **a**59–61 Symmetry: **d**25	Number line: **e**6, 46, 49; **f**12, 16, 21, 29; **g**54, 58; **h**35 Triangles, squares, rectangles, circles: **g**23–26 Segments: **g**27–28 Congruence: **g**28–30 Similarity: **g**31–32 Graphing: **h**48
K Special Topics		Telling time: **c**27–33 Money: **c**45–51 Let's have fun: **a**12, 24, 40, 50, 62; **b**14, 24, 38, 52, 62; **c**14, 26, 34, 44, 52, 62; **d**12, 24, 34, 44, 54, 62	Time: **e**33–39 Roman numerals: **e**32–39 Money: **e**25–31, **g**14 Magic squares: **h**8 Let's have fun: **e**12, 24, 32, 40, 52, 62; **f**10, 24, 36, 46, 62; **g**14, 22, 34, 48, 62; **h**8, 20, 30, 38, 48, 62

[Continued on inside back cove

Teachers' Edition to accompany

Investigating School Mathematics

ROBERT E. EICHOLZ

PHARES G. O'DAFFER

CHARLES R. FLEENOR

Collaborator, Teachers' Edition
THERESA BURKE

Collaborators, Reference Material and Metrication
JOHN BATES
DONALD IRWIN
J. NORMAN C. SHARP
JAMES SHERRILL

ADDISON-WESLEY (CANADA) LTD.
DON MILLS, ONTARIO • READING, MASSACHUSETTS • MENLO PARK, CALIFORNIA • LONDON

This book is in the
ADDISON-WESLEY MATHEMATICS SERIES FOR GRADES K–8

Investigating School Mathematics for Grades K–6

School Mathematics I and II for Grades 7 and 8

Success with Mathematics 1 and 2 for Grades 7 and 8

Supplementary Materials available for use with
ADDISON-WESLEY MATHEMATICS SERIES FOR GRADES K–8

Big Book Primer • Duplicator Masters for Grades 1–8
Supplementary Workbooks for Grades 3–8
Achievement Tests for Grades 1–8 • Diagnostic Tests for Grades 3–8
Getting Ready Books • Mathematics Manipulative Aids
Enrichment Booklets for Grades 1–2   Practice Booklets for Grades 1–2

The color sequence of the strips illustrated in this work is used with the permission of the CUISENAIRE COMPANY OF AMERICA.

ISBN 0-201-01131-X

CDEFGH 79876

Foreword

The *Investigating School Mathematics* series co-ordinates the precise concepts of modern mathematics with an approach that stimulates the child to actively participate in his own learning experiences. The series provides for the necessary mastery of basic number skills, and presents the material in a way that emphasizes the exciting, creative nature of mathematics. As the child becomes involved in exciting explorations and investigations, the structure and beauty of mathematics unfolds. The children are encouraged to investigate and discover ideas for themselves, to look for interesting patterns and relationships, and to develop their own generalizations. New and fascinating topics are explored not solely for their mathematical value, but also because they stimulate interest and motivate children to put forth their best efforts.

In our view, the development of a sound mathematical structure need not be hindered by an exciting, activity-oriented approach. Rather, the activity approach can and should reinforce the child's experiences as he investigates mathematical topics in an orderly, structured fashion. The same, sound mathematical structure that was called "modern" in the 1960's is present in *Investigating School Mathematics*. The important difference in this new series lies in its approach. The child learns through continual active participation in activities and investigations that lead to the unfolding and discovery of each new idea.

As each new concept unfolds, the child is given an opportunity to investigate the ideas by using a wide variety of manipulative materials and activities. Then, through guided discussion, he is led to a deeper understanding of the ideas and their relation to the overall structure of mathematics. Following the investigation and discussion, he is provided with sufficient problem-solving practice to develop speed and accuracy.

The *Investigating School Mathematics* series is unprecedented in its careful provision for individual differences. Throughout each text, the child is challenged to do what he *can* do, not what someone else *thinks* he can do. Each child has the opportunity to experience individual success in an environment that stresses co-operation and communication rather than competition. This careful provision for individual differences makes the *Investigating School Mathematics* series unusually adaptable to such diverse teaching situations as ungraded schools, individual or small group instruction, or whole-class instruction.

The essence of the *Investigating School Mathematics* series is reflected in the beliefs to which we are committed: that there are fundamental mathematical concepts which can be isolated and set forth with sharpness and clarity; that these concepts, when truly understood, provide powerful tools for extending knowledge; that children of every level should be encouraged to actively participate, to think, to question, and to seek understanding; that, although a certain body of knowledge must be passed on to each generation from preceding generations, the individual creativity of each new generation must not be stifled by pedagogy which forces upon its pupils patterns of thought which have served us well in the past but which may be inadequate for the future.

Mathematics can be successfully taught in this spirit. At every stage in the learning of mathematics, the discovery of new relationships can be a delight. It is in this spirit that *Investigating School Mathematics* has been written.

The authors wish to express their appreciation to Ball State University and to the Educational Research Council of Greater Cleveland, where many of the ideas were generated and tested for the *Elementary School Mathematics* series, which served as forerunner of *Investigating School Mathematics;* to Edith Biggs and the Nuffield Project in England, for their leadership in bringing the activity-oriented laboratory approach into prominence; to Mrs. Nancy Hildebrand, whose contributions to the teachers' manuals for *Elementary School Mathematics* are still reflected in this manual; to Theresa Burke, who assisted in the preparation of this manual by bringing, from a wealth of classroom experience, many of the activities and teaching suggestions found in each lesson; and finally, to the many teachers and children who have proved that studying mathematics can be an exciting and stimulating experience in the elementary school.

About the Teacher's Edition

The complete pupils' text is reproduced in this teachers' edition in order that you may have before you at all times, in one book, both the page being studied by the pupils and the pertinent manual material.

Throughout the manual, page numbers for the student text are given letter-numeral designation, while pages for the teacher text are referred to by numeral only.

Contents of Learning Units A, B, C, D

*Enrichment Module

The Book 1 Program

Design Features of Book 1

The Book 1 program is organized into four 64-page learning units. Each learning unit is divided into five or six single-concept modules, ranging from 8 to 16 pages each. The four learning units are labelled A, B, C, D, while the modules within these units are color coded in the upper right-hand corner of each sheet. The first module of each learning unit is color coded yellow, the second is orange, the third is red, the fourth is light green, the fifth is dark green, and the sixth is blue. Three of the modules in the Book 1 program have a special tinted top margin. These modules are considered as optional modules for the Book 1 program. The last module in Learning Unit C is optional and the last two modules in Learning Unit D are optional.

Each lesson throughout Book 1 is presented on the front and back of a sheet, rather than on facing pages. Thus the sheets may either be torn out and used one at a time or be used in the book itself.

Punchout materials are available as separate items for some of the units. Some of these materials are for use with one particular module, such as the clockface and hands to be used with the module on telling time. Other punchouts can be used with many modules. For example, centimetre strips are available as punchouts and suggested for use in more than one module.

Pre-book activities are suggestions to help develop a concept more completely. Materials required for these activities are given in the teacher's notes.

The first lesson in each module is made up of an investigation and a discussion page. The investigation page is labelled "Let's do" and the discussion page "Let's talk." The investigation phase of the first lesson is designed to provide the child with an opportunity to explore the germ of the concept developed throughout the module. This part of the lesson should be child-centred insofar as possible. That is, the children should be given considerable freedom to explore and investigate the particular concept involved. Following the investigation phase of the lesson, the reversed side of the sheet provides the children with an opportunity to discuss the ideas and the teacher with an opportunity to further develop the ideas and prepare the children for the various utilization phases of the module which follow. Following the investigation-discussion lesson in the module, other lessons are provided to give the child an opportunity to practice, develop, and extend the concepts and skills of the module. Each lesson has detailed teacher commentary notes to provide objectives, pre-book activities for further developing the concepts and follow-up suggestions. The last lesson in each module is made up of two parts. The front side of the sheet is labelled "Show you know" and this can be used as a module review and, possibly, as an evaluation instrument to determine the child's understanding of the concepts and ideas presented in the module. The back page of each of the final lessons is labelled "Let's have fun," and is intended as an interesting mathematical side trip to be treated with a light touch and in the spirit of having fun with mathematics. The final lesson in each of the 64-page learning units is labelled "Looking back" and is intended as a cumulative review for that learning unit.

The investigation phase of the first lesson is marked by a blue band on the left side of the page, and the discussion phase of the lesson is marked by a green band on the right side of the page. Each key lesson in Book 1 contains an art demonstration at the top of the page to assist you in introducing the activity required on the page itself. It also serves as a guide to the children to remind them of the particular task to be done on the page.

Mathematics of the Book 1 Program

A significant number of mathematical concepts are introduced for the first time in the Book 1 program for *Investigating School Mathematics*. The depth of coverage of these topics is often quite limited due to the age of the children involved. The following list indicates the major concepts introduced in the Book 1 program.

Classification and Sorting
Sets and Subsets
Union and Intersection of Sets
One-to-One Matching
Operations: Addition and Subtraction
Numeration and Place Value
Equality and Equations
Inequalities, Order, and Counting
Basic Principles of Addition
Introductory Geometric Concepts
Basic Concepts of Measurement

While this list of mathematical concepts may seem formidable for the Book 1 program, it should be understood that many of the concepts, for example, the ideas of subsets and union and intersection of sets, are introduced only on an intuitive level. The concept of a subset of a given set is vital to the idea of removing one set from a given set for the purposes of introduction to subtraction. This is done easily without ever mentioning the word subset. Also, the concept of the union of two sets comes up naturally when the child puts two sets together in working with the idea of addition of the numbers of the two sets. Unit A begins with primitive number concepts and introduces one-to-one matching and the concepts of more and less on a pre-number basis. The orange and red modules in Unit A introduce the numbers 0 through 9. By using the pre-number work from the first module, the numbers 0 through 9 are introduced as concepts associated with equivalent sets; that is, by matching two sets of three, the child begins to see that there is some-

thing common between these sets and it is that idea which we learn to refer to as "threeness" or the number three. The light green module in Unit A brings together the ideas introduced in the first three modules and by the use of the number line introduces the concepts of inequality and order. The dark green module introduces some basic geometric figures including circles, triangles, squares, and rectangles. The ideas of open and closed curves, segments, and congruent figures are also introduced.

The first four modules of Unit B introduce the basic addition and subtraction operations associated with sums to 9. At this stage, the child is encouraged to use counters, centimetre strips, and number lines to find sums and differences. The overriding objective at this level is the development of the power skill or understanding level of the basic concepts of the operations. The dark green module of Unit B deals with measurement. The child's understanding of the counting process in measurement is of prime importance to his understanding of the concept.

Learning Unit C introduces two-digit place value and counting to 99. These place-value and counting modules provide the children with many physical experiences in grouping sets of objects into so many tens and so many more. In this way, children have a greater opportunity to develop a basic understanding of the meaning of two-digit numerals. In addition, introductory work is provided for development of speed skills for sums and differences associated with sums to 9. A review of sums and differences initiates the work toward greater proficiency in finding sums and differences. At this stage, mastery or memory of the basic facts is not yet expected. Modules introducing telling time and money are also in Learning Unit C.

The final module of Unit C introduces the concept of the relationship between addition and subtraction, along with the idea of finding a missing addend in an addition equation. This module is considered optional. It is intended for introductory work that will eventually assist in the development of speed skills for subtraction. Subtraction speed skills will be developed through the concept of finding a missing addend.

Unit D provides additional practice with sums and differences through 10 and then extends the work to the sums and differences through 18 at the power skill or concept level. Unit D also introduces fraction concepts of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$. The final two (optional) modules of Unit D involve inverses, missing addends, and inequalities.

General Suggestions

Telling time is introduced in the red module of Learning Unit C. It is not intended that this unit be all-inclusive with regard to the teaching of telling time. Rather the module should be considered as a stimulus and impetus to assist the children in learning this important skill. The children should be working with basic concepts of telling time from the beginning of the school year as they discuss such things as the time for recess, lunch, reading period, and so on; that is, time telling should be an ongoing process which is practiced on a daily basis throughout the school year.

Work with money is introduced in the dark green module of Unit C. Again it is intended that this module provide an impetus to work with money which has probably occurred from time to time throughout the school year. Most children already will have had opportunities to work with and count various amounts of money.

Special topics which do not occur on the student worksheets but which should arise in a natural setting throughout the school year are work with the calendar and the language of our ordinal numbers. Again, these topics should be presented in an ongoing situation, rather than in a single lesson or module. For example, you should continually use language such as first, second, and third in various meaningful situations. The children will pick up these words naturally without undue stress. Also work with the calendar can occur as you discuss such things as special holidays, Saturdays and Sundays, Thanksgiving, Christmas, Spring Vacation, and so on.

Provision for Individual Differences

Since 6-to-7-year-old children are limited in their ability to work independently, provision for individual differences is particularly difficult; however, the investigations, discussions, and pre-book activities provide you with an unusual opportunity to give the more able children a chance to discover more, discuss more, and do more. In addition to these investigations and activities which are primarily teacher directed, there are accompanying enrichment materials available for *Investigating School Mathematics,* Book 1. Also, for more able children, you may want to make greater use of the follow-up activities suggested in this manual as well as the additional material which can be found in the three modules on tinted or colored paper. This should not indicate that these extension modules are not useful for all of the children. It merely indicates that extra care must be taken in working with average and below average children in these learning modules. There are many excellent outside sources for additional enrichment material. Among the best is the book titled *Workjobs* by Mary D. Lorton, Addison-Wesley Publishing Company. This book presents activity-centred learning for early childhood education.

Teaching Strategies for Book 1

All the pages in Book 1 student text are reproduced in this Teachers' Edition in full color, with annotated answers. Unit notes for each module of Book 1 are provided to orient you to the contents and objectives of the

module. Detailed page lesson notes and suggested activities appear next to each pair of pages (or lesson) in each module. Mathematics sections are included in both the module and page lesson notes to clarify the concepts presented and to provide a full explanation whenever a mathematical topic new to the children is introduced.

The general teaching strategy for the Book 1 program is based on a five-point plan, as follows: Preparation, Investigation, Discussion, Utilization, and Extension.

The actual physical design of each module as described in the section on design features of Book 1 reflects this five-point teaching strategy. The preparation, investigation, and discussion phase of the teaching strategy occurs at the beginning of each module. The preparation may be found in the teachers' guide and the investigation and discussion are a part of the actual design of the module and are labelled "Let's do" and Let's talk" and color coded with blue and green bands. When the child is working on the investigation phase of the teaching strategy, maximum effort should be made to encourage him to work on his own. That is, the child should be given an opportunity to investigate, explore, and discover the mathematical concepts of the module. The teacher should, in so far as possible, act as a resource person for this phase of the lesson. The preparation for an investigation should be very short, no longer than is required to prepare the child to begin the investigation independently. Following the investigation, the teacher has an opportunity through the use of the "Let's talk" page to develop and extend the mathematical concepts which were investigated. By using the various pictorials on the "Let's talk" page, the teacher has an opportunity to draw out the ideas and thoughts from the investigation and at the same time develop those concepts which are needed for the remaining lessons. The remaining lessons of the module constitute the utilization phase of the teaching strategy. In the teachers' manual, each of these utilization lessons is accompanied by a preparation or a pre-book activity as well as follow-up material for use after the children have completed the work on a given sheet. The follow-up material and the supplementary materials, as well as the last lesson in the module, may be thought of as the extension phase of the teaching strategy. Actually, each utilization lesson may be thought of as employing the entire five-point teaching strategy in itself. The preparation and investigation are accomplished through pre-book activity found in the teachers' manual. This discussion part of the teaching strategy can be accomplished as the teacher and children communicate the ideas of the suggested pre-book activity. The utilization phase is the work on the text page and the extension is provided by the teacher in the form of additional or enrichment follow-ups.

A part of the overall philosophy of *Investigating School Mathematics* is to encourage each child to become actively involved in his own learning experience. This is most often and best accomplished through the use of a variety of manipulative aids such as counters, geometric shapes, and measuring devices. In addition to these manipulative materials, each child should be provided with a set of the punchout centimetre strips. These strips are available from Addison-Wesley (Canada) Ltd. and may be used as aids in developing various basic mathematical concepts such as the order of the numbers, addition, and subtraction.

Although some of the techniques for teaching modern mathematics are highlighted throughout this manual, teaching children is your specialty. A manual cannot and should not attempt to dictate the day-to-day handling of your class and the individuals in it. The directions given in the teachers' manual should be used flexibly according to the needs and abilities of your children. Do *not* allow the lesson notes to stifle your own effective teaching methods and creative efforts. Consider follow-up activities as suggestions to inspire your own resourcefulness.

Evaluation of Progress

The broad objectives of any mathematics program are to promote the understanding of the mathematical concepts presented and to develop adequate skill in using these concepts. Certainly the development of meaningful skills in mathematics cannot be completely separated from an understanding of the mathematical concepts.

The problem of evaluating children in a modern mathematics program is quite complex when one attempts to analyze the various types of skills that are developed. A clear understanding of these skills can best be accomplished by categorizing them into two basic classes: power skills and speed skills. The role of understanding is most important in power skills. When sufficient emphasis has been placed upon understanding, the child is then prepared to attack new problems or skills independently. At every stage in the mathematics program, the child is encouraged to seek out and develop concepts for himself. This is what we describe as the power-skill stage. It is the understanding stage. Briefly, we can describe the power-skill as the ability to attack a problem and through some technique, however awkward or however time consuming, find the correct answer. Since power skill has to do with understanding a concept in order to find the correct answer and speed skill has to do with techniques and efficiency in finding correct answers, it becomes clear that most evaluation instruments test speed skills only. For this reason, any sound evaluation of progress in a modern mathematics program must be based on daily observation. In this way, you will be able to observe those children who are able to attack new problems and techniques with the power of understanding. It is very difficult to construct written test items which will reveal understanding and the child's ability in the area of power skills. Your job of evaluating speed skills is much simpler. For the most part, speed skills are those skills which we

have always evaluated through traditional types of testing programs. Therefore, any evaluation of achievement should include both your daily observation of a child's creativity and understanding along with the more routine results of speed-skill tests.

About Resources for Active Learning

With the investigative approach it is important to know what materials are available for activity-oriented classrooms. The lists in the module introductions and in most of the lessons offer you some suggestions. If one or two of these resources are available, hopefully you will be able to use them or adapt the ideas.

In the module introductions there are three kinds of resources named. The "General Activities" would be useful as ongoing activities throughout the module, as a review of concepts, and as practice for basic skills. The "Manipulative Devices" and "Commercial Games" can be used to support the lessons in the Module and throughout that Unit. The "Resource(s)" listed with a lesson are more clearly related to the "Objective" of that lesson. Choose one or two and try them out in a variety of situations.

At the time of this writing, those high quality resources which directly complement the active-learning approach were included. Become aware of those materials that were marketed since that time. Check with your principal, (mathematics) supervisor, or resource teacher for the more recent material that supports this type of learning environment. Several suppliers are noted to help you get started in the search.

Tips on technique: start gradually, choose discreetly, be flexible, experiment, relax, and then have fun and learn with the children.

Bibliography of Resources for Active Learning

Bates, John, Donald Irwin and Garry Hamilton, *Developmental Math Cards* (Don Mills, Ont.: Addison-Wesley, 1970).

Biggs, Edith, James MacLean, *Freedom to Learn* (Don Mills, Ont.: Addison-Wesley, 1969).

Buckeye, Donald, William Ewbank and John Ginther, *A Cloudburst of Math Lab Experiments*, Volume 1 (Birmingham, Mich.: Midwest Publications, 1971).

Cohen, Donald, *Maths Mini-lab*, Activity Cards and Teacher's Guide (Newton, Mass.: Selective Educational Equipment, 1971).

Dienes, Z. P., E. W. Golding, *Exploration of Space and Practical Measurement* (New York: Herder and Herder, 1966).

Dienes, Z. P., E. W. Golding, *Learning Logic, Logical Games* (New York: Herder and Herder, 1970).

Dienes, Z. P., E. W. Golding, *Sets, Numbers and Powers* (New York: Herder and Herder, 1966).

Dienes, Z. P., *Multibase Arithmetic Blocks*, Tasks and Manual (New York: Herder and Herder, 1961).

Dumas, Enoch, *Math Activities for Child Involvement* (Boston: Allyn and Bacon, 1971).

Elementary Science Study: *Attribute Games and Problems; The Balance Book; Geo-Blocks; Match and Measure; Mirror Cards; Pattern Blocks; Tangrams* (St. Louis: Webster Division, McGraw-Hill Book Co., 1968–1971).

Fletcher, Harold, Arnold Howell, and Ruth Walker, *Mathematics in Modules* (Sydney, Australia: Addison-Wesley, 1973).

Huff, M. Elizabeth, Donald Irwin, *Activities in Geometry for Primary Pupils* (Don Mills, Ont.: Addison-Wesley, 1973).

Lorton, Mary, *Workjobs: Activity-Centered Learning for Early Childhood Education* (Menlo Park, Ca.: Addison-Wesley, 1972).

Mathex: Matching and Graphing No. 1; Numeration No. 2; Operations No. 3; Geometry No. 4; Measurement and Estimation No. 5, (Toronto, Ontario: Encyclopaedia Britannica Publications Ltd., 1970).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (Cambridge, England: Cambridge University Press, 1967).

Neufeld, Evelyn, James Lucas, *Developing Number Experiences*, Kit A (New York: Holt, Rinehart and Winston, 1966).

Neufeld, Evelyn, James Lucas, *Number-blox* (Portola Valley, Ca.: Sanford Associates, 1969. Obtained from Creative Publications, Palo Alto, Ca.).

Neufeld, Evelyn, James Lucas, *Think and Color* (San Leandro, Ca.: Educational Science Consultants, 1971).

Nuffield Mathematics Project: *Environmental Geometry; Beginnings* 1; *Mathematics Begins* 1; *Computation and Structure* 2; *Pictorial Representation* 1 (New York: John Wiley, 1967–1969).

Read, Ronald, *Tangrams-330 Puzzles* (New York: Dover Publications, Inc., 1965).

Rockcastle, Verne, Frank Salamon and Victor Schmidt, *Elementary School Science*, Primer and Book 1, (Menlo Park, Ca.: Addison-Wesley, 1972).

Sealey, Leonard, *Early Number Multi-Group Lab*,

Turner, Ethel, *Teaching Aids for Elementary Mathematics* (New York: Holt, Rinehart and Winston, 1966).

Wirtz, Robert, Morton Botel, Max Beberman and W. W. Sawyer, *Math Workshop:* Games and Enrichment Activities (Chicago: Encyclopaedia Britannica Educational Corp., 1964).

Suppliers of Resources for Active Learning

Addison-Wesley (Canada) Ltd., Don Mills, Ontario
CCM School Materials, Inc., Chicago, Illinois

Childcraft Education Corp., New York, New York
Creative Publications, Palo Alto, California
Cuisenaire Company of America, Inc., New Rochelle, New York
Educational Playsystems, Inc., New York, New York
Educational Supply Co., Toronto, Ontario
Edmund Scientific Co., Barrington, New Jersey
Educational Teaching Aids, Chicago, Illinois
Gamco Industries Inc., Big Spring, Texas
J. L. Hammett Co., Braintree, Massachusetts
Jack Hood School Supplies Co., Ltd., Stratford, Ontario
Herder and Herder, Inc., (Methuen Publications, Agincourt,Ontario)
Holt, Rinehart and Winston of Canada Ltd., Toronto, Ontario
Ideal School Supply Co., Oak Lawn, Illinois
Lakeshore, San Leandro, California
Mafex Associates, Inc., Johnstown, Pennsylvania
Math Media Inc., Danbury, Connecticut
Metric Aids Ltd., Toronto, Ontario
Milton Bradley, Springfield, Massachusetts
Moyer-Vico Ltd., Weston, Ontario
Responsive Environments Corp., Englewood Cliffs, New Jersey
Sargent-Welch Scientific Co. of Canada Ltd., Weston, Ontario
Science Research Associates, Inc., Don Mills, Ontario
Scott, Foresman and Co. (Gage Educational Publishing Ltd., Agincourt, Ontario)
Selective Educational Equipment, Newton, Massachusetts
Sigma Division, Scott Scientific Inc., Fort Collins, Colorado
Teaching Resources, Boston, Massachusetts
TUF, Rowayton, Connecticut
Webster Division, McGraw-Hill Book Co. (McGraw-Hill Ryerson, Ltd., Scarborough, Ontario)
Wff 'N Proff, New Haven, Connecticut

Teacher's Bibliography

To gain a broader, more comprehensive feeling for the philosophy of *Investigating School Mathematics*, the list of books that follows should prove extremely helpful.

Biggs, Edith, and James MacLean, *Freedom to Learn: An Active Learning Approach to Mathematics* (Don Mills, Ont.: Addison-Wesley, 1969).

Boyer, Carl B., *A History of Mathematics* (New York: John Wiley & Sons, Inc., 1968).

Copeland, Richard, *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York: The Macmillan Co., 1970).

Dienes, Z. P., *Building Up Mathematics* (London: Hutchinson Educational Ltd., 1960. Can be obtained from Humanities Press, New York).

Elliott, H. A., James MacLean, and Janet Jorden, *Geometry in the Classroom: New Concepts and Methods* (Toronto, Ontario: Holt, Rinehart and Winston of Canada, Ltd., 1968).

Forbes, Jack, and Robert Eicholz, *Mathematics for Elementary Teachers* (Menlo Park, Ca.: Addison-Wesley, 1971).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (New York: Cambridge University Press, 1967).

National Council of Teachers of Mathematics, *Insights into Modern Mathematics* (23rd Yearbook, 1957); *The Growth of Mathematical Ideas, Grades K-12* (24th Yearbook, 1959); *Enrichment Mathematics for the Grades* (27th Yearbook, 1963); *Topics in Mathematics for Elementary School Teachers* (29th Yearbook, 1964); *More Topics in Mathematics for Elementary School Teachers* (30th Yearbook, 1969). Washington, D.C.: National Council of Teachers of Mathematics).

Newman, J. R., *The World of Mathematics* (New York: Simon and Schuster, 1956).

Nuffield Mathematics Project, *I Do, and I Understand* (New York: John Wiley & Sons, Inc., 1967).

School Mathematics Study Group, *Studies in Mathematics, Volume IX, A Brief Course in Mathematics for Elementary School Teachers*, Revised Edition (Stanford University: 1963).

The Schools Council, *Mathematics in Primary Schools* (Curriculum Bulletin No. 1). (Available from Selective Educational Equipment Newton, Mass., 1964).

Williams, E. M., and Hilary Shuard, *A Resource for Teachers: Primary Mathematics Today*, Grades 1-8 (Menlo Park, Ca.: Addison-Wesley, 1972).

Children's Bibliography

Alain, *One Two Three, Going to Sea* (New York: William R. Scott, 1965.) Counting.

Ambler, Gifford, *Ten Little Foxhounds* (Chicago: Children's Press, 1969.) Counting.

Ardizzone, Edward, *Johnny the Clockmaker* (New York: Walck, 1960). Time.

Atwood, Ann, *The Little Circle* (New York: Scribner's, 1967.) Shapes.

Barr, Cathrine, *Jeff and the Fourteen Eyes* (New York: Walck, 1958). Counting and Numerical Order; Multiples and Pairs.

———, *99 Ducks Plus 1* (New York: Walck, 1969.) Numeration.

———, *Seven Chicks Missing* (New York: Walck, 1962). Cardinal Numbers; Counting and Numerical Order.

Baum, Arline and Joseph, *One Bright Monday Morning* (New York: Random House, 1962). Number; Order; Days of the Week.

Behn, Harry, *All Kinds of Time* (New York: Harcourt Brace Jovanovich, 1950.) Clocks, time, seasons.

Berkley, Ethel, *Big and Little, Up and Down* (New York: William R. Scott, 1960). Patterns and Shapes; Size.

Bishop, Claire and Kurt Wiese, *The 5 Chinese Brothers* (New York: Coward-McCann, 1938.) Ordinal numbers.

________, *Twenty-Two Bears* (New York: Viking Press, 1964). Cardinal Numbers; Counting and Numerical Order.

Borten, Helen, *Do You See What I See?* (New York: Abelard-Schumann, 1959). Patterns and Shapes.

Branley, Franklyn M., *Big Tracks, Little Tracks* (New York: Crowell, 1960). Patterns and Shapes; Size.

Brenner, Barbara, *The Five Pennies* (New York: Knopf, 1963). Coins.

Buckley, Helen E., *The Little Boy and the Birthdays* (New York: Lothrop, Lee & Shepard, 1965). Calendar.

Budney, Blossom, *A Kiss Is Round* (New York: Lothrop, Lee & Shepard, 1954). Patterns and Shapes.

Caudill, Rebecca, *A Pocketful of Cricket* (New York: Holt, Rinehart and Winston, 1964). Number line.

Colman, Hila, *Watch That Watch* (New York: William Morrow, 1962). Time.

Duvoisin, Roger, *Two Lonely Ducks* (New York: Knopf, 1955.) Cardinal and ordinal numbers.

Elkin, Benjamin, *Six Foolish Fishermen*, Revised Edition (Chicago: Children's Press, 1965). Cardinal Numbers; Counting and Numerical Order.

______, *The True Book of Money* (Chicago: Children's Press, 1960). History and use of money.

Emberley, Ed, *The Wing on a Flea* (Boston: Little, Brown, 1961.) Shapes.

Fisher, Margery M., *One and One* (New York: Dial Press, 1963). Counting and Addition.

Françoise (Seignobore), Jeanne-Marie Counts Her Sheep (New York: Scribner's, 1951). Counting and Numerical Order.

________, *What Time Is It, Jeanne-Marie?* (New York: Scribner's, 1963.) Time.

Freeman, Mae, *Finding Out About Shapes* (New York: McGraw-Hill, 1969.) Shapes.

Friskey, Margaret, *Chicken Little, Count to Ten* (Chicago: Children's Press, 1946). Counting and Numerical Order.

________, *Mystery of the Farmer's Three Fives* (Chicago: Children's Press, 1963). Number sets.

Gans, Margaret, *Three Presents for Jamie* (Chicago: Children's Press, 1969.) Concept of 3.

Geisel, Theodor S. (Dr. Seuss), *One Fish, Two Fish, Red Fish, Blue Fish* (New York: Random House, 1960). Counting and Numerical Order; One-to-one Correspondence.

________, *Yertle the Turtle* (New York: Random House, 1958.) Number, comparison and measurement.

Hoban, Russell, *Bedtime for Frances* (New York: Harper and Row, 1960). Time.

Hoban, Tana, *Shapes and Things* (New York: Macmillan, 1970). Shapes.

Hoberman, Mary Ann and Norman, *All My Shoes Come in Twos* (Boston: Little, Brown, 1957). Multiples and Pairs.

Hutchins, Pat, *Clocks and More Clocks* (New York: Macmillan, 1970.) Time.

Ipcar, Dahlov, *Ten Big Farms* (New York: Knopf, 1958). Ordinal Numbers.

________, *Brown Cow Farm* (New York: Doubleday, 1959). Counting.

Jean Priscilla, *Pattie Round and Wally Square* (New York: Obolensky, 1965). Patterns and Shapes.

Jordan, Helene, *How a Seed Grows* (New York: Crowell, 1960). Numerals and Record Keeping.

Klein, Lenore, *Just a Minute* (Irvington-on-Hudson: Harvey House, 1966.) Time.

Kohn, Bernice, *Everything Has a Shape* (Englewood Cliffs, N.J.: Prentice-Hall, 1964). Patterns and Shapes.

Kuskin, Karla, *Square as a House* (New York: Harper and Row, 1960). Rhymes of Comparison.

"Little Owl Books" New York: Holt, Rinehart and Winston, 1963.)

Fehr, Howard, *Five is 5*. Sets and numerals.

________, *This Is My Family*. Sets and subsets.

Hall, William, *Captain Murphy's Tugboats*. Counting and adding.

Otto, Margaret, *Three Little Dachshunds*. Number.

Schuler, Ruby and Kate Considine, *One, Two, Three, Four*. Cardinal and ordinal numbers.

Sullivan, Joan, *Poems for Counting*. Counting poems.

________, *Round is a Pancake*. Shapes.

Wing, Henry, *Ten Pennies for Candy*. Coins.

________, *What is Big?* Size.

Wittram, H. R., *Going Up, Going Down*. Number sequence.

McGinley, Phyllis, *Wonderful Time* (Philadelphia: Lippincott, 1966). Time.

McLeod, Emilie W., *One Snail and Me* (Boston: Little Brown, 1961). Cardinal numbers; One-to-one correspondence.

Mollie Clark Books (available from Newton, Mass.: Selective Educational Equipment.) Some of the titles are *Beads; Numbers; Dominoes; What is Missing?; The Calendar; What is Inside?; Shapes; Buttons; Symmetrical Shapes.*

Myller, Rolf, *How Big Is a Foot?* (New York: Atheneum, 1962). Measurement.

Peppe, Rodney, *Circus Numbers* (New York: Delacorte, 1969.) Counting.

Podendorf, Illa, *How Big is a Stick?* (Chicago: Children's Press, 1971.) Measurement.

________, *Many is How Many?* (Chicago: Children's Press, 1970.) Classification and comparison.

________, *Shapes: Sides Curves and Corners* (Children's Press, 1970.) Shapes.

Rand, Ann, *Little 1* (New York: Harcourt Brace Jovanovich, 1962). Cardinal Numbers.

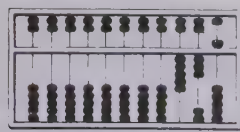
Ravielli, Anthony, *The World Is Round* (New York: Viking Press, 1963). Patterns and Shapes.

Rudolph, Marguerita, *I Like a Whole One* (New York: McGraw-Hill, 1968.) Fractions.

Russell, Solveig, *Lines and Shapes* (New York: Walck, 1965.) Geometry.
———, *One, Two, Three and Many* (New York: Walck, 1970.) Counting.
Schlein, Miriam, *It's About Time* (New York: William R. Scott, 1955.) Time.
———, *Shapes* (New York: William R. Scott, 1952.) Shapes.
Seignobore (See Francoise).
Slobodkin, Louis, *Millions and Millions* (New York: Vanguard Press, 1955,) Number.
Stengle, Hansgeorge, *Busy Builders* (Chicago: Children's Press, 1969). Color and Number.
Tudor, Tasha, *Around the Year* (New York: Walck, 1957). Calendar.
———, *1 is One* (New York: Walck, 1956). Counting.
Vogel, Margaret, *1 is No Fun, But 20 is Plenty* (New York: Atheneum, 1965). Counting.
Waller, Leslie, *A Book To Begin On:* Time (New York: Holt, Rinehart and Winston, 1959). Time.
Watson, Nancy, *Annie's Spending Spree* (New York: Viking Press, 1957.) Money.
Watts, Mabel, *Weeks and Weeks* (New York: Abelard-Schuman, 1962). Calendar.
Wildsmith, Brian, *Brian Wildsmith's 1, 2, 3's* (New York: Franklin Watts, 1965). Shapes.
———, *Brian Wildsmith's Puzzles* (New York: Franklin Watts, 1971). Teasers and braintwisters.
Winn, Marie, *The Fishermen Who Needed a Knife* (New York: Simon and Schuster, 1970.) Uses of money.
Wolff, Janet and Bernard Owett, *Let's Imagine Numbers* (New York: Dutton, 1964). Numerals and Counting.
"Young Math Books," (New York: Crowell)
Charosh, Mannis, *Straight Lines, Parallel Lines, Perpendicular Lines,* (1970). Geometry.
Dennis, J. Richard, *Fractions are Parts of Things* (1971). Fractions.
Fey, James, *Long, Short, High, Low, Thin, Wide* (1971). Linear Measurement.
Froman, Robert, *Bigger and Smaller* (1971). Relativity of size.
Linn, Charles, *Estimation* (1970). Estimation.
O'Brien, Thomas, *Odds and Evens* (1971). Number.
Sitomer, Mindel and Harry, *Circles* (1971). Circles.
———, *What is Symmetry?* (1970). Symmetry.
Srivastava, Jane, *Weighing and Balancing* (1970). Weight and comparison.
Ziner, Feenie, and P. Galdone, *Counting Carnival* (New York: Coward-McCann, 1962). Counting rhymes.

Glossary

abacus A device used for calculating, usually involving sliding beads or counters along a wire.



addend Any one of a set of numbers to be added. In the equation $4 + 5 = 9$, the numbers 4 and 5 are addends.

addition An operation that combines a first number and a second number to give exactly one number. The two numbers are called addends, and the one number that is the result of combining the two numbers is called the sum of the addends.

algorithm Generally used in elementary mathematics to mean one of the various procedures used for computing sums, differences, products, quotients, square roots, etc.

angle Two rays from a single point.

approximation One number is an approximation of another number if the first number is suitably "close" (according to context) to the other number.

area The area of a closed figure or region is the measure of that region as compared to a given selected region called the unit, usually a square region in the case of area.

associative principle *See* grouping principle.

average (arithmetic mean) The arithmetic mean of a set of numbers is the quotient resulting when the sum of the number in the set is divided by the number of addends.

bisect To divide in half or to find the midpoint.

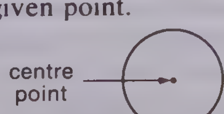
borrow A commonly used term for the regrouping process involved in certain types of subtraction.

Example:
$$\begin{array}{r} \overset{3}{\cancel{4}}\ \overset{13}{\cancel{3}} \\ -1\ 7 \\ \hline 2\ 6 \end{array}$$

carry A commonly used term for the regrouping that is involved in addition.

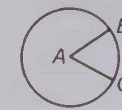
Example:
$$\begin{array}{r} \overset{1}{5}7 \\ +26 \\ \hline 83 \end{array}$$

centre point A given point in the interior of a circle, such that all the points on the circle are the same distance from this given point.

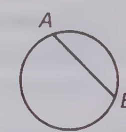


centimetre A unit of length. One centimetre is $\frac{1}{100}$ metre.

central angle In the figure below, angle BAC illustrates a central angle with respect to a given circle with centre A.



chord A line segment that has its endpoints on a given circle.



circle A set of points, all of which are a specified distance from a given point called the centre or centre point.

circumference The distance around a circle.

circumscribed circle A circle is circumscribed about a polygon when each vertex of the polygon is a point of the circle. In the figure, the circle is circumscribed about the triangle.

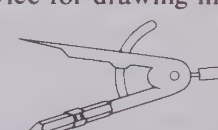
closed figure Intuitively, a figure is closed if you can begin at any point and trace the figure, returning to the initial point.

common factor When a number is a factor of two different numbers, it is said to be a common factor of the two numbers.

common multiple A number is a common multiple of two numbers if it is a multiple of each of the numbers.

commutative principle *See* order principle.

compass A device for drawing models of a circle.



composite number Any whole number greater than 1 that is not prime.

congruent angles Two angles are congruent if they are the "same size."

congruent segments Two segments are congruent if they are the "same size."

construction Used in this program relative to drawing models of particular geometric figures, using ruler and compass only.

count To name numbers in regular succession.

cube A rectangular prism (box) such that all faces are squares.

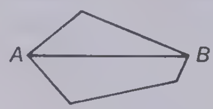
decimal Any base-ten numeral that uses place value to represent a rational number.

decimal point The dot that is used in the decimal symbol.

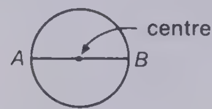
decimetre One tenth of a metre. Ten centimetres.

denominator The number indicated by the numeral below the line in a fraction symbol.

diagonal A segment joining two nonadjacent vertices of a polygon. In the figure, the diagonal is segment AB.



diameter A chord that passes through the centre point of the circle.



difference The number resulting from the subtraction operation.

digits The basic Hindu-Arabic symbols used to write numerals. In the base-ten system, these are the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

dimensions The lengths of the various sides or parts of a particular geometric figure.

disjoint sets Disjoint sets have no common elements. That is, when S and T are two sets such that $S \cap T$ is empty, we say that the sets are disjoint with respect to each other.

distributive principle *See* multiplication-addition principle.

division An operation related to multiplication, as illustrated:

$$3 \times 4 = 12 \quad \begin{cases} 12 \div 3 = 4 \\ 12 \div 4 = 3 \end{cases}$$

dividend In the problem $33 \div 7$, 33 is called the dividend.

Example: $7\overline{)33}$ with quotient 4, then 28, remainder 5 — 33 is the dividend.

divisor In the problem $33 \div 7$, 7 is called the divisor.

Example: $7\overline{)33}$ with quotient 3, then 28, remainder 5 — 7 is the divisor.

edge An edge of a space figure is one of the segments making up any one of the faces of the space figure.

empty set The set that has no objects in it.

equality (equals, or =) A mathematical relation of being exactly the same. The statement $4 + 5 = 6 + 3$ claims that the number $4 + 5$ is exactly the same as the number $6 + 3$.

equation A mathematical sentence involving the use of the equality symbol. Examples: $5 + 4 = 9$; $7 + \square = 8$; $\mathbf{n} + 3 = 7$.

equivalent fractions Two fractions are equivalent when it can be shown that they can be used to represent the same amount of a given object. Also, two fractions are equivalent if these two products are the same:

$$\frac{3}{4} \times \frac{6}{8} \qquad \rightarrow 4 \times 6 \qquad \rightarrow 3 \times 8$$

equivalent sets Two sets that may be placed in a one-to-one correspondence.

estimate To find an approximation for a given number. (Sometimes a sum, a product, etc.)

even numbers The whole-number multiples of 2 (0, 2, 4, 6, 8, 10, 12, · · ·).

exponent In the symbol 10^3, the "3" is an exponent. It indicates that 10 is used as a factor three times. Thus:

$$10^3 = 10 \times 10 \times 10 = 1000$$
$$5^4 = 5 \times 5 \times 5 \times 5 = 625$$
$$2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$$

face The face of a given space figure is any one of the plane geometric figures (regions) making up the space figure. For example, in a cube each of the square regions is a face of the cube.

factor *See* multiplication. The equation $6 \times 7 = 42$ illustrates that both 6 and 7 are factors of 42.

factor tree The example shows two factor trees for the number 30.

```
2 × 3 × 5        2 × 3 × 5
 \ /   /         |   \ /
  6 × 5          2 ×  15
   \ /            \  /
   30              30
```

formula A general fact or rule expressed using symbols. Example: $A = \ell \times w$ (A = area of rectangle, ℓ = length of rectangle, w = width of rectangle).

fraction A symbol for a rational number.

Example: $\frac{2}{3}$, $\frac{5}{8}$, $\frac{7}{2}$.

graph (1) A set of points associated with a given set of numbers or set of number pairs. (2) A picture used to illustrate a given collection of data. The data might be pictured in the form of a bar graph, a circle graph, a line graph, or a pictograph. (3) To draw the graph of.

greater than (>) One of the two basic inequality relations. Examples: $8 > 5$, $28 > 25$, $80 > 50$.

greatest common factor The greatest number that is a factor of each of two numbers.

grouping principle (associative principle) When adding (or multiplying) three numbers, you can change the grouping and the sum (or product) is the same.

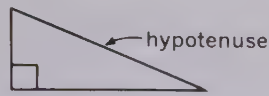
Example: $2 + (8 + 6) = (2 + 8) + 6$
$3 \times (4 \times 5) = (3 \times 4) \times 5$.

hexagon A six-sided polygon.

higher terms A first fraction is in higher terms than a second fraction if the first fraction is equivalent to the second fraction and if the denominator of the first fraction is greater than the denominator of the second fraction.

Example: $\frac{9}{12}$ is in higher terms than $\frac{6}{8}$.

hypotenuse The side opposite the right angle in a right triangle.

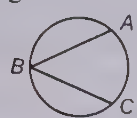


improper fraction A fraction in which the numerator is greater than or equal to the denominator.

Examples: $\frac{8}{5}$, $\frac{6}{6}$, $\frac{12}{3}$.

inequality ($>$, $\neq$, $<$) In arithmetic, a relation indicating that the two numbers are not the same, or that one is greater (or less) than the other.

inscribed angle Angle ABC in the figure illustrates an angle inscribed in a given circle.



integers, the set of The whole numbers together with their negatives:

$\{\cdots, {}^{-}3, {}^{-}2, {}^{-}1, 0, 1, 2, 3, \cdots\}$.

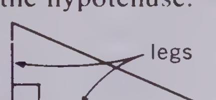
intersection of two sets The set containing those objects and only those objects that are in both of two sets. Example: Set $A = \{c, d, e, f\}$; Set $B = \{e, f, g\}$. The intersection of two sets is the set $\{e, f\}$. We write: $A \cap B = \{e, f\}$.

inverse A number and its negative are additive inverses of each other. A number and its reciprocal are multiplicative inverses of each other. Addition and subtraction are referred to as inverse operations, as are multiplication and division.

least common denominator The least common multiple of two denominators. For $\frac{1}{4}$ and $\frac{5}{6}$, the least common denominator is 12.

least common multiple The smallest number that is a multiple of each of two numbers. For 4 and 6, the least common multiple is 12.

legs of a right triangle The two sides of a right triangle other than the hypotenuse.



length (1) A number indicating the measure of one line segment with respect to another line segment, called the unit. (2) Sometimes used to denote one dimension (usually the greater) of a rectangle.

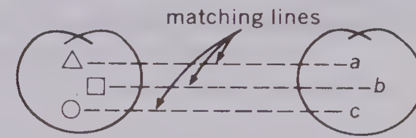
less than ($<$) One of the two basic inequality relations. Examples: $5 < 8$, $25 < 28$, $50 < 80$.

lower terms A first fraction is in lower terms than a second fraction if the first fraction is equivalent to the second fraction and if the denominator of the first fraction is less than the denominator of the second fraction.

Example: $\frac{6}{8}$ is in lower terms than $\frac{9}{12}$.

lowest terms A fraction is in lowest terms if the numerator and denominator of the fraction have no common factor greater than 1.

matching lines Lines used to indicate or denote the correspondence between the objects in two sets.



measure (1) A number indicating the relation between a given object and a suitable unit. (2) The process of finding the number described above.

metre A unit of length in the Metric System, 100 centimetres.

minus ($-$) Used to indicate the subtraction operation, as in $7 - 3 = 4$ (read, "7 minus 3 equals 4").

mixed numeral A symbol given for a fractional number greater than 1 that is a combination of a whole-number symbol and a fraction symbol.

Examples: $2\frac{1}{2}$, $3\frac{2}{3}$, $5\frac{1}{4}$.

multiple A first number is a multiple of a second number if there is a whole number that multiplies by the second number to give the first number. Example: 24 is a multiple of 6 since $4 \times 6 = 24$.

multiplication An operation that combines a first number and a second number to give exactly one number. The two numbers are called factors, and the one number which is a result of combining the two numbers is called the product of the two numbers.

multiplication-addition principle (distributive principle) This principle is sometimes described in terms of "breaking apart" a number before multiplying.

Example: $6 \times (20 + 4) = (6 \times 20) + (6 \times 4)$.

negative integer An integer other than those integers which are the whole numbers.

number line A line on which specified points are given number labels or names. The following example illustrates the whole-number line.

number pair Any pair of numbers. In this program, usually a pair of whole numbers.

numeral A symbol for a number.

numerator The number indicated by the numeral above the line in a fraction symbol.

odd number Any whole number that is not an even number.

one-to-one correspondence A one-to-one correspondence exists between two sets when the elements of one can be matched with the elements of the other in such a way that each element of the first set is matched with exactly one element of the second set, and each element of the second set is matched with exactly one element of the first set.

opposite Referring to the relation between two integers whose sum is zero. Example: 2 is the opposite of $^-2$; $^-8$ is the opposite of 8.

order principle (commutative principle) When adding (or multiplying) two numbers, the order of the addends (or factors) does not affect the sum (or product).

Example: $4 + 5 = 5 + 4$, $2 \times 3 = 3 \times 2$.

parallel lines Two lines which lie in the same plane and do not intersect.

parallelogram A quadrilateral with opposite sides parallel.

parentheses (( )) Symbols used to indicate grouping or order of performing operations. Examples:

$$(5 \times 4) - 2 = 18; \quad 5 \times (4 - 2) = 10.$$

pentagon A five-sided polygon.

percent (%) Per 100; for each 100; $\frac{1}{100}$.

perfect number A number that is half the sum of its factors. Examples: 6, 28, and 496.

perimeter The sum of the lengths of the sides of a given polygon.

perpendicular Two lines that intersect in right angles are perpendicular to each other.

pi (π) The ratio of the circumference to the diameter of a circle; approximately 3.14.

placeholder In this program, this term is used to indicate the small box in which you write the solutions to equations.

place value A system used for writing numerals for numbers, using only a definite number of symbols or digits. The system permits a given digit to stand for different numbers, depending upon its location or position within a given numeral. The number a given digit stands for in a symbol is determined by its position in the numeral and by the base being used in the particular system. In the ordinary base-ten system, for example, a numeral 2 in the third place from the right would stand for 200. Other examples are as follows: in the numeral 3257, the 5 stands for 5 tens or 50; in the numeral 36289, the numeral 6 stands for 6000 (or $6 \times 10 \times 10 \times 10$).

plus (+) Used to indicate the addition operation, as in $4 + 3 = 7$ (read, "4 plus 3 equals 7").

polygon A closed geometric figure made up of line segments.

positive integer Any whole number other than zero.

power The following examples illustrate powers of 10: 10^2, 10^5, 10^3, 10^7. These are powers of 7: 7^2, 7^4, 7^3, 7^5.

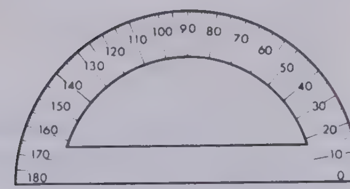
prime factor A factor that is a prime number.

prime number A number greater than 1 whose only factors are itself and 1.

product The result of the multiplication operation. In $6 \times 7 = 42$, 42 is the product of 6 and 7.

proper subset Each subset of A except A itself is called a proper subset of A.

protractor A device used for measuring angles.



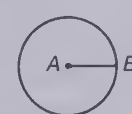
Pythagorean Theorem For any right triangle, the sum of the squares of the lengths of the two legs is equal to the square of the length of the hypotenuse.

quadrilateral A four-sided polygon.

quotient The number (other than the remainder) that is the result of the division operation. It may be thought of as a factor in a multiplication equation.

radian A unit for measuring angles. A radian is approximately 57.3 degrees.

radius (1) Any segment from the centre point to a point on the circle. (2) The distance from the centre point to any point on the circle.



ratio A pair of numbers used for certain types of comparisons.

ray A point on a line and all the points on one side of that point that are on that line.

reciprocal Two numbers are reciprocals of one another if their product is 1. Example: $\frac{4}{7}$ and $\frac{7}{4}$.

rectangle A quadrilateral that has four right angles.

regrouping A procedure commonly used in manipulating place-value symbols in adding or subtracting.

remainder

Example:

$$\begin{array}{r} 6 \\ 7\overline{)47} \\ \underline{42} \\ 5 \end{array} \longleftarrow \text{remainder}$$

repeated addition Finding the sum of a set of numbers, each of which is the same.

Example: $5 + 5 + 5 + 5$.

repeated subtraction Starting with a number and repeatedly subtracting the same given number from each difference that is obtained.

repeating decimals A decimal that repeats a given digit or group of digits over and over without end.

Example: $.333 \cdots$.

right angle An angle that has the measure of 90 degrees.

A right angle

right triangle A triangle that has one right angle.

Roman numerals Numerals used by the Romans. Used primarily to record numbers rather than for computing. Examples: IV, IX, XIV.

rounding Giving an approximation for a number.

scale drawing A drawing constructed so that the ratio of all the dimensions in the drawing to those of the actual object is the same.

scientific notation A number is said to be written in scientific notation if it is indicated as a number between 1 and 10 times a power of 10. Example: Speed of light in kilometres per second,

$$300\,000 = 3.0 \times 10^5.$$

segment Two points on a line and all the points on that line that are between the two points.

sequence A collection or set of numbers given in a specific order. Such numbers are commonly given according to some rule or pattern.

set A group, collection, family, or aggregate of objects. At the heart of the concept of set is man's ability to think of a collection of objects as a single entity.

similar triangles Two triangles are similar to each other if their sides can be matched so that the ratio of the length of each pair of sides is the same.

skip count To count by multiples of a given number. Example: Counting by fives—0, 5, 10, 15, 20, · · · .

solution The number or numbers that result from solving an equation or a given problem.

solve To find the number or numbers that, when substituted for the variable or placeholder, make the given equation true.

square A quadrilateral that has four right angles and four sides that are the same length.

subtraction An operation related to addition as illustrated:

$$7 + 8 = 15 \begin{cases} \nearrow 15 - 8 = 7 \\ \searrow 15 - 7 = 8 \end{cases}$$

surface area The sum of the area of each face of a figure.

times (×) Used to indicate the multiplication operation, as in $3 \times 4 = 12$ (read, "3 times 4 equals 12").

triangle A three-sided polygon.

union of two sets The set consisting of those objects which are in one or the other or both of two sets. Example: Set $A = \{c, d, e, f\}$; Set $B = \{e, f, g\}$. The union of the two sets is the set $\{c, d, e, f, g\}$. We write: $A \cup B = \{c, d, e, f, g\}$.

unit An amount or quantity adopted as a standard of measurement.

vertex The point that the two rays of an angle have in common.

volume The measure, obtained using an appropriate unit (usually a cube), of the interior region of a space figure.

whole number Any number in the set

$$\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, \cdots\}.$$

YELLOW MODULE, UNIT A

Primitive Number Concepts

Pages a-1 to a-12

General Objectives

To introduce the concepts of more and less

To develop the method of one-to-one matching to show equivalent sets

To develop an intuitive idea of cardinal number

To establish the concept of one set having exactly one more element than another set

The fundamental objective of this module is to lead the children through a carefully planned development of number concepts before introducing the names or numerals for the numbers.

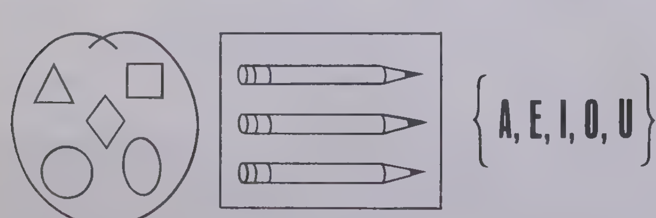
Mathematics

Some concepts that are basic to all mathematics underlie the mathematics of this module. One such concept is that of a *set*. The notion of a set is intrinsically related to the ideas of number and counting.

It is very difficult to define the word *set* in terms of simpler concepts. The following statement is an intuitive description of a set rather than a precise definition.

> A *set* is a group or collection of objects considered as a single entity. The objects are called *elements,* or *members,* of the set.

The key phrase in the above description is "considered as a single entity." It is our ability to think of a group as a single thing that makes the set concept useful. The following figures illustrate sets. To indicate just which objects are contained in a given set, we often draw outlines around them, as shown in the first two figures, or enclose them in braces, as shown in the third figure.

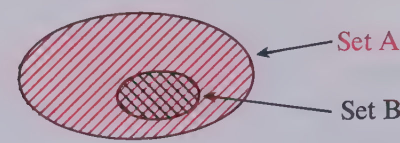


A Set of Geometric Shapes | A Set of Pencils | A Set of Letters

We say, for example, that *A* is an element, or member, of the set of letters shown in the illustration.

Often we need to discuss sets that are "part of" a given set, that is, sets whose elements are also elements of a given set. Such sets are called *subsets* of the given set.

The idea of a subset is pictorially represented by the diagram. Set *B* in the diagram is contained entirely within set *A*, and nothing outside *A* is in *B*. This shows that *B* is a subset of *A*.

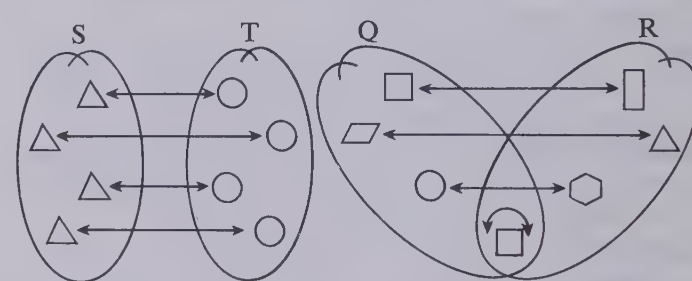


The following is a precise definition of equivalent sets.

> If two sets *S* and *T* have the property that the elements of *S* can be paired with the elements of *T* so that each element in *S* is paired with exactly one element in *T*, and vice versa, then the sets *S* and *T* are said to be *equivalent* to each other.

The relation shown by the pairing of elements is called a *one-to-one correspondence* between the sets.

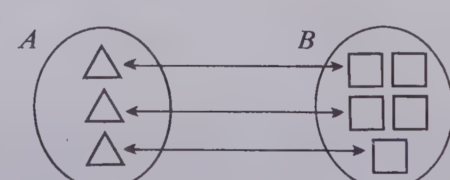
Examples of pairs of equivalent sets are shown in the following figures.



Note that the sets *S* and *T* are disjoint with respect to each other and that the sets *Q* and *R* are not disjoint. In the case of *Q* and *R*, we think of the square, which is common to both sets, as being paired with itself.

The concept of equivalent sets plays an important role in developing the concept of number. The child hears the word "two" as he encounters different sets: two eyes, two legs, two blocks, and so on. Gradually, he begins to recognize that "two" does not refer to color, size, shape, position, and so on, but to a common property of the sets which allows them to be matched one-to-one.

When two sets cannot be matched one-to-one then one of the sets must contain more elements.



Although each element in set *A* is matched with exactly one element in set *B*, there are elements in set *B* that are not matched with any element in set *A*. Also, there

is no way to match each element of set B with exactly one element of set A. Thus set B contains more elements than set A.

Let us list some of the subsets of a given set A:

$$A = \{m, n, o, p\}.$$

Some of the subsets of A are

$$\{m, n\}, \quad \{m, n, o\}, \quad \{n\}, \quad \{m, n, o, p\},$$
$$\{n, o, p\}, \quad \{n, o\}, \quad \{o, p\}, \quad \{\ \}.$$

The last subset of A is the empty set. This is not a complete list of the subsets of A. There are eight more. It would be instructive for you to list them. Note that the set $\{m, n, o, p\}$ is a subset of A. *It contains no element that is not also in* A. This example illustrates the fact that *each set is a subset of itself.*

It is often desirable to refer to the subsets of a given set which are different from the set itself. The following definition gives us special terminology with which we can describe these sets.

> Each subset of A other than A itself is called a *proper subset of* A.

Teaching Yellow Module, Unit A

Approximate Time: 6 to 8 days

Materials

boxes of small items such as: pieces of yarn, large buttons, pieces of felt, pencils
cards showing sets of circles, stars, etc.
colored chalk
construction paper
counters (about 20 per child)
crayons
newsprint in large sizes
old magazines
paste
scissors
yarn

Vocabulary

bottom	last	one-to-one matching
circles	least	row
equivalent	less	the same number as
fewer	lesser	second
fewest	matching lines	set
first	middle	third
fourth	more	top
greater	most	
greatest	one more	

We do not expect the children to master all of these vocabulary words, but we list those that might arise in class discussion. The names of a few ordinal numbers are included in the vocabulary list. The concept of ordinal numbers and the ordinal number names are ideally taught by repeated use in the classroom.

To develop the ideas in this module, children will need many experiences matching objects in sets. Often children of this age are not able to separate the notions of "how many" in a set from "how much space" elements in a set occupy. It is important to provide varied opportunities for the child to match equivalent sets and compare the same pairs of sets in various arrangements. Note that this matching requires no counting. It is desirable to avoid counting and naming the number of a given set.

One of your greatest problems in this first module will be coping with individual differences within your class. You should avoid unnecessary references to the number of a set. Nevertheless these number names will arise despite your own avoidance of them. In such a case, point out that it is not necessary to know the number name for each set, since the objects can be matched to find that one set has more than another.

Evaluation of Progress

Stress the skills required to work on paper and to follow the organizational format of the text page. Be sure that each child understands such concepts as positional terms and the sequential order of working problems in rows or columns.

The fact that a child can efficiently match two sets one-to-one in no way implies that he understands the ideas involved. You should evaluate the daily oral and written work of the children informally. The essential, mathematical ideas should be developed in each day's pre-book activity and each page can be used to evaluate the ideas presented. However, the most valid evaluation will come from talking with a child.

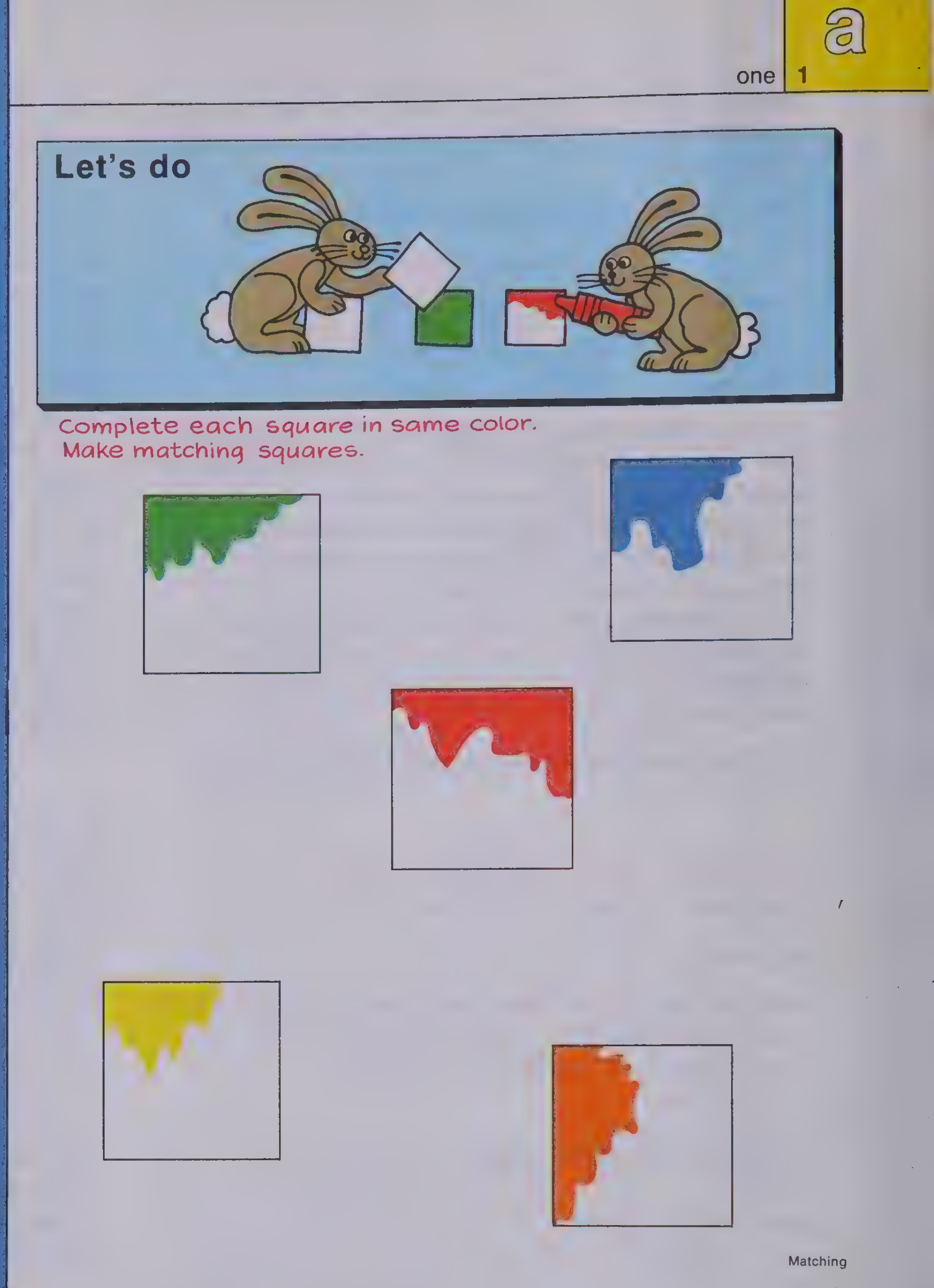
Resources For Active Learning

General Activities:
SETS, NUMBERS AND POWERS, "Lessons and Games . . . Sets," pp. 71–96, Herder and Herder
DEVELOPMENTAL MATH CARDS, A[1]7, 9, A[3]11, Addison-Wesley
Nuffield Project: MATHEMATICS BEGINS 1, "Sorting," pp. 10–26; ". . . Correspondence," pp. 27–31, Wiley

Manipulative Devices:
Beads and string (CCM School Materials; Childcraft; Ideal)
Sorting and Order Kit (Mafex Associates)
Sorting and Sets cards (Teaching Resources)

Investigation
Page a-1

Since this is the first investigation in the text, children will need considerable guidance. Notice the demonstration art in the box at the top of the page. The first page of most lessons includes such art and should be helpful to you as an aid in introducing the page. For example, on page a-1, point out the two rabbits in the box. Discuss how one is matching an uncolored square with the green square and how the other rabbit is completing the coloring of the red square. Then explain the directions for the investigation for this page. Give each child a set of 5 uncolored squares. Explain that they should match an uncolored square to each square on the page. They should then take their crayons and match a crayon to each square according to color. They should complete the coloring of each square on their page and then color their uncolored squares similarly. Thus to complete the investigation, they should have completed the coloring on each square on the page and they should have a matching set of squares, now colored as those on the page. Notice that reference to the number five need not arise in this activity. If someone mentions that there are five squares or five colors, praise him for being correct, but do not emphasize. The stress during this investigation should be on the matching activity.



Purpose

To provide a variety of activities that involve matching the objects in two sets, thus setting the stage for the more detailed introduction of the related concepts in this module

Preparation

Materials

crayons
uncolored 3-cm squares, 5 per child

Since subsequent lessons develop the matching method more extensively, this lesson should simply be considered an introduction to the general matching activities of the module. Thus your preparation should consist in making 5 white squares, each 3-cm-by-3-cm, for each child and in distributing these uncolored squares.

a 2 two

Let's talk

Sample answers: I see 4 bicycles, 5 balloons, 4 children.

Matching

Discussion
Page a-2

Direct the children's attention to the illustration on page a-2. Ask them to describe what they see. Encourage as much freedom as possible in the children's discussion of the picture. Draw from them where the children might be going or coming from. (Point out the basketball held by one of the boys.) During this discussion stress as many matchings as possible. For example, ask questions such as: "Will each child be able to sit on a stool at the ice cream counter?" "Does each child have a ball to play with?" "Are these as many bicycles as there are children?" that is, "Does each child have a bicycle to ride?"

As time permits extend these ideas to sets of objects in the classroom. For example, discuss the matching of chairs and children, or pencils and children, or coats and coat hooks and so on. Begin to involve the children as much as possible in matching activities.

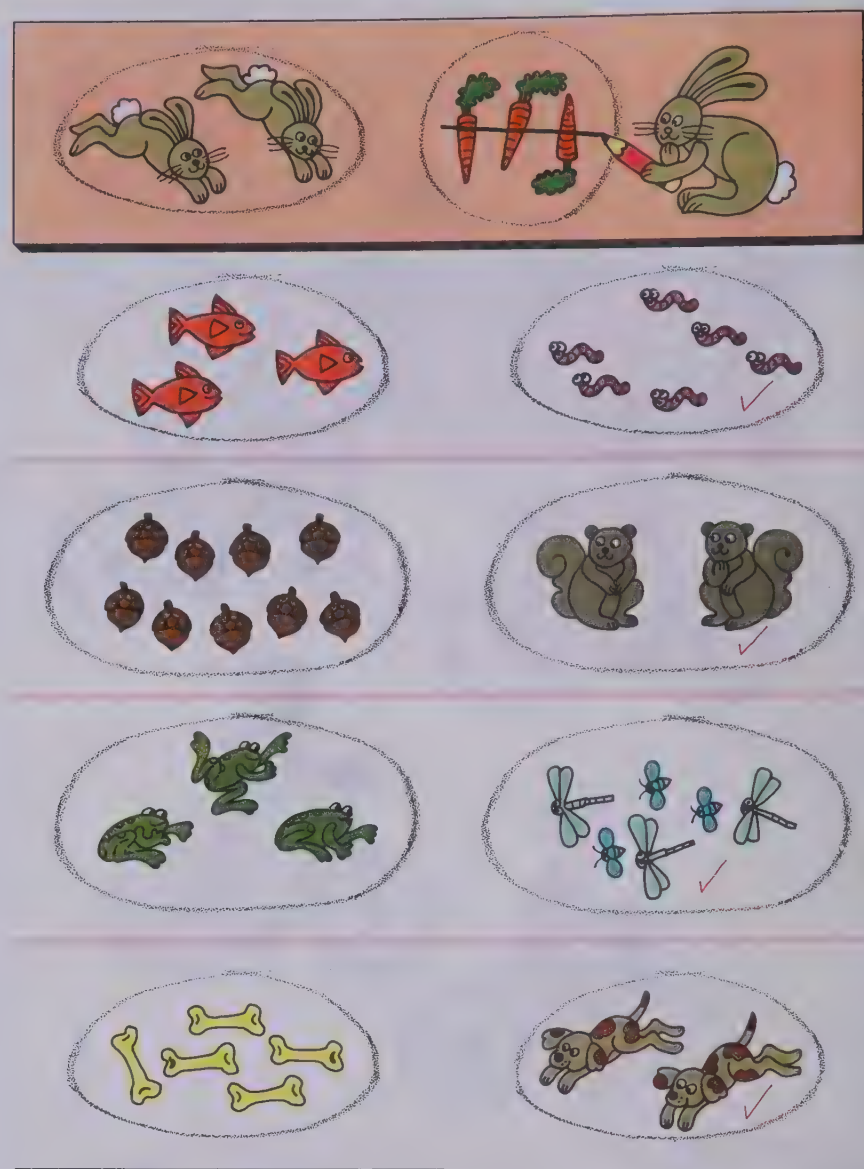
Follow-up

Oral activities to accompany matching exercises might be used at various times throughout this module. Make use of available sets in the classroom to play "Guess and Prove It." For example, before a cut-and-paste activity, ask the children whether there are more scissors or more children. When a child volunteers an answer, ask him to try to prove he is correct. Other sample situations might be: whether there are more chalkboards than erasers, more chairs than desks, more crayons than pencils, and so on.

Children would benefit from an opportunity to choose materials from a math corner. Make available objects from a variety of related sets which children can use for matching experiences. Suitable materials might be cutouts such as: shoes and socks, dogs and bones, cups and saucers, dolls and doll dresses, cats and balls of yarn, cowboys and horses, drivers and cars, and so on. Simply direct children to make two sets that have the same number in both sets.

Teaching
Page a-3

Tell the children that they need not count the objects in each set, nor know the number of each set. It is intended that they just look at the two sets and decide which has more and which has less. Ask them to look at the top frame and put a mark on the set that has more objects, or the greater number of them. When the children have completed marking the first row, direct their attention to the second frame. This time, ask them to mark the set that has less or fewer objects. You may need to say, "Mark the set that does not have as many objects as the other set." and explain the words less and fewer. Continue similarly with the last two frames.



Objective

Given two obviously non-equivalent sets, the child will be able to compare the sets by visual inspection (not counting) and mark the set which has more, or fewer objects, according to directions.

The emphasis in this lesson is on the comparison of sets by visual inspection; subsequent lessons will require drawing matching lines to tell which set has more. Basic vocabulary for comparing sets (more, greater, less, and fewer) is introduced informally and excessive drill with these terms should be avoided.

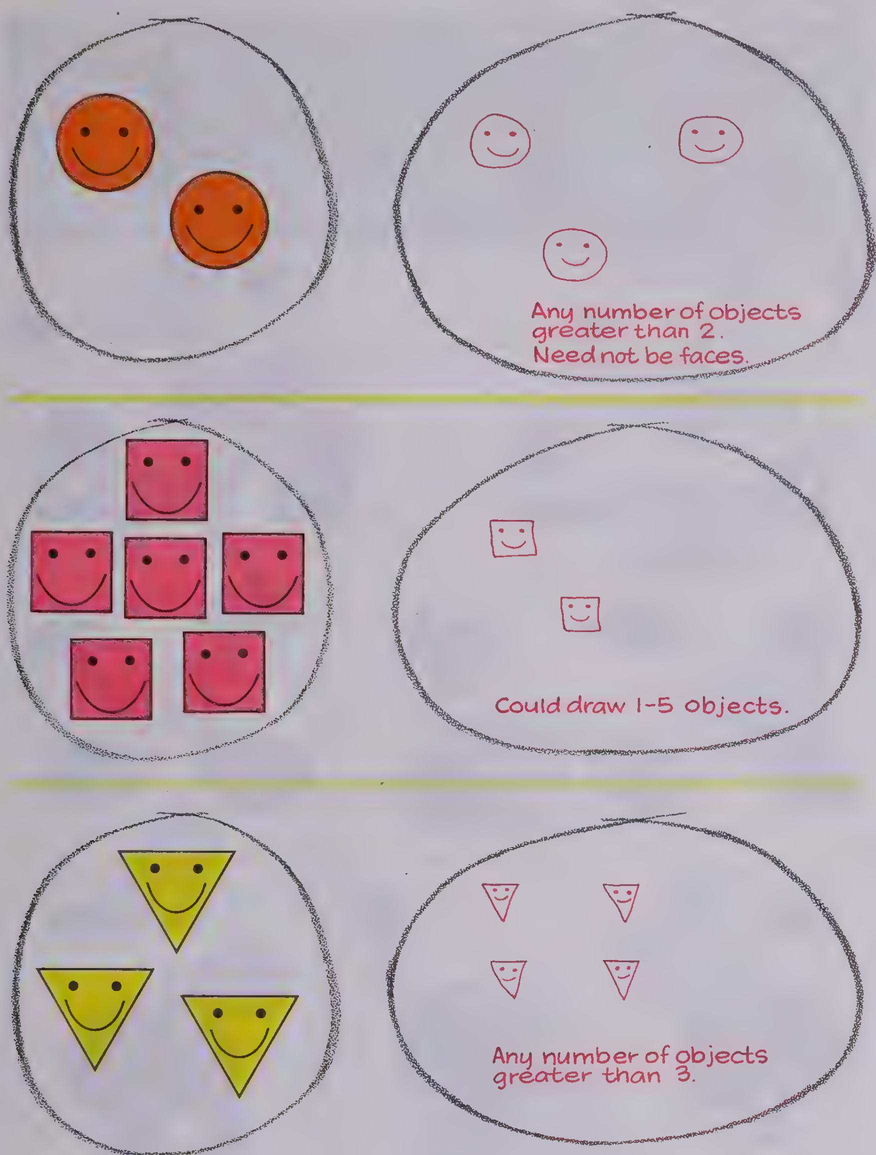
Pre-Book Activity

Materials

counters

It will be helpful to conduct readiness activities in which the answer to the question "Which set has more?" is obvious.

For example, distribute about ten counters to each child. Then on a flannelboard, the overhead projector, or the chalkboard, show various parts of nonequivalent sets and ask the children such questions as: "Which set has more?" "Which set has less?" "Which has the greater number?" "Which has fewer?" After several examples of this type, ask the children to try to build a set on one side of their desk or table and another set on the other side so that one set has more objects.



More and less

Teaching
Page a-4

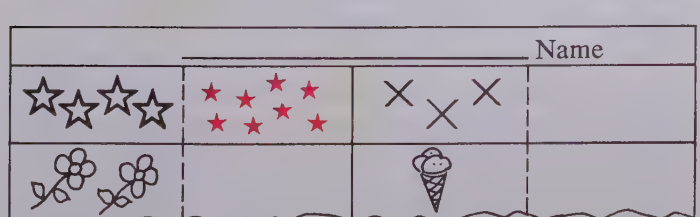
Call the children's attention to the smiling faces in the first, or top, frame. Tell them to look at these and then at the outlined space which has no objects in it. Ask them to draw, without counting, a set of objects that has "more" objects than the smiling faces. Do not say how many more; simply instruct the children to draw a set that has more. It is not necessary to speak of the shapes, circles, squares, and triangles, by name if you choose not to. Children can draw a set of anything they choose.

Ask the children to look at the set of squares in the second, or middle frame. Instruct them to draw a set that has less or fewer objects than the given set. Caution them to draw only a few objects so they will not have as many as, or more than, the given set.

In the third, or last frame ask the children to draw a set that has more objects than the given set.

Follow-up

To extend this lesson you might give the children worksheets similar to the sample shown. Supply each child with scissors, paste, and an old magazine. For each frame, direct the children to find and cut out a set that has more objects than the set shown on their sheets and to paste that set next to the given set. Or give them each an envelope of pictures from which to choose the larger sets.



If you prefer, adapt the above directions for each child's use on a large sheet of newsprint. In this activity, guide the children in folding the sheet in half three times to make eight sections.

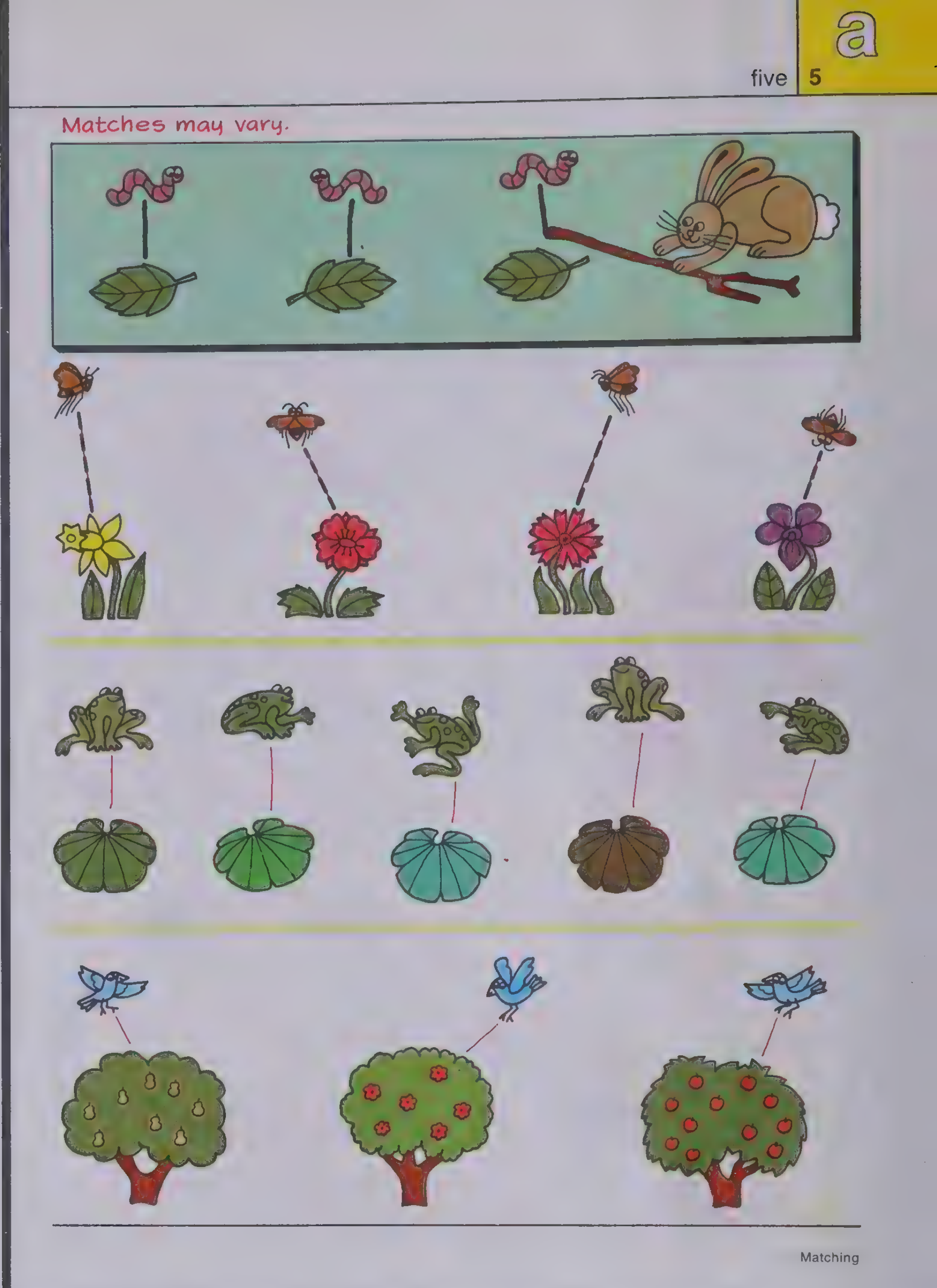
Mathematics

Because the sets on these pages should be compared by visual inspection alone, the mathematical ideas will not stand out as clearly as they will when the sets are compared by the more formal method of drawing matching lines. Notice, however, that a basic mathematical idea is beginning to unfold. When a child says that one set contains more objects than another, he is claiming that the number of one set is greater than the number of the other. This is the first step in ordering the set of cardinal numbers.

Teaching
Page a-5

Call attention to the illustrated worms and leaves, pointing out how the rabbit has matched each worm to a leaf. Observe with the children that in the first frame the bees and flowers have been matched with dashed lines. Here they will show the matching by tracing over these lines with their pencils or crayons. When they finish see if the children realize that there are as many bees as there are flowers, that is, that the set of bees is equivalent to the set of flowers, by pointing to the connecting lines they have traced. Then move on to the second frame, which shows the set of frogs and the set of petals. Instruct the children to find out whether or not each frog can be matched with a petal by drawing lines from the frogs to the petals.

For the last frame, ask the children to compare the set of birds with the set of trees by drawing matching lines between the objects in the two sets.



Objective

Given two equivalent sets in which a one-to-one matching is obvious, the child will be able to show that the sets are equivalent by drawing matching lines from the objects of one set to the objects of the other.

It is important for children to realize that two sets can have the same number of objects even though the objects of one set may be spread farther apart than the objects of the other. For reference see *How Children Learn Mathematics* by Richard W. Copeland (Macmillan).

Pre-Book Activity

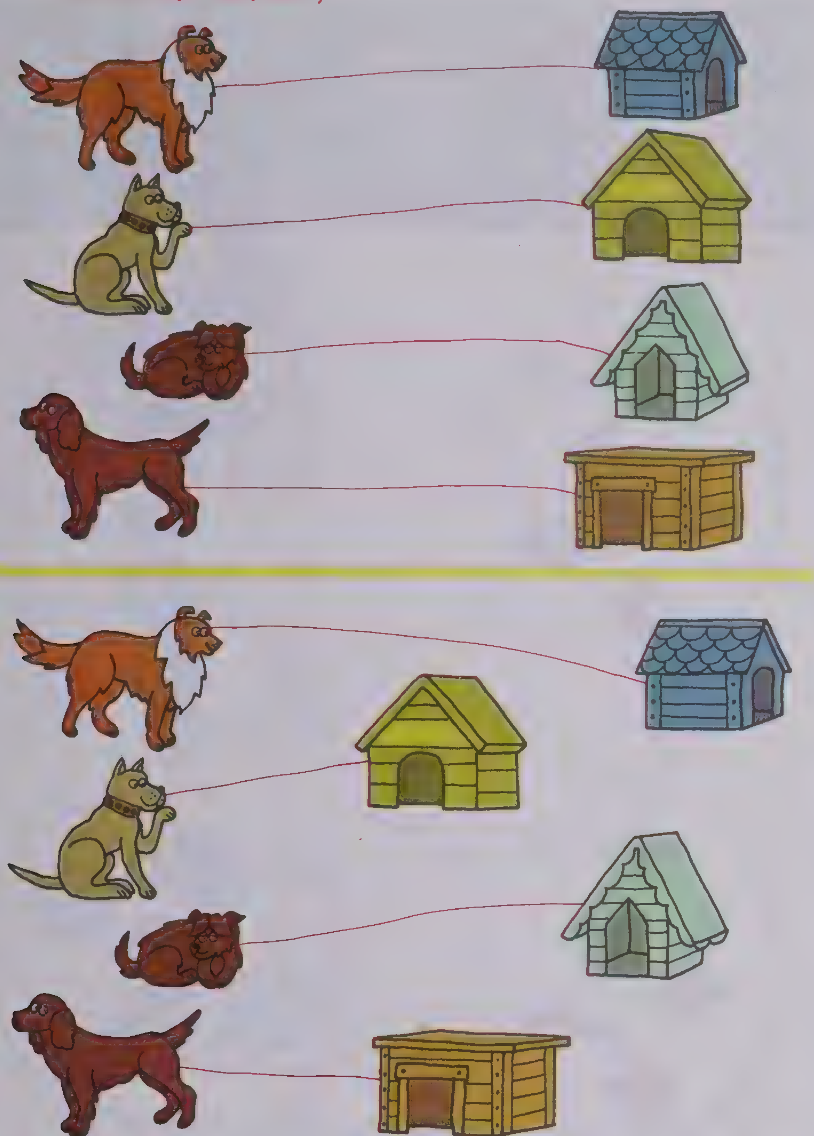
Materials

sets of objects suitable for matching

Let the children practice one-to-one matching in actual situations by matching equivalent sets. Have them use a variety of related objects such as sets of red counters and sets of blue counters, pencils and pieces of paper, chairs and children. For example, group 3 chairs next to your desk and talk about a set which has the same number as the number of chairs. You might ask: "How can we find a set of children that has the same number as the set of chairs?" Have 3 children sit on the chairs and point out that since each child has a chair, the set of children and the set of chairs has the same number.

Take advantage of daily opportunities for matching, such as passing out straws with the daily milk, or determining whether there is a pair of scissors for each child in an art lesson.

Matchings may vary.



Matching

Teaching
Page a-6

Ask the children to match the set of dogs with the set of doghouses. Then ask them if they notice anything different about the second group of dogs and doghouses. Point out the change of position of the doghouses in the bottom frame. For each frame, ask them if each dog has a doghouse even though the doghouses are spread out differently.

If some of the children use the number names for these sets, explain that even if they do not know the number of the sets they can use the matching method to show that the sets have the same number. Use the terms "equivalent sets" and "the same number as" frequently to help children understand their meaning.

Follow-up

Have children compare objects in various shaped containers. Show a tall thin glass and a short wide glass. Suggest that they put the same number of objects (counters, beans) in each by matching: if one counter is put in the tall glass, then one should be put in the short glass. No counting need be involved. Ask: "Do both containers have the same number of counters in them?" "Does one container have more?" "If so, which one?"

Mathematics

Two sets are in one-to-one correspondence if there is a matching such that for each element of the first set, there is exactly one element of the second set, and visa versa. Children show this by drawing lines. They must be careful not to match two objects of one set with one object of the other set. If two sets are in one-to-one correspondence, they are equivalent. Children should understand that "equivalent" means "having the same number of objects." Discourage use of "equal sets." Reserve the word "equal" to mean "exactly the same as." Then $A = B$ (A equals B) for sets A and B means that A is the same set as B and that A and B are different names for that set.

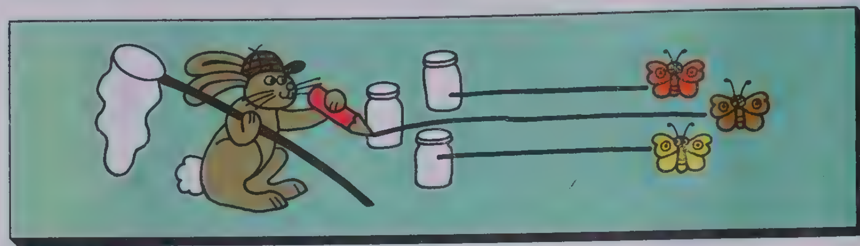
Resources For Active Learning

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "One-to-one correspondence," Nos. 1–3, Responsive Environments Corp.

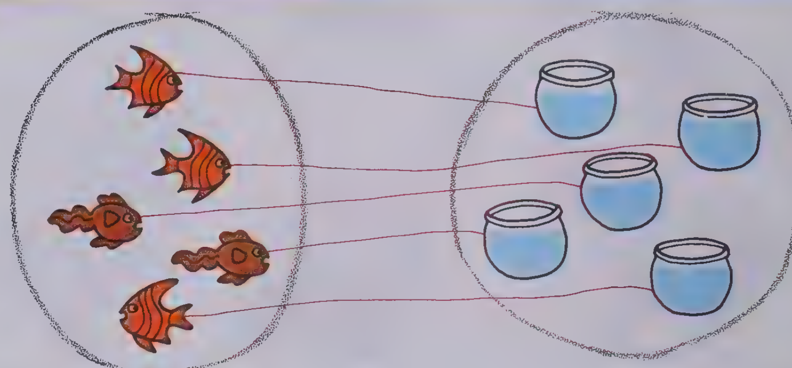
MATHEX: Matching and Graphing No. 1, "Game 1," p. 7, Encyclopaedia Britannica Publications Ltd.

Teaching
Page a-7

Ask the children to look at the first frame of page a-7 and explain that they are to draw lines to match the fish in the one set to the fish bowls in the other set. When the children have finished drawing their lines, ask them what the lines show. Elicit from them that, since each fish can be matched with a different fish bowl, the two sets have the same number of objects or are equivalent. Continue with the next frame. Help the children see that, even though the bowls might be spread out differently, each fish can be matched with a bowl and each bowl can be matched with a fish, so the two sets are equivalent.



Matchings may vary.



Matching

Objective

Given pairs of equivalent sets the child will show that they are equivalent by drawing matching lines between the objects of each set.

In this lesson, the sets are well-defined by a ring around each set. The children might recognize a variety of ways in which the objects of one set may be matched with those of another. Any of the possible one-to-one matchings are correct, since all such matchings show that each of the two sets has the same number of objects.

Pre-Book Activity

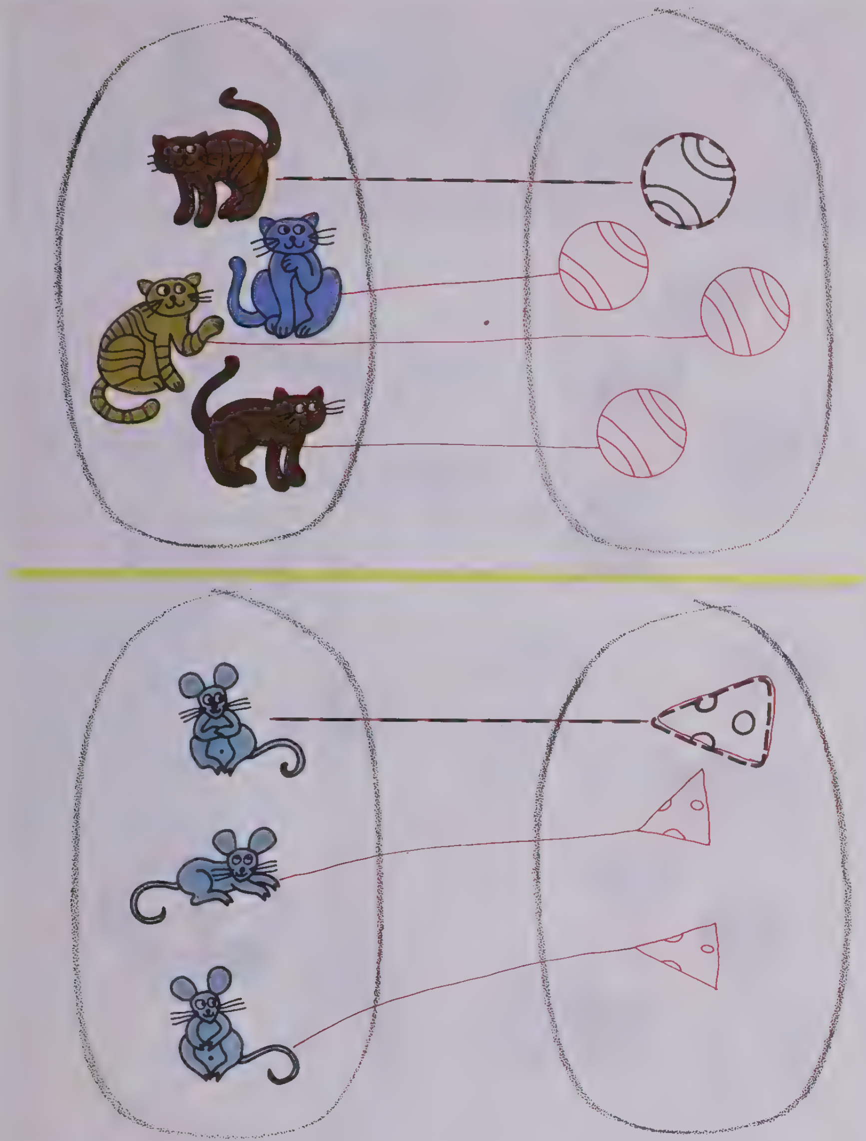
Materials

counters

boxes of small items such as pieces of yarn, large buttons, pieces of felt, pencils

Distribute a pile of counters to each child. Then hold up one of your hands showing four fingers. Ask the children to hide their thumb and show the fingers of one hand on the top of their table. Then ask them to make a set of counters so that a counter matches each finger. Also have available a group of items as listed above. Then ask a volunteer to make a new set from these materials which has the same number as the set of counters on his desk. If some children use "four" to describe how many they have, fine, but stress that the sets have the same number and that their objects are matched one to one.

Children might demonstrate matching sets with felt objects on the flannelboard and other classroom sets. For example, suggest the set of all the children sitting



Matching

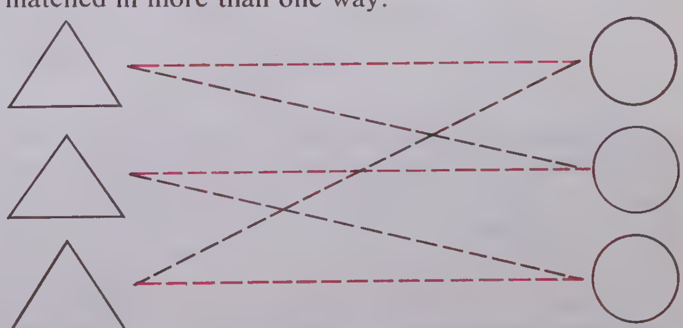
Teaching
Page a-8

Ask the children if they can draw a set of balls so that each kitten will have a ball. Also ask them to draw in the matching lines. Give similar directions for the mice and cheese. Encourage children to color their pictures also. As they work, ask them if they can draw the matching lines. You might want to use the suggestion in the follow-up with one of the frames on these pages.

at a table and ask them to show a set to match this. One way would be for each child to put a pencil in the centre of the table. Stress that as long as there is a one-to-one matching between children and pencils the sets have the same number and we say that they are equivalent sets.

Follow-up

Draw two sets, each containing three elements, on the chalkboard. Ask one child to draw matching lines with one color chalk, and ask another child to match them in a different way with a different color. While the pair of sets remains on the board, draw a third set containing the same number of objects as the other two. Ask a child to match the objects in this set with those of one of the other sets. Discuss whether or not the third set has the same number of objects as the other two sets. Notice the illustration which shows that equivalent sets may be matched in more than one way.



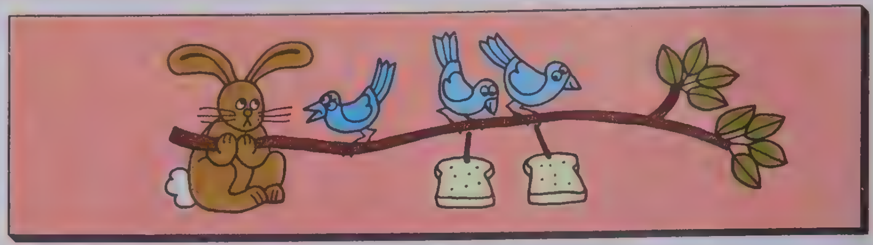
Resources For Active Learning

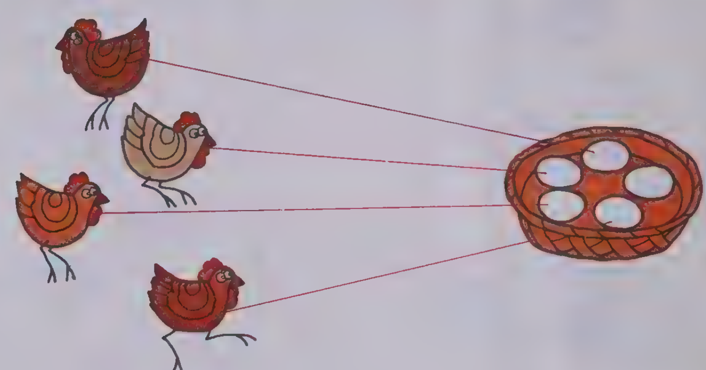
MATHEX: Matching and Graphing No. 1, "Mapping Experiences," pp. 8–11, Encyclopaedia Britannica Publications Ltd.

Teaching
Page a-9

Try to relate the illustration at the top of the page to some of your pre-book activities involving the one-more-than concept. Then ask the children to look at the first frame. Suggest questions such as: "Are there enough nests for each hen?" "How many more are needed?" Stress the one-more-than concept in this lesson. Ask the children to draw matching lines and help them realize that one hen is "left over." You might have them try to draw the nest needed to make a one-to-one matching, but this is not necessary.

For the second frame, ask: "Are there as many hens as there are eggs?" As the children respond, continue to stress the one-more idea. Again ask them to draw matching lines and tell how they can use them to show that there is one more egg than hens.





Concept of one more

Objective

Given two sets whose cardinal numbers differ by one, the child will be able to identify the set which has one more member than the other by matching.

This lesson develops the concept of nonequivalent sets with emphasis on the more exact idea of one-more-than. Again the method of one-to-one matching of objects in the sets is helpful.

Pre-Book Activity

Use any of the matching activities which children enjoyed and adapt them to the study of nonequivalent sets. Stress the use of the matching method to find out if two sets are equivalent. For example, ask all the children to be seated and take a seat yourself. It is likely that some chairs in the room will remain empty. Ask the children to look around and decide whether there are more chairs or more people in the room. If there are empty chairs in the room when everyone is seated, the children should immediately observe that there are more chairs then people. When asking the children to explain why they think there are more chairs, try to draw out the response that there are more chairs because everybody has a chair and some are "left over." If you do not have the same number of girls and boys in your room, you can demonstrate the matching of sets that are not equivalent by having the girls and boys pair off to see which set has more. When possible, use sets which will show the concept of one-more-than.

Concept of one more

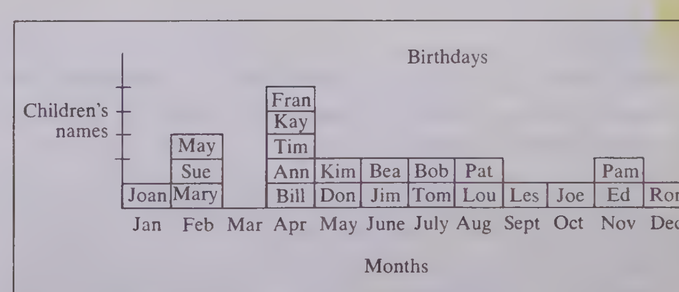
Teaching
Page a-10

Ask the children to look at and identify the first set in the first row as the pink set. Direct them to make a large check mark (✓) on each set in the row that has the same number as the pink set. Then ask them to put an X on the set that has one object more than the pink set. Work through each of the other rows similarly. Avoid, if possible, mention of the actual numbers one, two, three, and four. Stress instead the idea of same number and one more.

Follow-up

As a follow-up, you might use graphing as suggested below. On occasion, you might want such an activity to comprise the major part of a day's lesson. In such case, you might devote most of the next day's lesson to use of text pages with less pre-book or follow-up activity.

Suggest that children build a "picture," or graph to show in what months their birthdays come, or how they travel to school, or what their favorite colors are, or whether or not they like chocolate ice cream, and so on. Choose one subject and label a graph on a large sheet of chart paper. Give each child a piece of colored paper to use to represent himself on the graph. Have the children mark the paper so that each child can identify the slip of paper which stands for himself; or prepare the named slips of paper yourself.

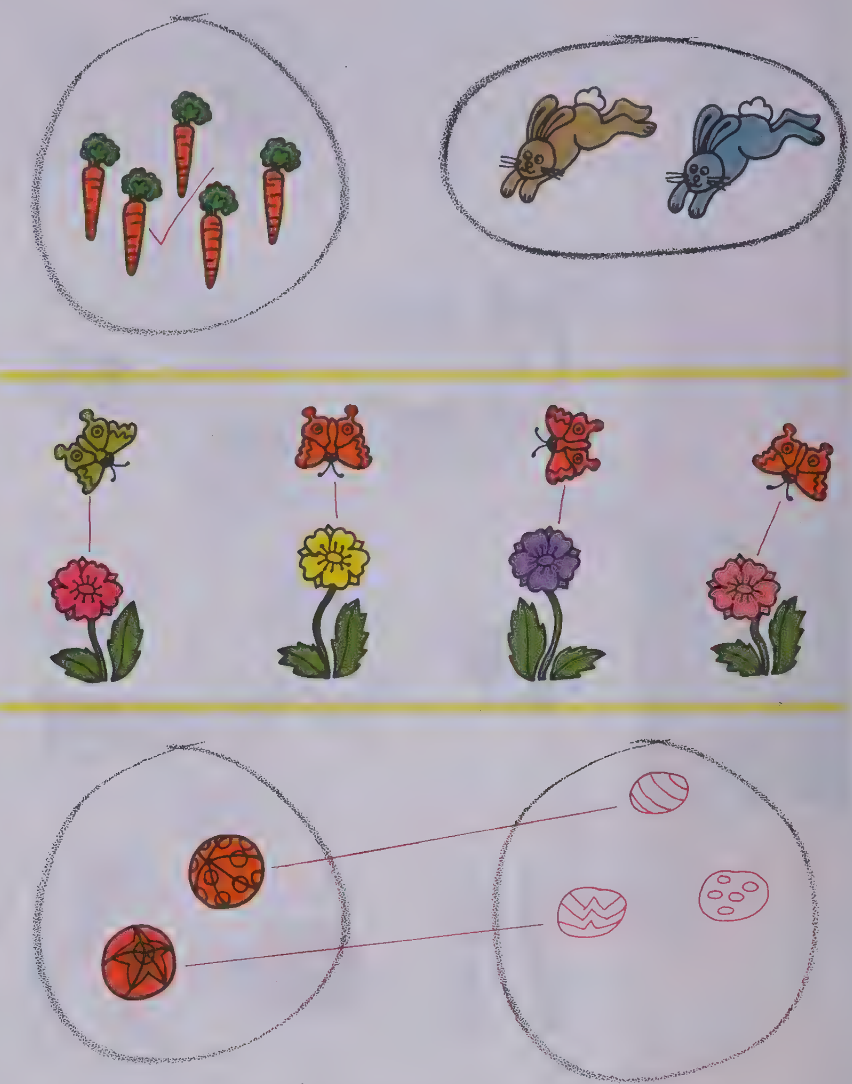


Use the charts as a basis for questions such as: "Since everyone has a slip of paper for himself on the chart, what kind of matching is there between the chart and the class?" (Stress one-to-one matching.) "Are there more birthdays in April or June?" (Help children compare the various columns. If a certain column has exactly one more than another, use it to stress the one-more-than idea.)

Teaching
Page a-11

Children will need specific directions for each frame on page a-11. Call attention to the first frame and ask them to look at the two sets. Then ask them to put a check mark (✓) on the set that has more. When you see that they are finished, ask them to look at the frame with the butterflies and flowers. Direct them to show a one-to-one matching between these two sets. When they have had sufficient time to draw their lines, continue to the last frame and ask children to draw a set of balls that has one more than the set on the page, and then draw matching lines to show that their set has one more.

Show you know



Objective

The child will demonstrate his ability to work with the concepts presented in this module by marking sets according to the teacher's directions.

Evaluation of a first grader's understanding of basic number concepts should not be limited to his performance on a single page. Consider page a-11 as a summary of the module and page a-12 as a change of pace. Page a-12 is intended to help children build positive attitudes toward math and provide a stimulating and enjoyable math exercise.

Pre-Book Activity

Since page a-11 reviews ideas previously treated, use a variety of matching activities which will review these ideas. If you prefer a demonstration activity, you might prepare some large cards on which you draw sets of stars, squares, circles, etc. Make two or three cards for each number from one to ten. On cards for the same number, space the figures differently.

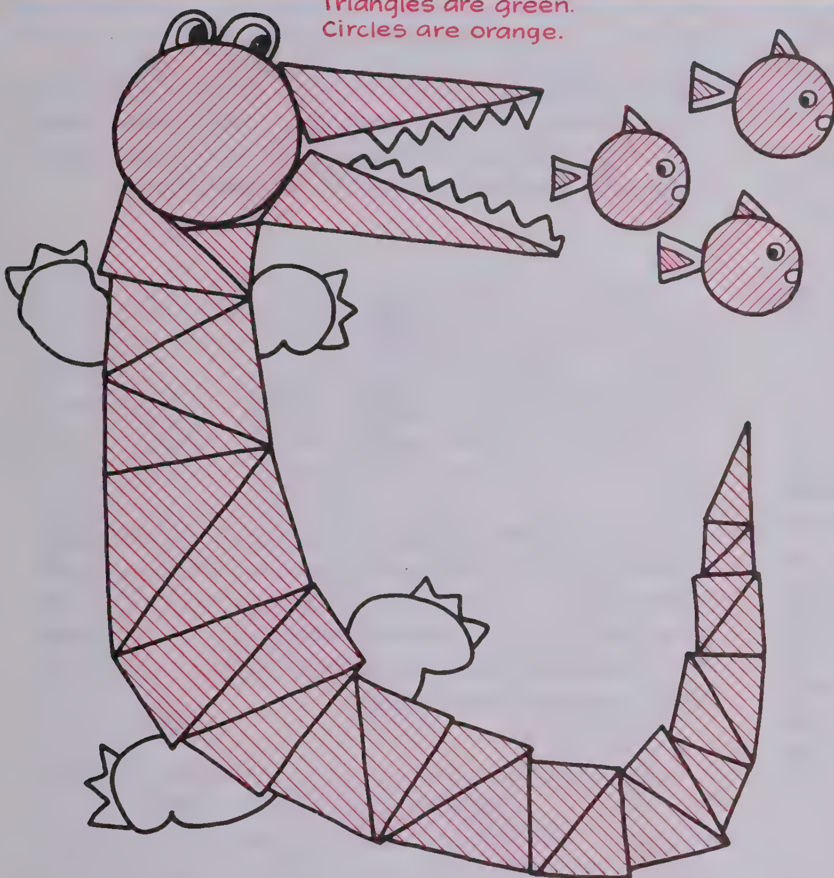
Use these cards in a matching game. Hold up two cards and have children respond "Yes" or "Match" if they are equivalent and "No" if they are not. When two sets differ by one, ask the children how many more in one set than another.

Follow-up

If you wish to provide children with exercises to reinforce any of the concepts which caused difficulty, prepare worksheets like the following. Give clear directions to the children.

Let's have fun

Triangles are green.
Circles are orange.

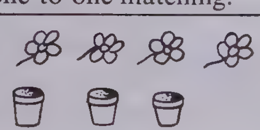


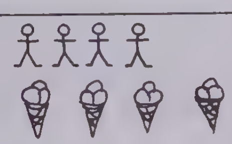
Circles and triangles

Teaching Page a-12

Page a-12 should be handled with a light touch. Use your own discretion in referring to the shapes as circles and triangles. Similarly guide the children in deciding how to color the paws which are not so clearly circles or triangles. You might use the terms yourself, but do not make an issue of them. Children may be directed simply to color orange all the shapes that match the orange shape at the top of the page, and to color green all the shapes that match the green shape at the top.

Draw matching lines. Color the sets which have one-to-one matching.





For each set draw a set which has one more.

For a "Clown Game" draw or cut out a picture of a clown holding a group of balloon strings. Use flannel on the back of the picture if a flannelboard is available; if not, draw it with colored chalk. Cut out several sets of balloons of various colors and shapes: some round, some shaped like animals, some sausage-shaped, some twisted, and so forth. One set could be equivalent to the set of strings, one could have one more balloon than the set of strings, and one could have one less. After you have provided some direction, let small groups of children experiment freely with these materials to find out for themselves which sets are equivalent and which are not.

Resources For Active Learning

THINK AND COLOR, "Matching," pp. 83–87, Educational Science Consultants

ORANGE MODULE, UNIT A

Numbers and Numerals 0 to 4

Pages a-13 to a-24

General Objectives

To introduce the cardinal numbers 0 through 4

To introduce and learn to write the numerals 0 through 4

To provide practice in identifying the number of a set containing four or fewer objects

In this module, the concept of cardinal number is treated more formally, and names and numerals are given for the cardinal numbers zero through four.

The fact that most children can recognize the number of a set of four or less without counting provides us with a "pure" environment for the development of cardinal-number ideas. In order to stress these basic number concepts in the beginning pages, we require the children to determine the number of a set by simply looking at the set and, without counting, to give the number.

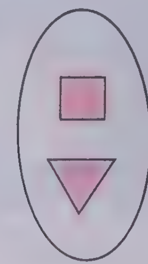
An on-going objective of this module is to teach the children to write the numerals 0 through 4 for the numbers they are studying. In the early pages of the unit we begin to develop skill in writing numerals by having the child write large-sized numerals. As he proceeds through the unit, he will find that the size of the numerals is being reduced gradually, so that when he finishes this unit, he will be prepared to write these numerals in the standard space required for the rest of the work at this level.

It is extremely difficult to maintain the correct usage of the words "number" and "numeral." Although it is desirable for you to use precise terminology whenever possible, sometimes you will not know whether to say "number" or "numeral." For example in discussing inequalities, you would most likely say: "Ring the larger number." or "Ring the greater number." Obviously, you want the child to ring the numeral, but you can communicate more effectively in this case by saying number. A rule of thumb for using the number-numeral terminology is: Whenever you are sure that you are referring to the symbol itself and not to the number, you should say "numeral"; in all other cases, say "number." Keep in mind also that it is better to abuse the language slightly than to make an issue of it and confuse the children. Always remember, the purpose of language is to communicate.

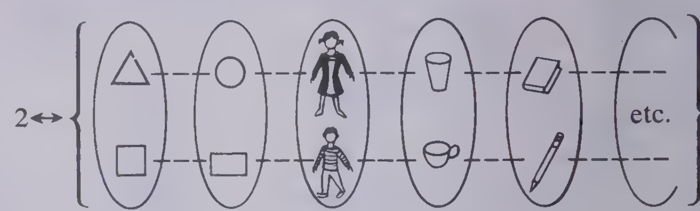
Mathematics

In this module, the chief emphasis is upon the cardinal numbers 0, 1, 2, 3, and 4 and the numerals for these numbers.

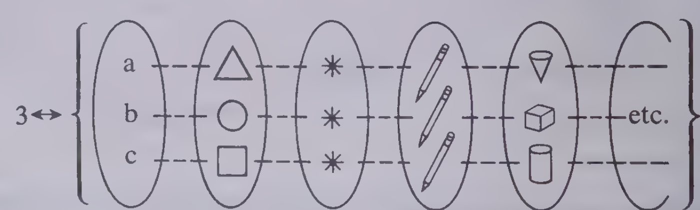
The idea of cardinal number 2 can be acquired by considering sets of objects that can be matched one-to-one with a set like the following one.



Thus a mathematical description (not a precise definition) of the cardinal number 2 might take the following form: The cardinal number 2 is the class of all sets equivalent to the set above as shown in the following figure.



As a general description of the idea of cardinal number, let us say that a *cardinal number* refers to a *class* (or *set*) *of equivalent sets*. For example, cardinal number 3 is the class of all sets equivalent to the set $\{a, b, c\}$. (This statement is depicted in the next figure.) Cardinal number zero is the number of the empty set.



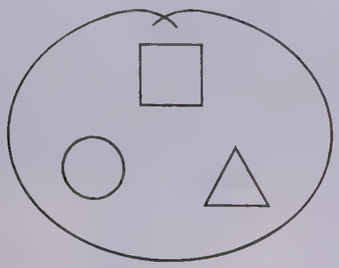
Since we think of each cardinal number as a *class* of sets, we can speak of *choosing* a set *A from* a cardinal number. If we choose a set from cardinal number 2, we have a set of two elements. Usually, when considering a set from cardinal number 2, we say that the cardinal number of this set is 2, or that it is a set *with* the cardinal number 2.

Using the ideas above, we can introduce the set of cardinal numbers

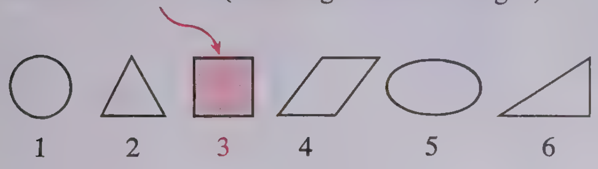
$$\{0, 1, 2, 3, 4, \ldots\}.$$

This set of numbers is usually called the set of *whole numbers*. However, both the *cardinal* and *ordinal* interpretations of the whole numbers should be considered. The following figures illustrate how these two ideas may be applied to "3."

Cardinal number 3 is the NUMBER of this set.



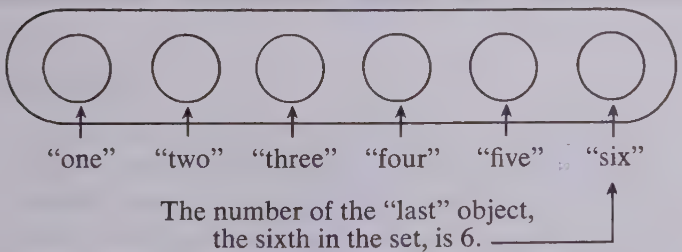
Ordinal number 3 is associated with this OBJECT (counting from left to right).



A cardinal number refers to a *quantitative* aspect of a set of objects, whereas an ordinal number refers to the *position* of one object in an ordered set. The common names for the ordinal numbers are "first," "second," "third," and so on, referring specifically to position.

Cardinal-number arithmetic is the mainstream of the mathematics presented in the elementary school. Ordinal numbers are brought in as a convenience and as an aid in shedding light upon certain topics in cardinal-number arithmetic. For example, we determine the cardinal number of a set by counting. To visualize this, think about counting a set of objects that are scrambled together. We would probably touch or point to each object as we count it, saying "one, two, three, four, . . ." thus assigning a number to each one. When we have completed the counting, the number of the last element of the set is the cardinal number of the set. This relationship is pictured in the next figure.

The cardinal number of this set is 6.



Teaching Orange Module, Unit A

Approximate Time: 6 to 8 days

Materials

counters, such as discs or checkers for each child
felt pens
flannelboard and felt objects such as: circles, triangles, hearts, bunnies, stars, and other shapes
newsprint
objects for set demonstrations (blocks, pencils, crayons, erasers, straws, pipe cleaners, etc.)
overhead projector and clear plastic discs or objects (if available)
paper plates, five for each child
rubber cement

Vocabulary

count	four	one	three
empty set	number	ring	two
	numeral		zero

The children are not expected to use all the words in the vocabulary list correctly. Do not belabor number-numeral terminology; just attempt to use the words correctly yourself whenever possible.

You might expect the number zero to be introduced first to preserve the order of the set of cardinal numbers. But it is important to expose the children to less abstract number concepts first, and then, after careful development, to introduce them to the more abstract concept of zero.

Evaluation of Progress

The children's ability to recognize the number of a set of objects fewer than five should be evaluated. It is also important to determine whether the children see the various relationships between the numbers. For example: Just as one is the number of the set which matches the set {1}, so zero is the number of the empty set.

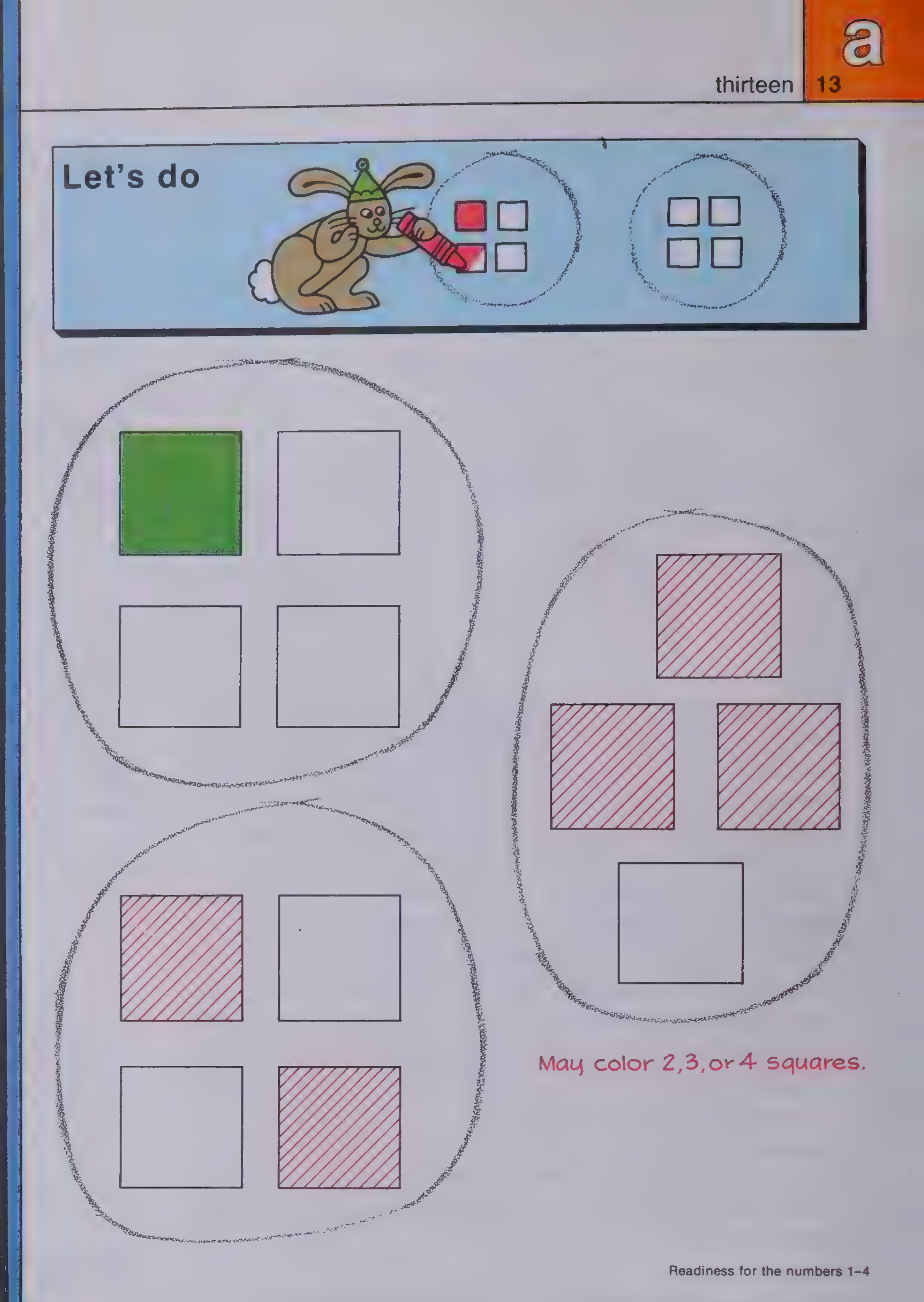
Resources for Active Learning

General Activities:
EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Counting," Nos, 12–15, Responsive Environments Corp.
MATH ACTIVITIES, Materials 2/52, p. 45, Allyn and Bacon
MATHEX: Numeration No. 2, ". . . Cardinal Number Concept," pp. 6–8, Encyclopaedia Britannica Publications Ltd.
SETS, NUMBERS AND POWERS, "Lessons and Games . . . Numbers," pp. 97–103, Herder and Herder
WORKJOBS, pp. 129–145, 156–157, 162–179, Addison-Wesley
Manipulative Devices:
Number/numeral apparatus (Childcraft; Teaching resources; school supplier)
Unifix material (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

INVESTIGATION
Page a-13

Observe with the children that one of the squares has been colored in the first ring. Then ask them if they can color a different number of squares in each of the other rings. Do not give any guidance; simply be sure the children understand that in each ring they should color a different number of squares. This gives them the opportunity to experience choice in building sets with different numbers, each less than 5. Don't attempt to identify the number they colored in each set at this time. It may come up, but is not a part of the lesson. Just be sure that they color different numbers of squares and understand that they have done so.

You might show on the chalkboard, or flannelboard, the way a child colored the squares. Point out a set that has one more square colored than another. Continue to emphasize *one-more-than* if you discuss other examples.



PURPOSE

To provide readiness for development of the concept of cardinal numbers 0–4

PREPARATION

Materials

crayons

Due to the simplicity of the investigation you might begin with it immediately. However, if you prefer a warm-up activity, use a matching activity which children enjoyed in the previous module.

Let's talk

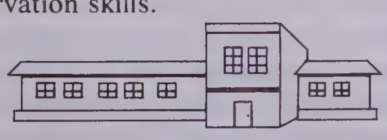
Readiness for the numbers 1–4

Discussion
Page a-14

Direct the children's attention to the picture. Encourage them to talk about it and to observe the various groups that are shown. For example, during this discussion elicit from them that there are more children than adults shown. That more children are standing than those sitting or kneeling. That at the bottom of the page there are more girls than boys. Stress that if they think of the children in pairs one girl would not have a partner. Notice that the emphasis should be on the ideas of pairs and one-more-than rather than on the mention of the specific numbers of one, two, three, or four. Also observe that the birthday theme is continued artistically throughout the module. A chart such as that suggested in the follow-up for page a-10 might stimulate further discussion.

Follow-up

After a walk around the school or playground, make co-operative charts of the sets of one and two that the children observed during their walk. Specific ground rules agreed upon beforehand, such as not including people in the sets, may be necessary to sharpen the children's observation skills.

1 school 

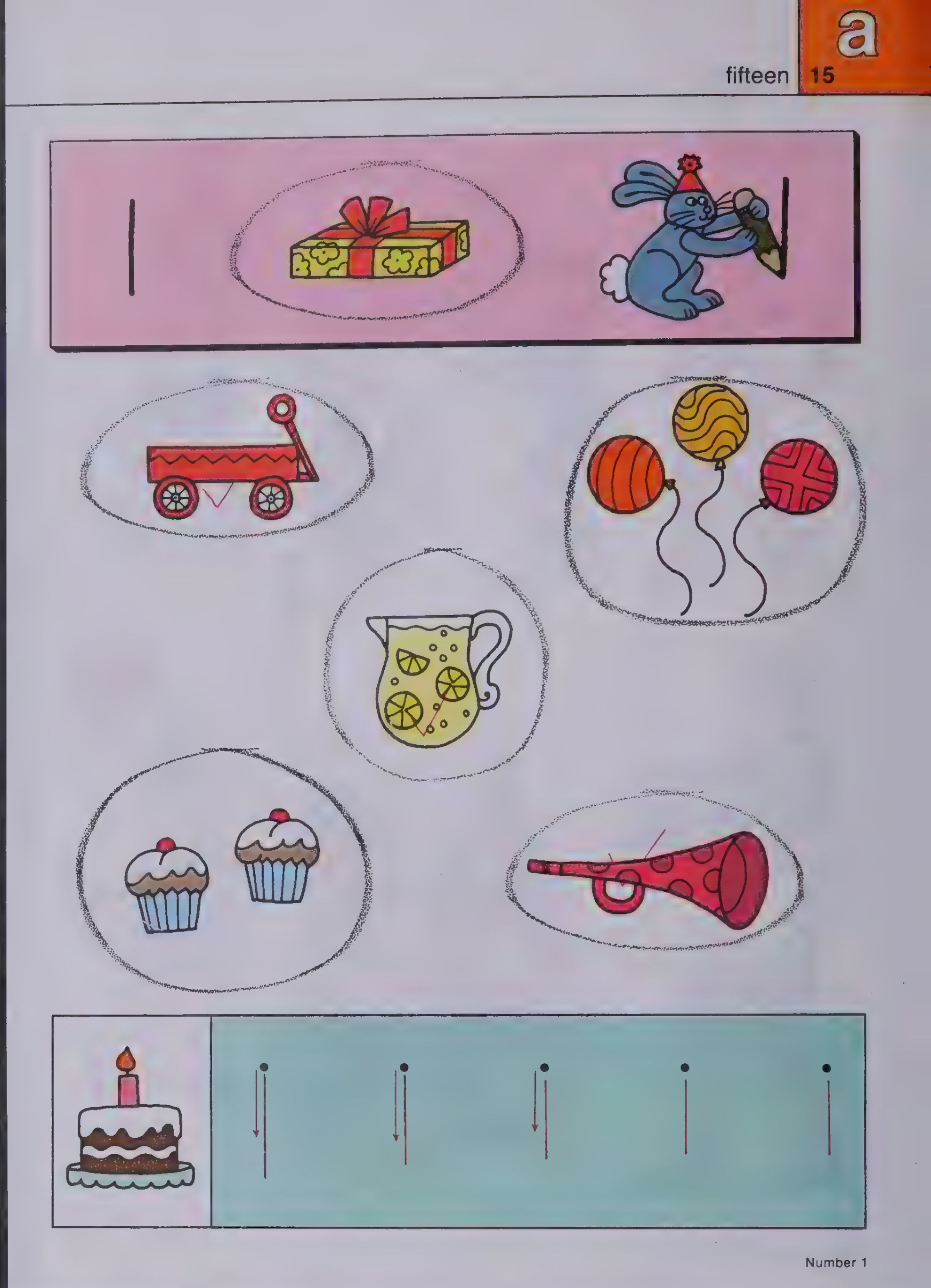
2 flags 

Resources for Active Learning

WORKJOBS, "The Mailman," pp. 64–65, Addison-Wesley

Teaching
Page a-15

Discuss the set on the top of page a-15 and point out that some of the sets on the page are equivalent, or have the same number as this set of one. Direct the children to mark with a check mark those sets which contain one element. When they have finished, write the numeral 1 on the chalkboard and ask children to trace through the air the way they would print the numeral 1, from top to bottom. Point out the one candle on the cake at the bottom on page a-15. Then have children practice writing the numeral.



Objective

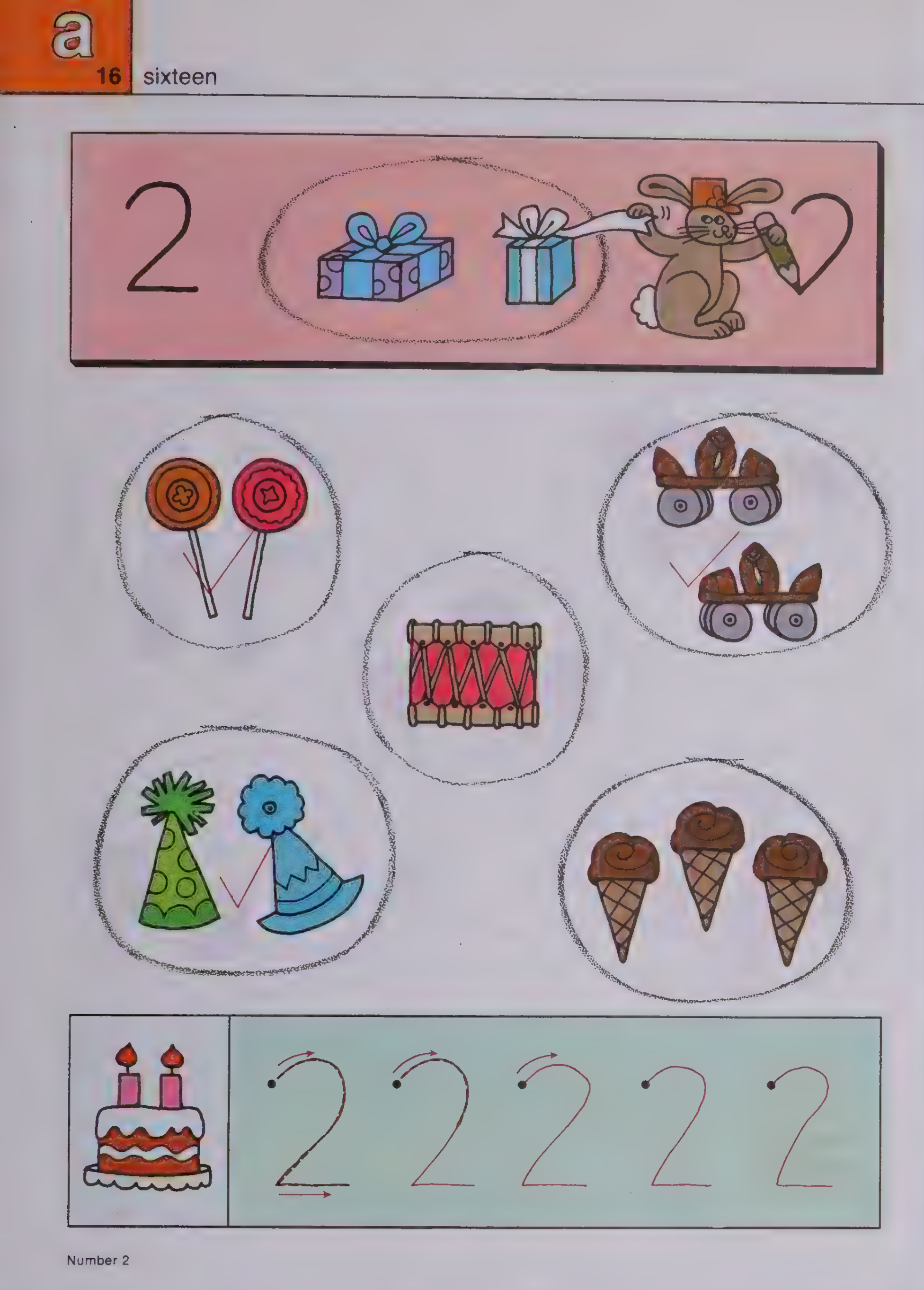
Given a variety of sets which contain one or two objects, the child will be able to identify the number of the set and write the appropriate numeral.

Throughout this and the following module, keep in mind that one of the on-going objectives is for children to learn to write the numerals, but that mastery is not expected immediately. That is, you should continue progressively with the lessons even though complete mastery has not been achieved. The emphasis of this lesson should be on the number concepts of one and two. Most children will have a fairly well-developed idea of the numbers one and two, so adapt your emphasis to their needs and background. There is no need to use the word "cardinal" when you are discussing numbers with the children.

Pre-Book Activity

Since some children are already familiar with the numbers one and two, you might begin this lesson by holding up two pencils and asking how many there are. Use a variety of items and ask children to identify sets of one and two. Also ask them to show sets of one and two with materials.

Those children who have had very little number background should be able to catch on to these early number concepts from the responses of the others. When appropriate during this discussion, write the numerals 1 and 2 on the chalkboard and be sure all the children can identify them.



Teaching
Page a-16

Work through page a-16 in a manner similar to page a-15. Ask children to explain how many gift boxes are in the set at the top of the page. Then have them put a check mark on sets in the centre that are equivalent to the set of gift boxes. Again ask children how many objects are in each of these equivalent sets. Write the numeral 2 on the chalkboard and guide the children in forming the numeral with their hands in the air. You might choose to follow the method used in your printing lessons if it is different from that presented here. In either case, give the children the necessary guidance to help them learn the correct formation of the numeral.

Follow-up

Arrange an activity that will encourage the children to think about different sets of two. Ask the children to stand in a circle. Appoint one of them to be "leader," and explain that as the leader points to a child, the child must name a set of two. Preferrably, this set of two should be a set visible in the classroom so that the child can point to the two things he is referring to. In some cases, you should encourage the child naming the set to actually pick up the two objects of his set to be sure the idea of "two" is clear for the rest of the children listening.

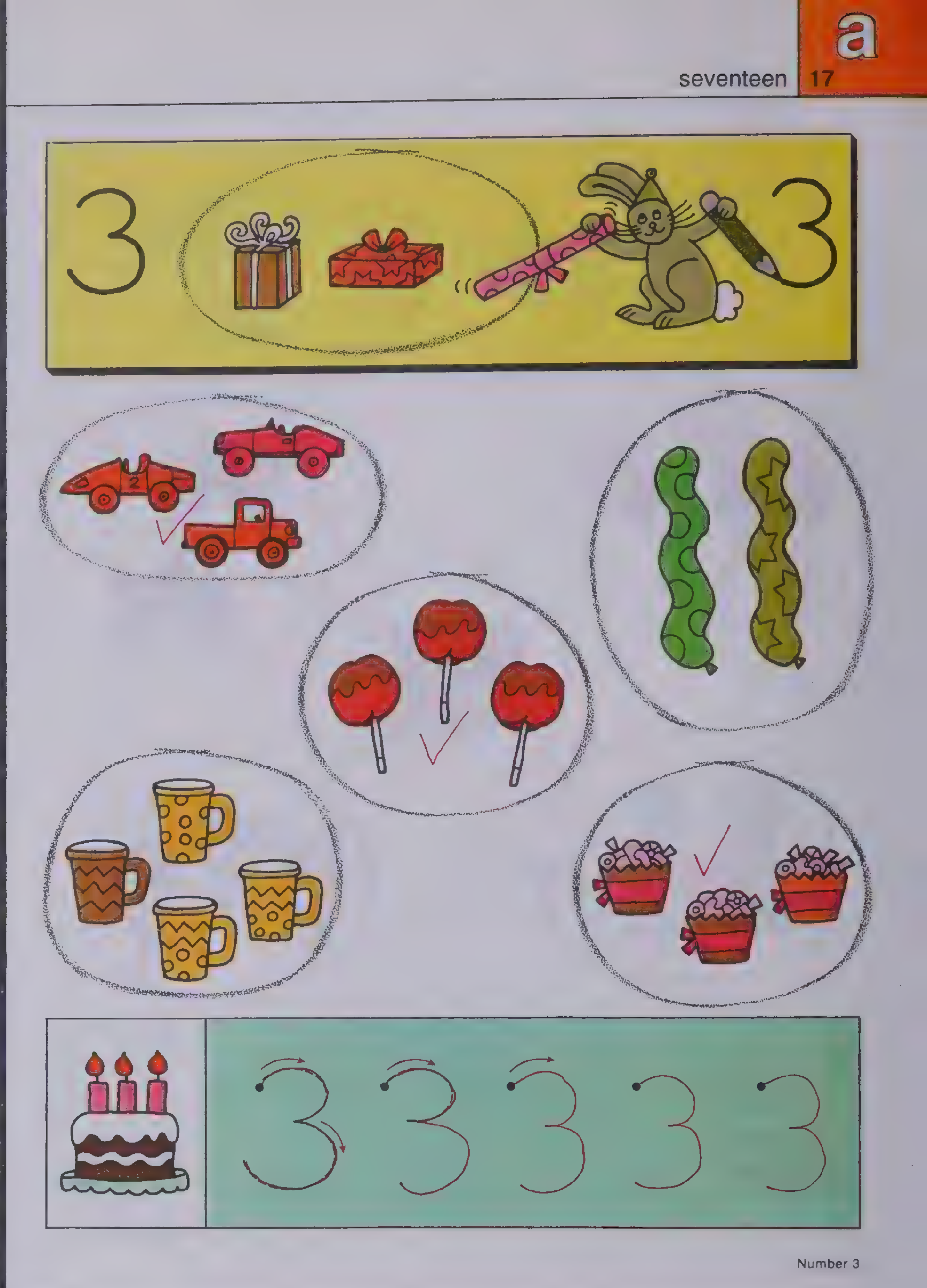
Mathematics

The pre-book activity should emphasize sets belonging to the given cardinal number taught on the page. The equivalence of the sets can be shown by matching the objects in the sets. In this way the idea that a cardinal number is a class of equivalent sets is stressed. Ask the children to suggest other sets that belong to the class being discussed. This activity should help strengthen the idea of cardinal number.

Teaching
Page a-17

In order to identify sets of one and two, it is unlikely that children will be tempted to count. However, some may want to count to identify sets of three and four. Discourage counting on both pages a-17 and a-18. Ask the children just to look at the set and then tell its number.

For page a-17 direct children's attention to the numeral 3 and its relation to the set of three gifts. Explain that 3 is what we write down when we want to answer the question how many gifts there are. Direct them to mark with a check mark the sets of three which they can find on this page. When they have finished, help children realize that the sets they chose are equivalent to the set at the top. Then explain the writing of the numeral 3; guide them first to form it with their hand in the air, then to trace the numerals in the book.



Objective

Given a set which has three or four members, the child will be able to identify the number of the set and write the numeral.

This lesson deals with the number concepts of three and four, and the writing of the numerals 3 and 4. Children should be able to identify the number of a set with one, two, three, or four members by visually inspecting the set, without counting each member.

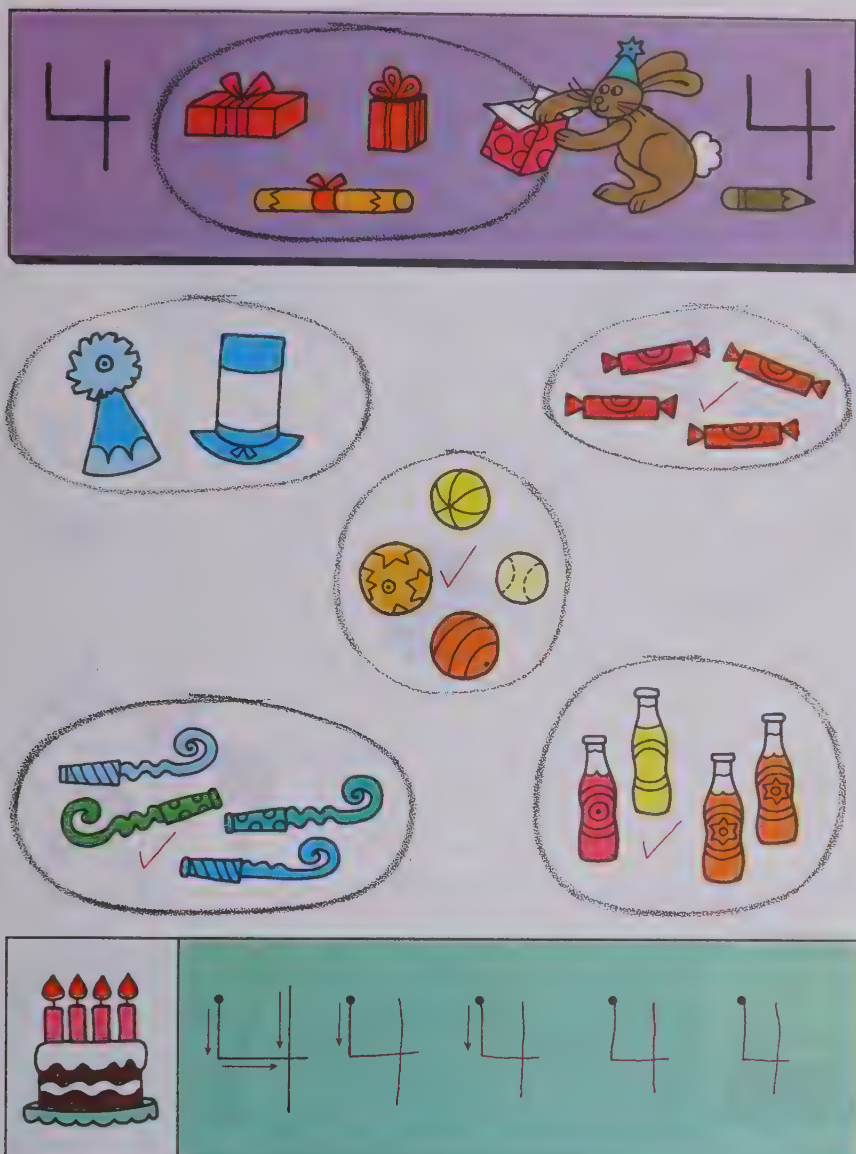
Pre-Book Activity

Since most children will already have a fairly well developed concept of the numbers three and four, you might position around the room sets which have one, two, three, or four objects. Use these to review the numbers one and two. Ask questions such as: "Can you find a set that makes you think of two?" "Point out all the sets that have one object." Then ask if anyone can show a set that has one more than two and give its number name. Ask them to point out examples of sets of three and then continue similarly with sets of four. Write the numerals 1, 2, 3, and 4 on the chalkboard. Stress the idea that three is one more than two, and four is one more than three, but for many children the only new thing in this lesson will be the writing of the numerals 3 and 4.

Follow-up

To give them further practice, both with the number concepts and with following directions, guide the children in folding a large sheet of newsprint into four sections.

a 18 eighteen

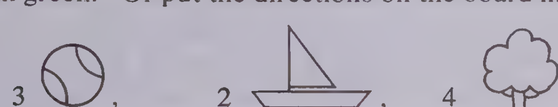


Number 4

Teaching
Page a-18

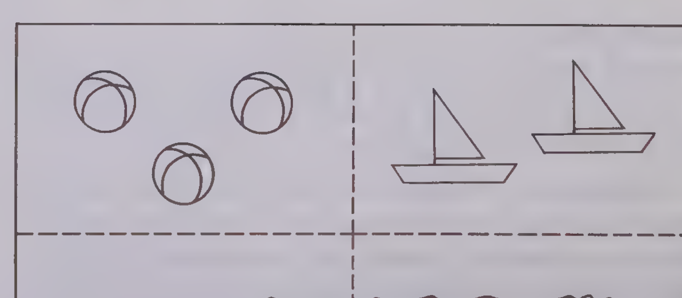
Develop page a-18 as you did page a-17. Again, stress that 4 is the symbol we use to write how many gifts are in the set at the top of page a-18. Have the children identify the sets of four and mark each with a check mark. Again guide them in the formation of the numeral. Notice that throughout this module, the child is asked to write a numeral while he is given a model to follow; only later will he be expected to write the numerals on his own.

Then give oral directions such as: "Draw three orange balls." "Make two red boats." "Draw four trees. Make them green." Or put the directions on the board like this,



and so on, instructing them to color the objects appropriately.

If you prefer, prepare a page the children can work on at their desks which shows objects in sets of 1 to 4. Ask the children to color the sets of four red, the sets of three green, the sets of two yellow, and the sets of one blue. You may choose to put the directions on the board with the numeral next to a simple patch of the appropriate color chalk to help the children remember the proper colors.



Child's paper

Resources for Active Learning

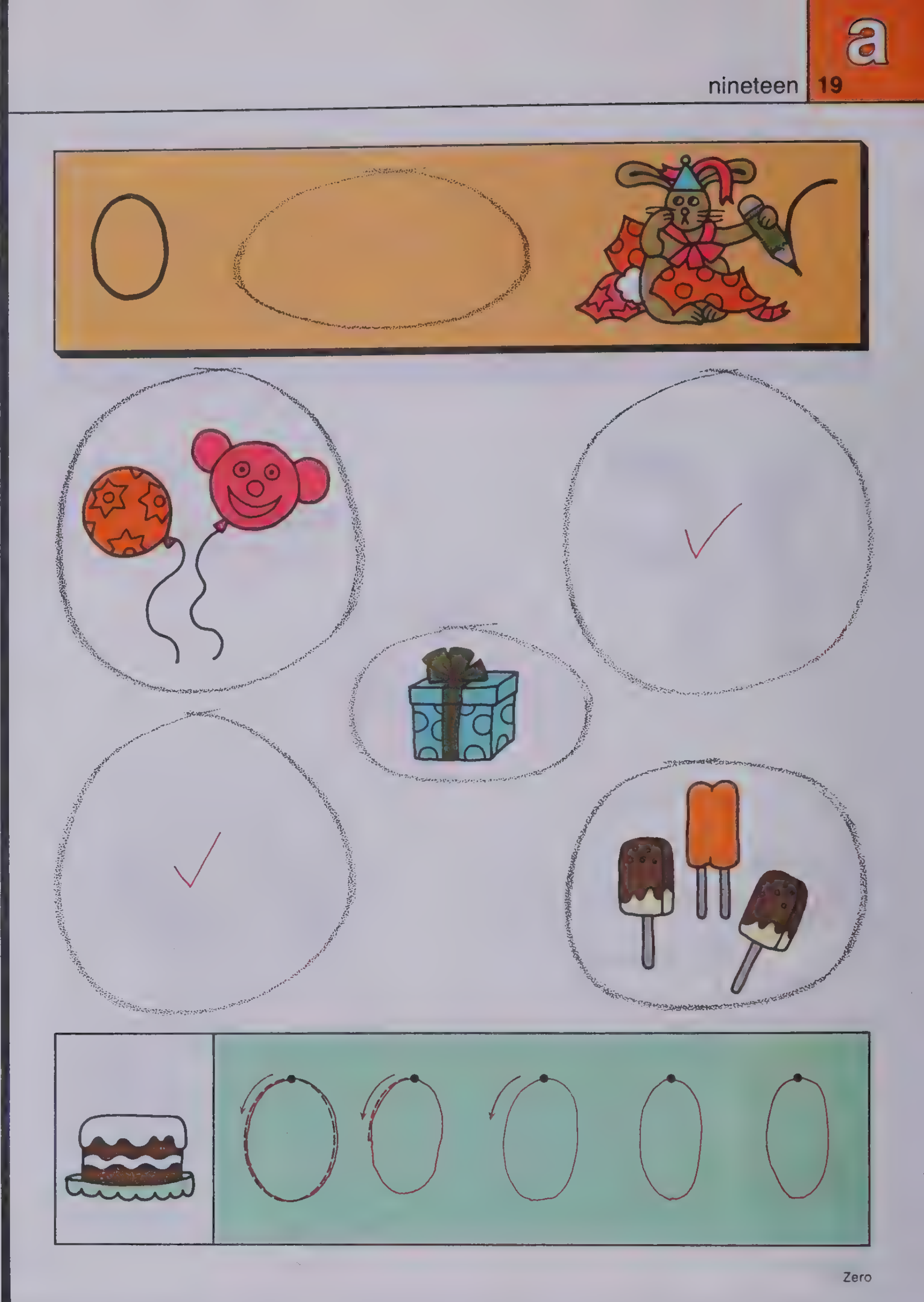
EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "One-to-one correspondence," No. 5, Responsive Environments Corp.

TEACHING
Page a-19

Call attention to the numeral 0 and to the empty set ring at the top of page a-19. Ask the children how many objects are in the ring. If they respond "none," emphasize that the *number* of objects in the set is zero. (Zero is the *number* which expresses the idea that there are "none," that is, zero is the cardinal number of the empty set.) Point out the numeral for zero.

Instruct the children to find and mark all the sets of the page which contain zero objects. Be sure to use the word zero in this particular direction. Do not say, "Find the sets containing no objects." or "Find all the empty sets."

Finally point out that there are zero candles pictured next to the zero numerals at the bottom of the page and guide children in the formation of 0. Direct them in writing the numeral in the air and then in tracing the dashed lines in the text following the direction of the arrows.



OBJECTIVE

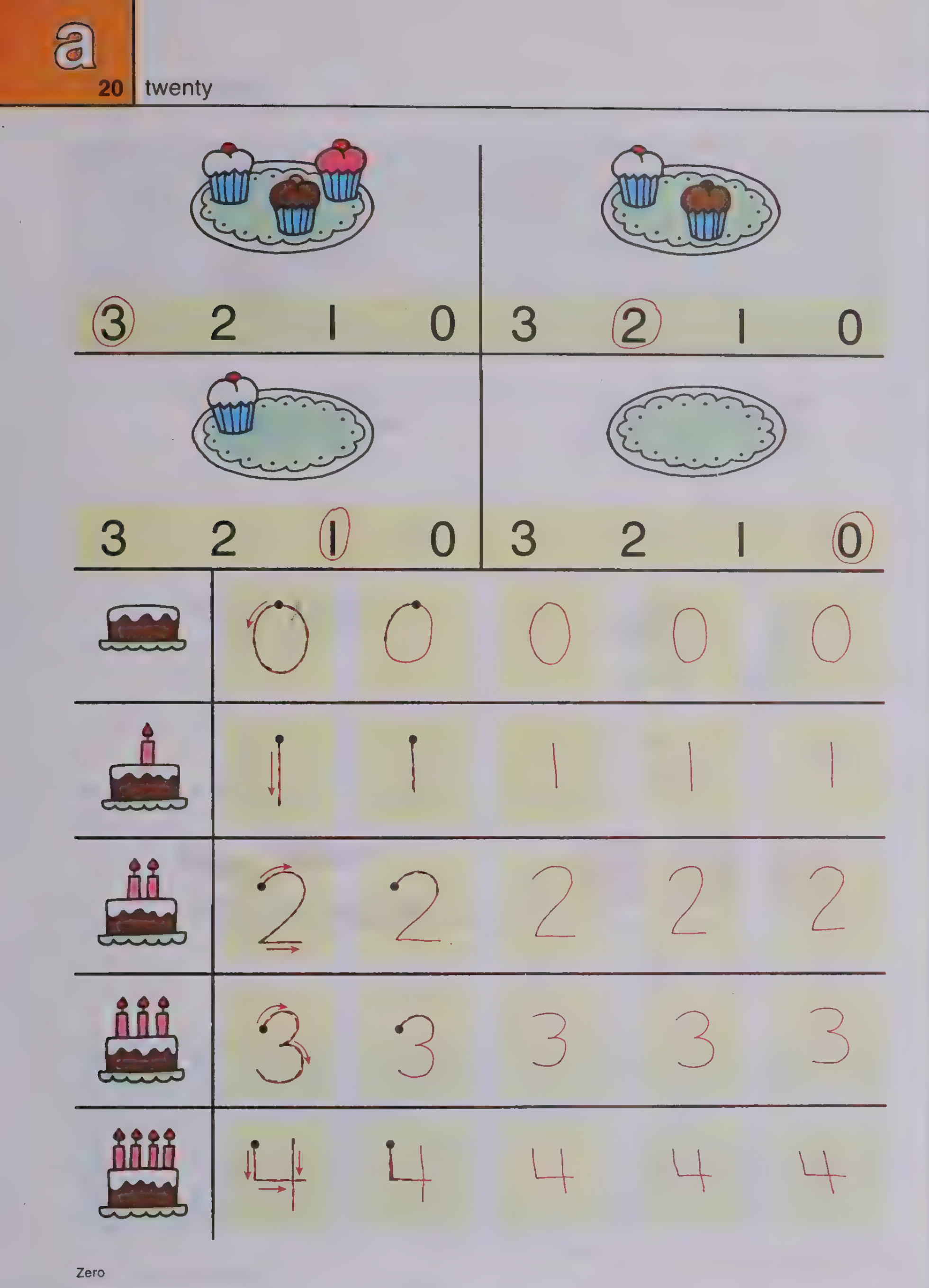
Given some representation of the empty set, the child will be able to identify the number of the set and write the numeral 0 for the number.

Children may find the concept of zero more difficult than that of other numbers. One, two, three, or four objects can be felt, moved, and manipulated. But, zero describes the situation of having *no* objects. Preparation for the concept of zero will need to be imaginative and mostly verbal.

PRE-BOOK ACTIVITY

Develop the idea of zero by asking such questions as: "How many of you are taller than your father?" "How many elephants do we have in this room?" "How many wild tigers have you captured in your lifetime?" and so on.

Another activity might be to distribute three or four counters to each child. Instruct them to put these on their table area or desk. Ask how many they have on their desk, and write their answers on the chalkboard. Then ask them to pick up as many as they can with one hand. Again ask how many they now have on their desk. Some children might be able to pick up all three or four counters in one hand so encourage discussion of how to describe that they have "none" or "not any" on their table. Continue by asking children to pick up any remaining counters with their other hand and asking, now, how many do they have on their table. Discuss the idea that the number which describes how many are now on their table is zero and write the numeral 0 on the chalkboard.



Teaching
Page a-20

As you give directions for page a-20, you could tell a story like this one: "Sally's mother made some cupcakes. She brought them out for Sally and her friends. Look at the first frame at the top of the page. How many cupcakes did Sally's mother take to Sally? Ring the correct numeral." Continue the story in this way, "Sally wanted to know whether the cupcakes were white or chocolate, so she ate one. She was surprised to find it was yellow. How many cupcakes were left for Sally's friends?" Continue with a plausible story until all the cupcakes are gone. Remind the children that we can say zero cupcakes are left.

The lower section of the page provides practice in numeral writing, but it may also be used to point out the sequence of the numbers one through four.

Follow-up

Have children draw sets for each number on paper plates. Each child will need five paper plates and felt pens or crayons. Draw circles like paper plates on the chalkboard, and write the numerals from 0 through 4 in them.

Circles on the board

Individual sets of paper plates

Ask the children to draw one set on each paper plate and to make a set for each numeral shown on the board. You might prefer that they paste on pictures you provide for them. Be sure they make one plate for each numeral.

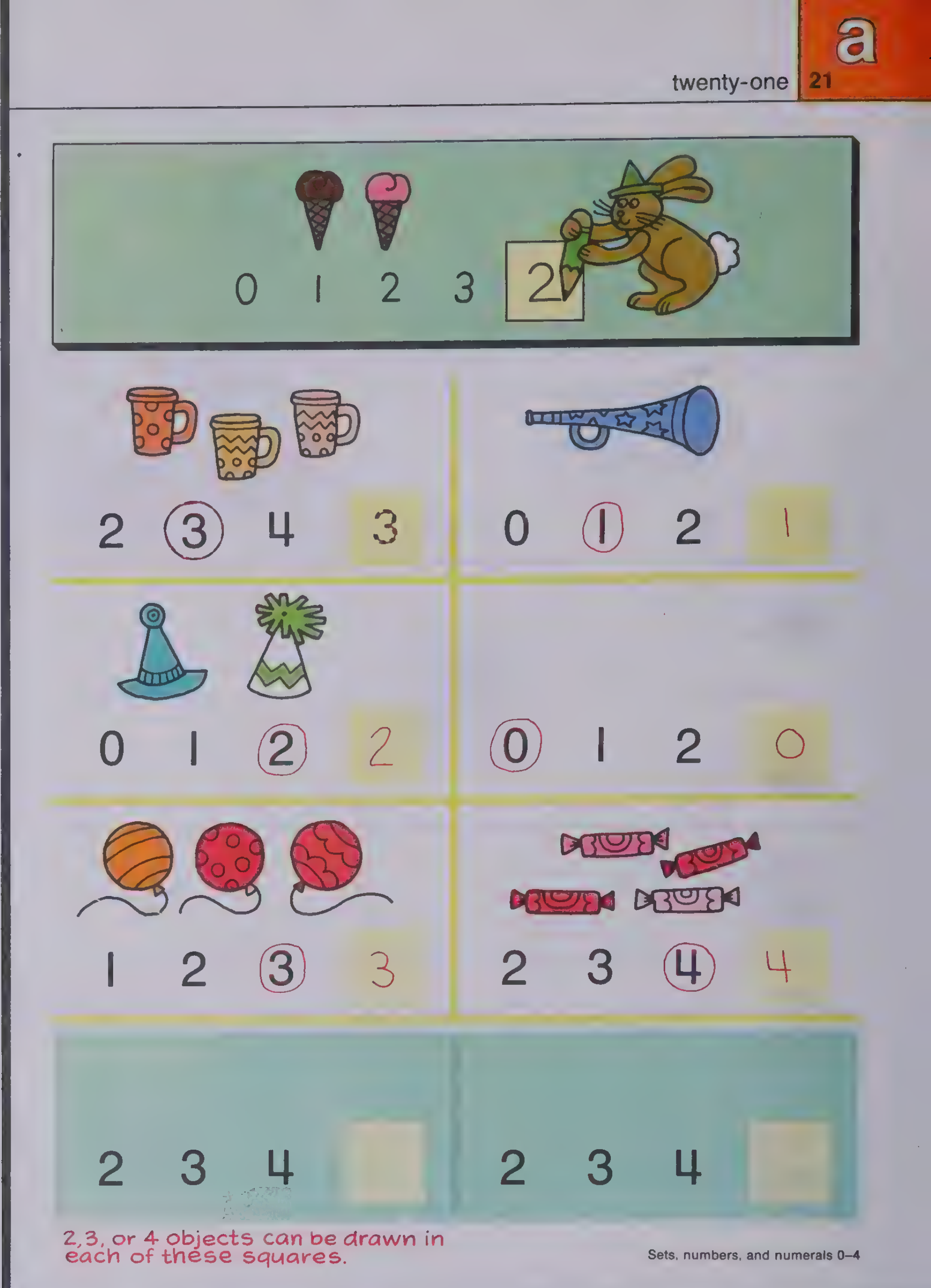
Collect the plates, shuffle them, and show them to the class one at a time. Ask "How many?" for each plate. Keep the pace as rapid as possible.

Mathematics

Zero is the number of the empty set. Several pictured empty sets are merely different illustrations of the one empty set. The empty set can be described in many ways. For example, you can talk about the set of all wild tigers in your classroom. One and only one set is indicated in such descriptions–the empty set. The most important mathematical idea to stress on these pages is that zero is a number–the number of the empty set.

Teaching
Page a-21

Many instructions should not be needed for page a-21. Directions as simple as: "Ring the numeral that tells the number of objects in a set and then write it in the box." will be sufficient for most children. However, you will have to explain that *the bottom two frames provide space in which they can choose the number and the kind of things they want to draw*. Then they should circle how many they've drawn and write the numerals in the yellow box.



Objective

Given a set of zero, one, two, three, or four objects, and a choice of numerals, the child will identify the number of the set and ring the corresponding numeral.

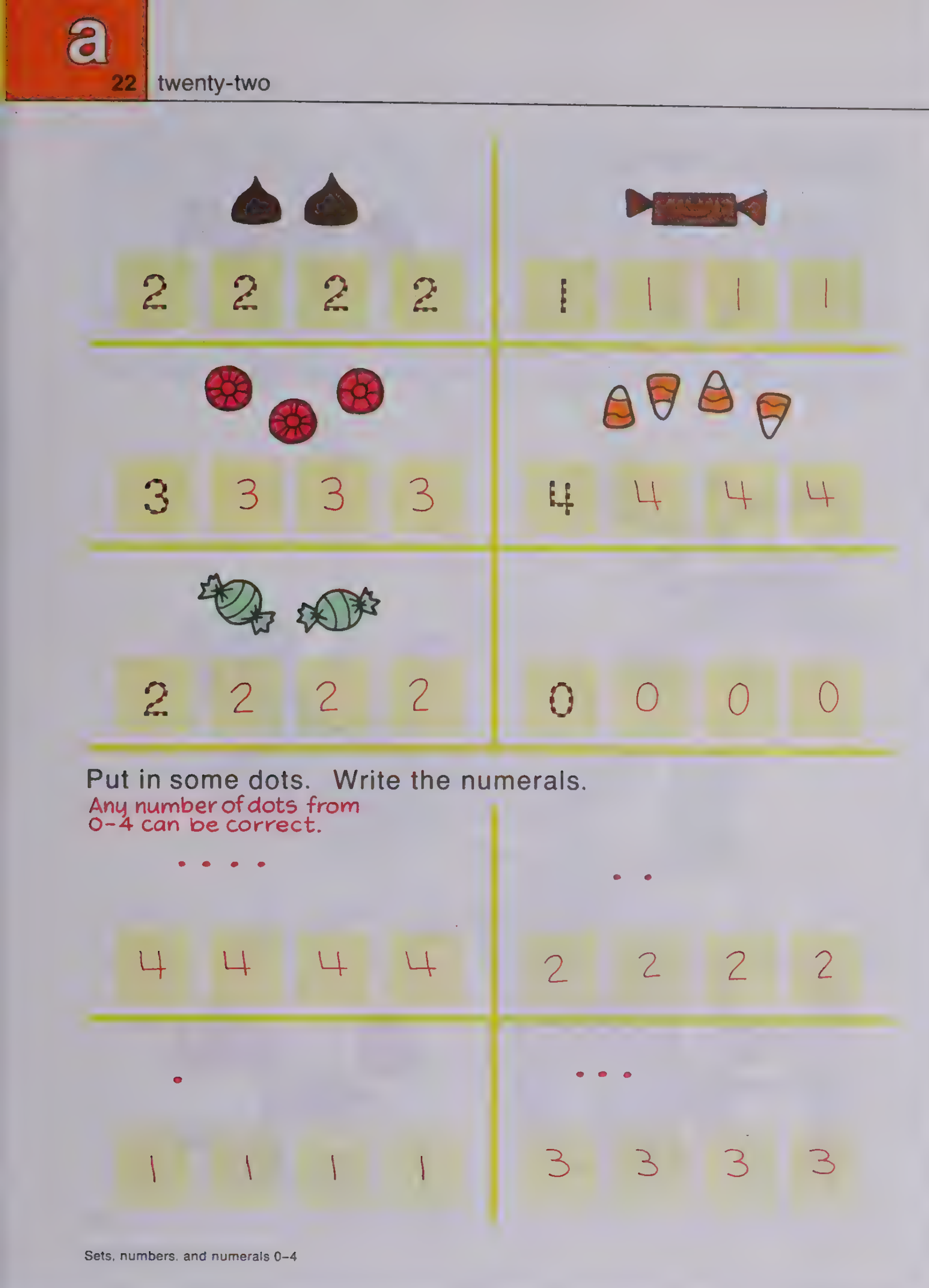
This lesson simply provides practice in identifying the number of a set and its correct corresponding numeral. The extent of practice you want to give children will depend on your evaluation of their number background and need.

Pre-Book Activity

The paper plates suggested on page a-20 may be used in a variety of matching games. For example, divide the class into two teams. Ask a child from "Team A" to choose a paper plate that has a set pictured on it and to name the number of the set pictured. Then ask a child from "Team B" to find the correct numeral plate to match it. Next let a child from "Team B" choose one of the pictured plates, and a child from "Team A" one of the numeral plates. Be sure plates for the numerals 0, 1, 2, 3, 4 are included.

Follow-up

For some children you might duplicate practice sheets like the one shown below with pictures of sets and guidelines for practice in writing numerals.


a 22 twenty-two

2 2 2 2 | 1 1 1 1

3 3 3 3 | 4 4 4 4

2 2 2 2 | 0 0 0 0

Put in some dots. Write the numerals.

Any number of dots from 0–4 can be correct.

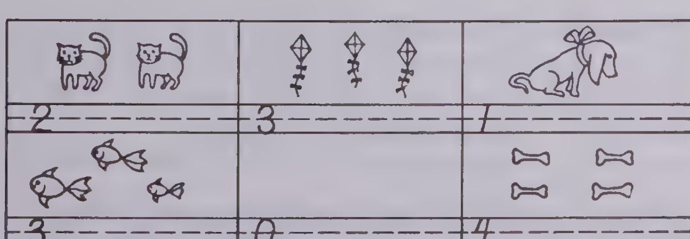
4 4 4 4 | 2 2 2 2

1 1 1 1 | 3 3 3 3

Sets, numbers, and numerals 0–4

Teaching
Page a-22

Direct the children to trace the numeral shown and then practice writing it in the space provided. Be sure they realize the relationship between the numeral and the sets pictured. Again, help them with the directions at the bottom of the page. The last four boxes are for the children to put in any number of dots they choose and to write the numerals. You might want to suggest 4 or fewer dots, but if you don't, be prepared for some children to be able to do higher numbers.



For those who need more practice in identifying numbers of sets, you may want to use duplicated sheets similar to the lesson pages. Have the children mark the sets according to your directions. Ask them to mark each set of four with a check mark, make a green circle around each set of three, and so on, marking each set differently.

Also, you can work informally with the ordinal numbers by giving directions such as: "Draw a set of four in a row on the chalkboard, and put a check mark on the second object from the left." Or say: "Draw a set of three, and put a ring around the third object from the right." Be sure to point out ordinal numbers and counting from both ends, but avoid confusing the positional name, such as fourth, with the cardinal number four.

Resources for Active Learning

Games to provide practice in recognizing numerals and their number values:

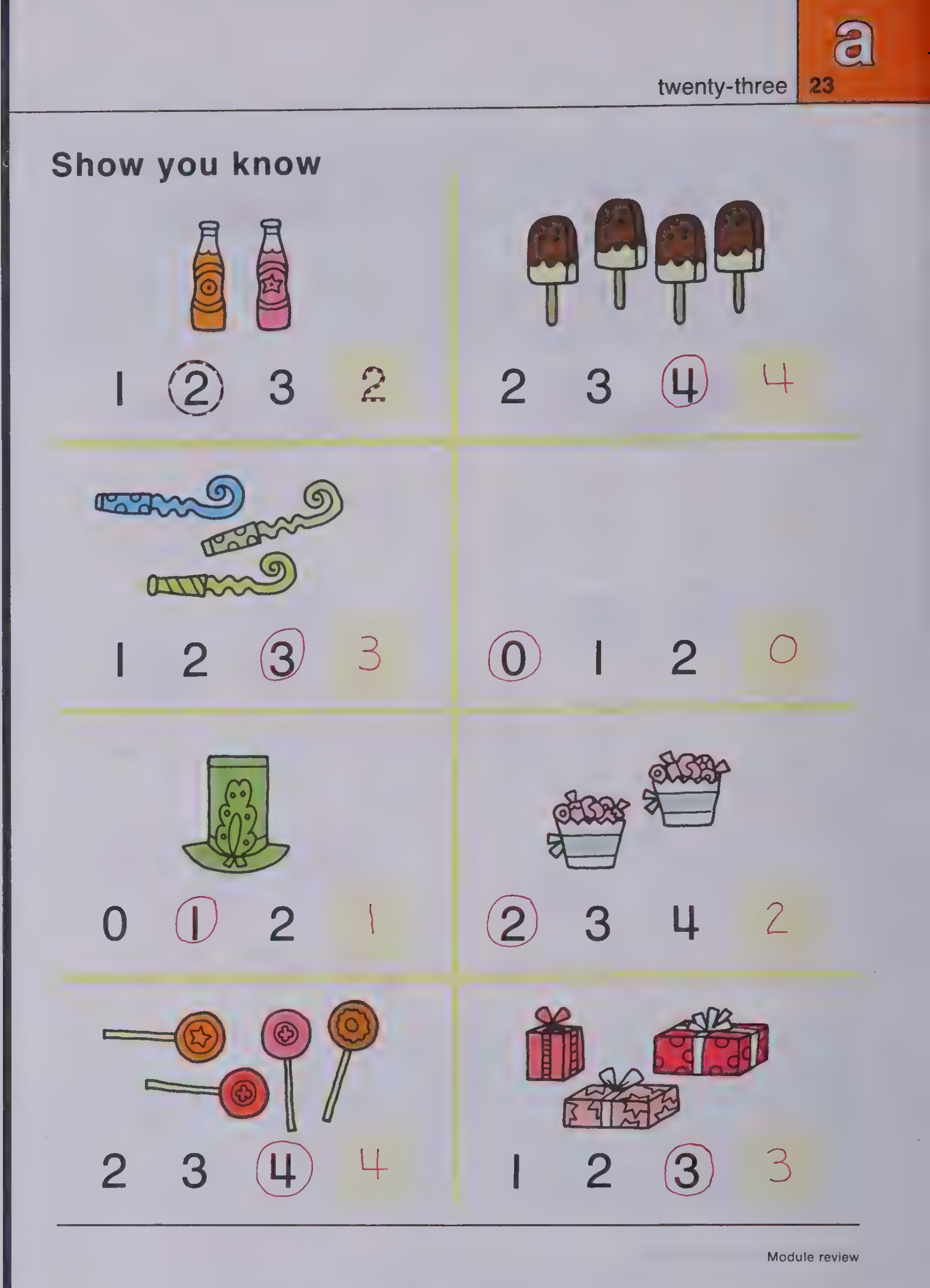
DEVELOPMENTAL MATH CARDS, A-10, Addison-Wesley

MATH ACTIVITIES, Games 2/58–61, 2/63, pp. 48–51, Allyn and Bacon

MATHEX: Numeration No. 2, pp. 26–27, Encyclopaedia Britannica Publications Ltd.

Teaching
Page a-23

Give children clear directions such as "Ring the numeral that tells how many are in each set and then write it in the yellow box." However, since this page is similar to a previous page, children should not have difficulty in knowing what to do. It will still be important to move around the room to give extra guidance to those who need it.



Objective

The child will demonstrate his ability to work with the concepts developed in this module by identifying the number of a set and recognizing and writing the appropriate numeral.

Since evaluation of a first grader's understanding of number should be on a daily basis, page a-23 may be considered an extension of identifying numbers of a set and writing numerals, rather than as a strict evaluative instrument. Page a-24 is intended as a change of pace and should be treated with a light touch.

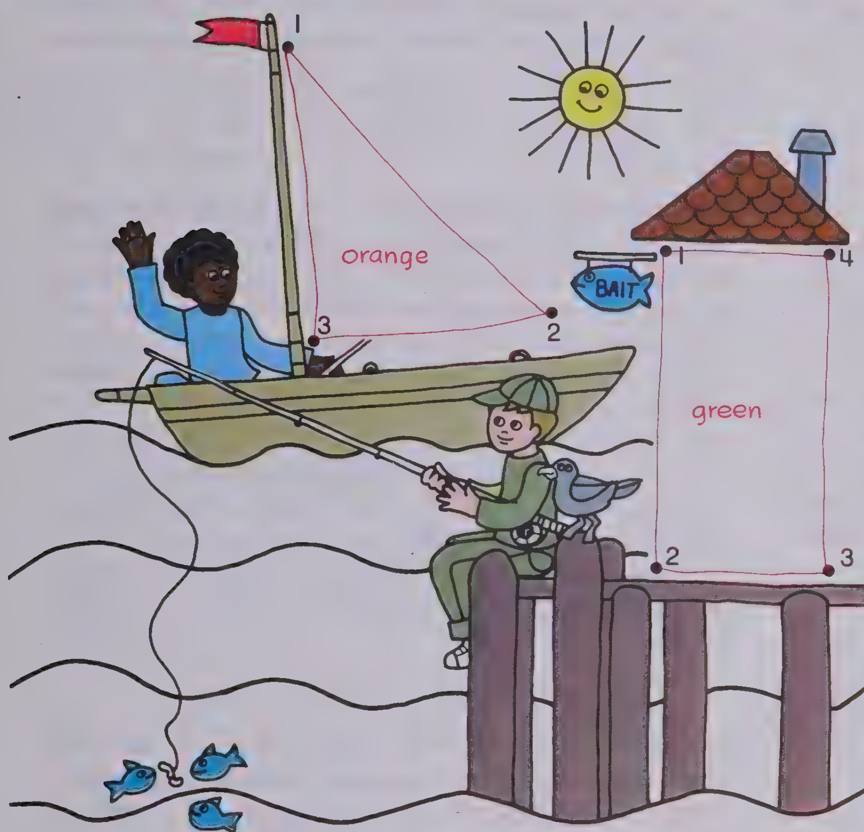
Pre-Book Activities

Materials

counters	*pieces of felt*	*sticks*
pencils	*pipe cleaners*	

Preparation for this lesson should include a review of the number concepts zero through four and recognition of the numerals. You might direct the children in playing a "Silent Number Game." Write the numerals 0, 1, 2, 3, and 4 on the chalkboard and ask a child to point to one of the numerals. He should then call on a second child who should use whatever materials you have made available to show a set which matches the numeral the first child pointed to. A box of pencils, ice cream sticks, blocks, counters, or pipe cleaners should be available. Note that neither child need say the number, the first

Let's have fun



Order of the numbers 1–4

Teaching
Page a-24

Page a-24 is intended as a change of pace page. Explain to the children that by connecting the dots they will be able to complete the picture. Be sure they realize that for each figure they are to start at 1, then go to the numeral which is one more than that, namely 2, then 3, and so on. Noticé that the two sets of numbers are fairly close together. Help the children begin with the 1 nearest the sail boat and complete the sail before they start the next set. Observe the children carefully so you can give help if necessary. When they have connected the two sets of dots, they should color the sail and dock house to complete the picture.

just points to the numeral and the other finds a matching set. This game may also be played with teams, having the first child come alternately from each team.

Follow-up

Individual number books can be made by folding A-3 (297 mm × 420 mm) sheets of newsprint in half and stapling them together on the line to make an A-4 (210 mm × 297 mm) book. Tell the children that they will be able to add to the book and take it home later for their parents to see.

Write the numeral 1 on the board. Ask the children to begin with the first page and draw a set that matches the numeral. Continue writing the numerals through 4, having the children draw sets to match the numerals on subsequent pages of their books. They may use space at the bottom of their pages to write the appropriate numeral.

Resources for Active Learning

Nuffield Project: MATHEMATICS BEGINS 1, "Class or group booklets," p. 38, Wiley

Numbers and Numerals 5 to 9

Pages a-25 to a-40

General Objectives

To introduce the cardinal numbers five through nine

To introduce and learn to write the numerals 5 through 9

To provide practice in identifying the number of a set containing nine or fewer objects

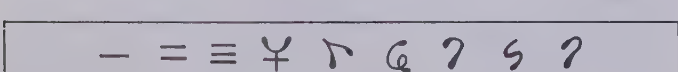
We present the numbers five through nine by introducing each new number as one more than the one before. At this time, counting should be introduced since the number of a set containing five or more objects cannot easily be determined by quick recognition.

Notice in particular that the number ten is not introduced in this unit. Actually, introducing the concept of the number ten presents no problem whatsoever. It is the numeral, or symbol, for ten which causes the problem. Therefore, we delay introducing this new symbol until we can develop the idea of place value carefully. Although it is true that most of the children already know the symbol for ten, few, if any, understand the place-value meaning of this symbol. For this reason it is most desirable to avoid mentioning this symbol at this time.

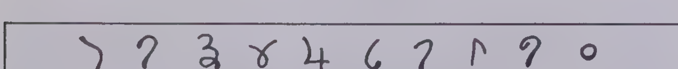
As in the previous module, numeral writing is introduced as each number is presented. Throughout this module the concept of the number that the numeral represents is continually reinforced by presenting the child with sets to look at as he writes the numerals.

Mathematics

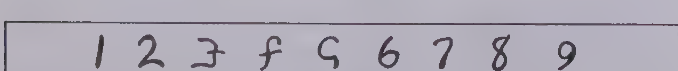
An important fact which simplifies calculations and makes it easy to learn arithmetic is that we can represent any whole number with only ten symbols—the Hindu-Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.



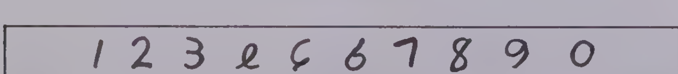
Brahmi, 300 B.C.



Indian, after 500 A.D.



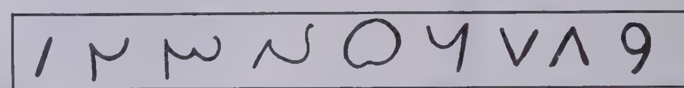
West Arabic



15th Century

The search for the origin of the Hindu-Arabic numerals has occupied men for many years. Even today, archaeologists, with the aid of mathematicians, continue to study ancient manuscripts in an attempt to clear away the mist that obscures our view of the past. In the preceding diagram we suggest only a bare outline of the development of these symbols.

Originally, there were two styles of Arabic numerals, Eastern and Western. We follow the West Arabic style, which was used in Moorish Spain. East Arabic numerals are still used today in Turkey, Egypt, Arabia, and nearby countries. The East Arabic numerals are shown below.



The evolution of the numerals we use today took more than 2000 years. As recently as 500 years ago, most of the people of Northern Europe did their calculations on the abacus and recorded the results in Roman numerals. Only after 1600 A.D. can it be said that the Hindu-Arabic numerals had really won their place in European civilization.

To determine the cardinal number of a set the child uses a *counting process*. Although the counting process is somewhat complicated, many children have already mastered it by the time they enter the first grade.

In order to be able to count, the child must first memorize the names of the cardinal numbers *in a specific order:*

one, two, three, four, five, six, seven, . . .

To determine the cardinal number of a set, the child matches the objects in the set in a one-to-one manner with a subset of the natural numbers *in order, starting with one* as shown below.

$$
\begin{array}{ccccccc}
T = \{ & x & x & x & x & x & \} \\
 & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \\
\{ & \text{one,} & \text{two,} & \text{three,} & \text{four,} & \text{five} & \}
\end{array}
$$

The last number name used to complete the matching, five, is the name for the cardinal number of the set T.

Instances of incorrect counting procedures sometimes used by very young children who are just learning to count are illustrated below.

(A)

$$
\begin{array}{ccc}
\{x & x & x\} \\
\updownarrow & \updownarrow & \updownarrow \\
\text{one} & \text{three} & \text{four}
\end{array}
$$

(B)

$$
\begin{array}{cccc}
\{x & x & x\} & \\
\updownarrow & \updownarrow & \updownarrow & \\
\text{one} & \text{two} & \text{three} & \text{four}
\end{array}
$$

In example A, the child has not paired the number names in their correct order with the objects in the set. In example B, the child has counted one member of the set twice, that is, there is not a one-to-one correspondence of the objects in the set and the set of ordered number names.

Teaching Red Module, Unit A

Approximate Time: 8 to 11 days

Materials

cards (A-6 (105 mm × 148 mm) tagboard; at least two for each number zero to nine)
counters
flannelboard and felt objects
objects for set demonstrations
paper plates, 5 for each child
tagboard
unit strips (felt or paper strips marked in units), ten strips of six units

Vocabulary

eight	seven
five	six
nine	

The arrows that guide the children in forming the numerals should be treated as suggested aids. Some children may make their numerals other ways. For example, some may find it easier to make an 8 by exactly reversing the method on page a-33. Children should have considerable freedom in using a method best suited to their abilities. Some children will naturally need more practice than others. Watch them carefully so that those who master the mechanical skills quickly will not be burdened with too much routine practice.

Evaluation of Progress

We offer a word of caution about evaluating a child's progress in numeral writing. Certainly, we want children to write numerals easily and neatly; however, because efficient numeral writing is a matter of motor skill rather than mathematical ability, we feel strongly that this skill should not unduly influence your overall evaluation of a child's achievement in this unit.

Resources for Active Learning

General Activities:
MATH ACTIVITIES, "Number Bingo," Game 2-57, p. 48; "Taps," Game 2/16, p. 30; Materials 2-52, p. 45, Allyn and Bacon

Sets, counting, ordering:
WORKJOBS: pp. 129–145, 156–157, 169–179, Addison-Wesley
SETS, NUMBERS AND POWERS, "Lessons and Games . . . Numbers," pp. 97–103, Herder and Herder
TEACHING AIDS FOR ELEMENTARY MATHEMATICS, "Cut-up Numerals," p. 115, Holt, Rinehart and Winston

Manipulative Devices:
Chips (Educational Teaching Aids; Selective Educational Equipment)
Foundations for Mathematics units (Teaching Resources)
Hundred peg board and cylinders (Educational Teaching Aids; Responsive Environments Corp.)

Commercial Games:
Candyland (CCM School Materials; Lakeshore; school supplier)
Dominoes games (school supplier)
First Game (Educational Playsystems)
Spot the Set (Childcraft; Selective Educational Equipment)

Investigation
Page a-25

This investigation extends the one-more-than concept from sets of five to sets of nine. Due to the simplicity of the investigation, children should be given as little guidance as possible. Simply note with them that the strips are made up of squares and that in the first strip, five of these are colored. Then present them with the following question: "Can you color a different number of spaces in each of the other strips so that each has more colored spaces than the first one? If children describe how many they color and use the numbers five, six, seven, and eight correctly, praise them, but do not stress the number names.

When the children have colored the squares, ask them to make matching sets with their counters. For example, ask them to place one counter on top of every square that they colored in the second or brown strip. As children do this at their places, you might have one child place felt objects on the flannel-board to make another matching set. Use phrases such as: "Show a set with your counters that has *one more than* this set of brown squares." or "Show a set with your counters that has one more than the set of red squares." Notice that by asking children to build a set of counters one more than the tallest set of squares, they will be building a set of nine.

Let's do

Readiness for the numbers 5–9

Purpose

To provide readiness for introduction of the numbers five through eight

Preparation

Materials

counters *crayons*

Depending on the needs of the children, you might exhibit sets of one, two, three, or four objects and review the "one more than" concept. For example, ask someone to point out a set that has one more than two; another child might then tell how many this set has and write the numeral on the chalkboard. You will also want to be sure that each child has at least nine counters.

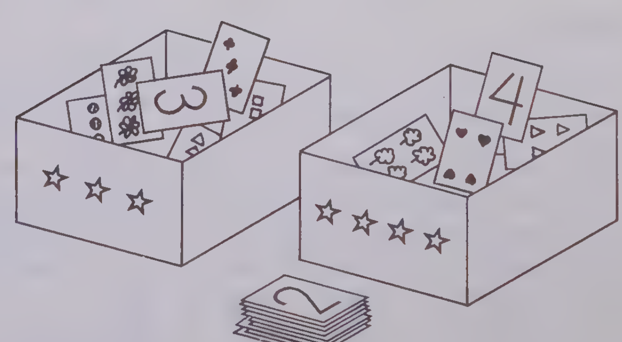
Let's talk

Discussion
Page a-26

Again, an illustration is provided to serve as a basis for discussion. Encourage children to discuss the picture. Help them notice that each child has a dog, but there is one more dog than children. Have them use their counters to match the sets in the picture. For example, say: "Make a set of counters that has the same number as the set of dogs." or "Build a set of counters so you have as many counters as the picture has children." Also, you might ask them to pretend that their counters are bones; then they might show enough bones so that there is one for each dog and one extra. If children begin to correctly use the number names for these sets praise them, but do not introduce the number names yourself.

Follow-up

To give children who have a meagre number background further practice in identifying numerals and number sets, you might set up a "number-box" for each number already introduced and include boxes for the numbers introduced on the next few pages. Make a set of at least five cards for each number. On one write the numeral; on each of the other four cards draw sets showing the number of elements indicated. Position the objects in various ways. Shuffle all the cards together and place them beside the number boxes which may be labelled with stars. Let the children pick from the shuffled deck and match the card with the correct number box.



Resources for Active Learning

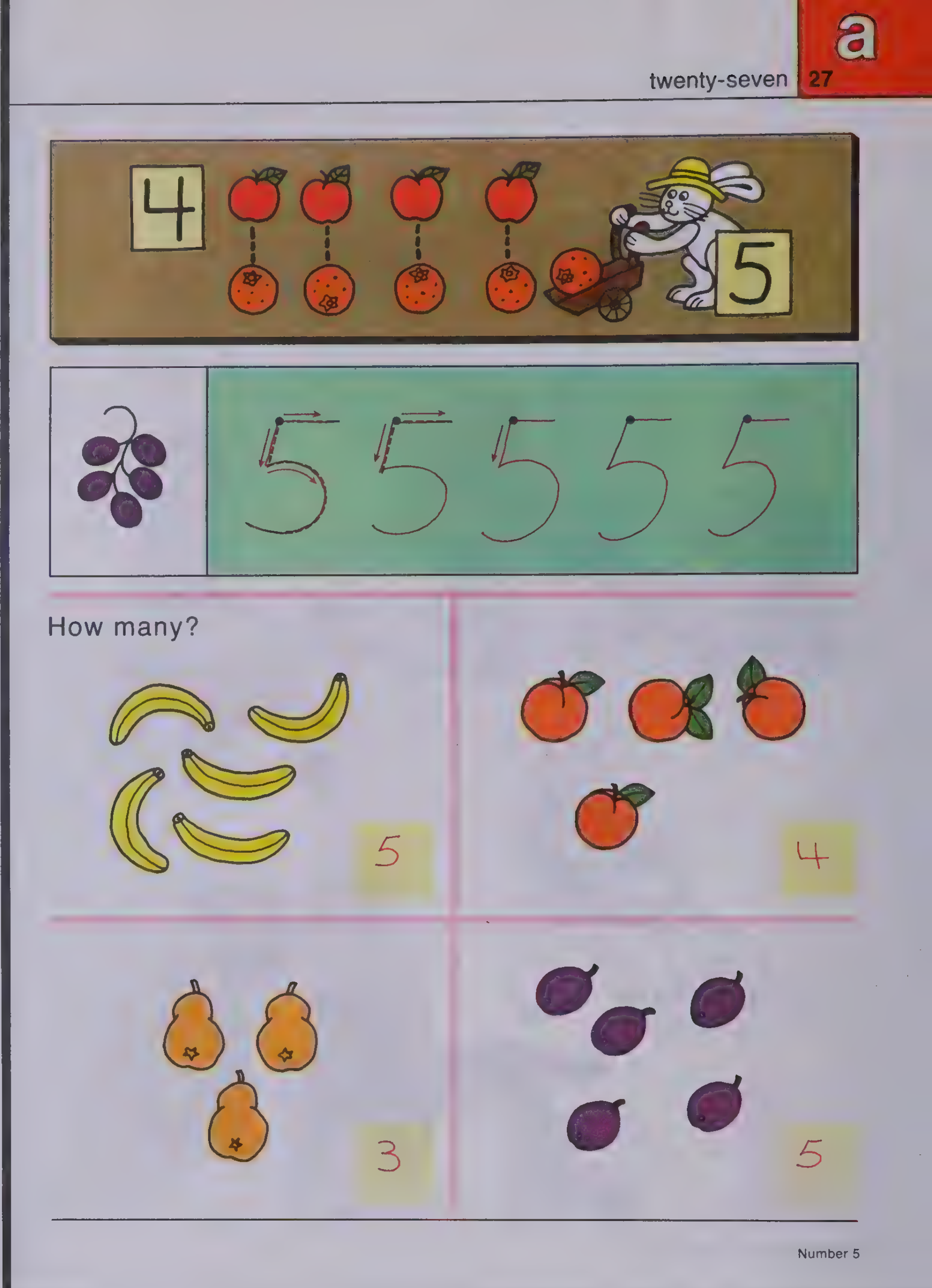
WORKJOBS, "The Mailman," pp. 64–65, Addison-Wesley.

Teaching
Page a-27

For page a-27, ask the children how many apples there are, then tell them to trace the dashed matching lines to compare the set of apples with the set of oranges. Ask, "Are there more apples than oranges?" Emphasize that the set of oranges contains "one more" than the set of four apples and that the number of the set of oranges is five. Point out the numeral 5 at the right.

Explain the formation of the numeral 5. Ask them to look at the dashed line 5 and point out the dot where they should begin their strokes. You might describe their strokes as "down, around, and put on its hat."

Then direct the children to look at the first box and decide how many bananas there are. Tell them to count them if they want. They should then write the correct numeral in the yellow box. Work through as many frames with the children as you decide are necessary. This exercise is continued on page a-28.



Objective

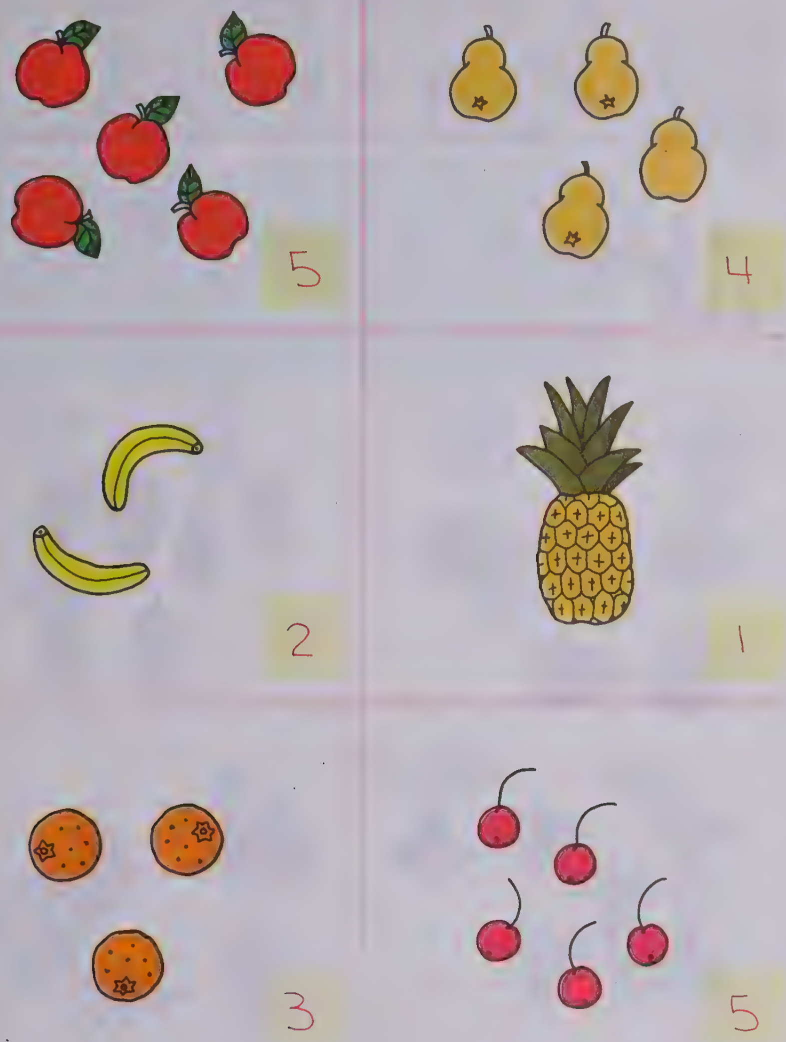
Given a set with five members, the child will be able to identify the number of the set and write the numeral 5.

Children can often identify sets of one, two, three, and four without counting, but as the sets of greater number are introduced, counting will become increasingly more important. Notice that in this lesson the number five is presented as *one more than four*.

Pre-Book Activity

Use a counting activity to begin this lesson. Although many children already know that counting yields the cardinal number of the set, stress this idea here and throughout the other lessons of this module. Use sets of three and four elements to demonstrate counting techniques. Count very slowly and touch each element as you name the number. Give the children an opportunity to count sets of three and four. Display a set of four objects and have them count to four with you. Observe that there are four objects in this set. Now put another object with the set of four, count together to five, and ask the children how many objects there are in this new set. When someone concludes that there are five objects in the new set, explain that five is one more than four. Display the numeral 5.

How many?



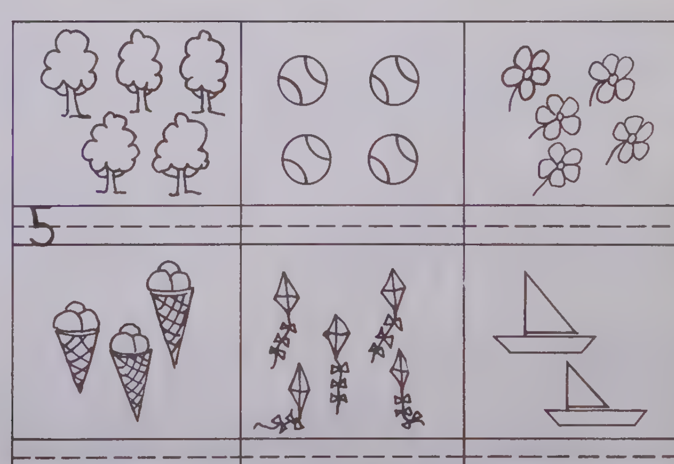
Number 5

Teaching
Page a-28

For page a-28, be sure children understand that they are to do the same as with the exercises on page a-27. Continue to emphasize counting, although for some exercises they will be able to identify the number of the set by visual inspection.

Follow-up

Some children will need further practice in writing the numeral 5. You might distribute a worksheet such as the one shown at the right.

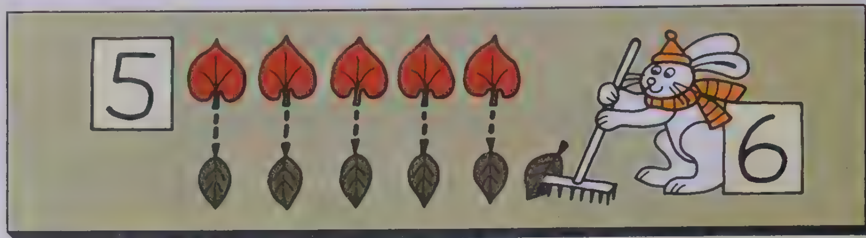


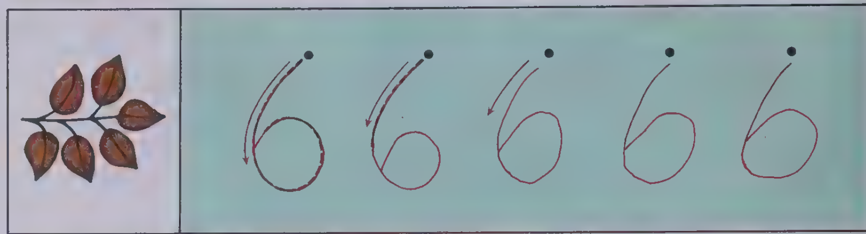
Teaching
Page a-29

Call the children's attention to the two sets of leaves. Ask the children, "How many leaves are there in the set at the top?" When someone says "five leaves," call attention to the numeral 5 beside this set. Ask the children to compare the set of orange leaves with the set of brown leaves by tracing over the matching lines. Observe with them that there is one more brown leaf than there are orange leaves. Ask someone to point out that there are six brown leaves, and call attention to the numeral 6 beside the set of brown leaves.

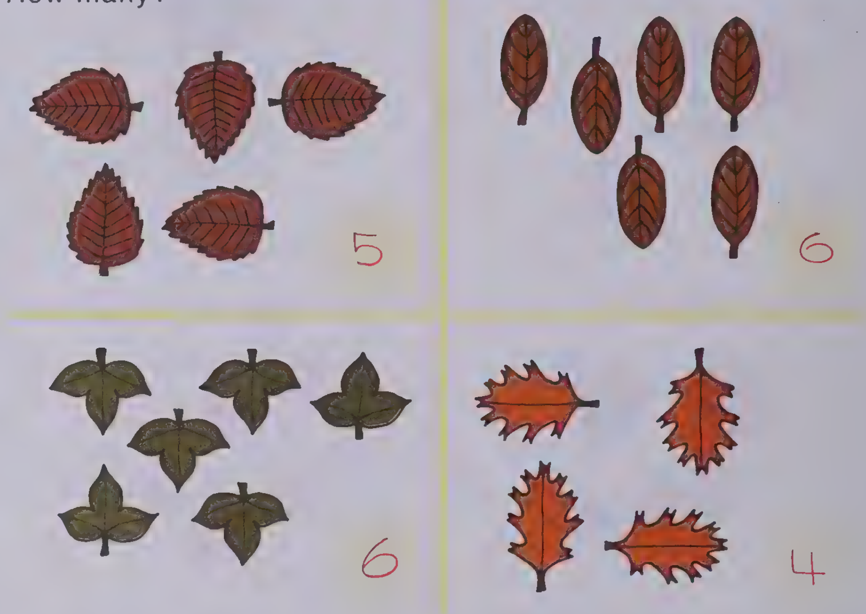
Make the formation of the numeral 6 in the air and help children to do the same. Then call their attention to the dashed line numerals in the text. Ask them to trace them and continue writing sixes across the page.

Then ask the children to look at the first frame with leaves. Ask them how many leaves there are and have them write the numeral in the box. You might also ask: "When you count, what number comes before five." Work through the other frames similarly. With the sets of six, stress that six is one more than five.





How many?



Objective

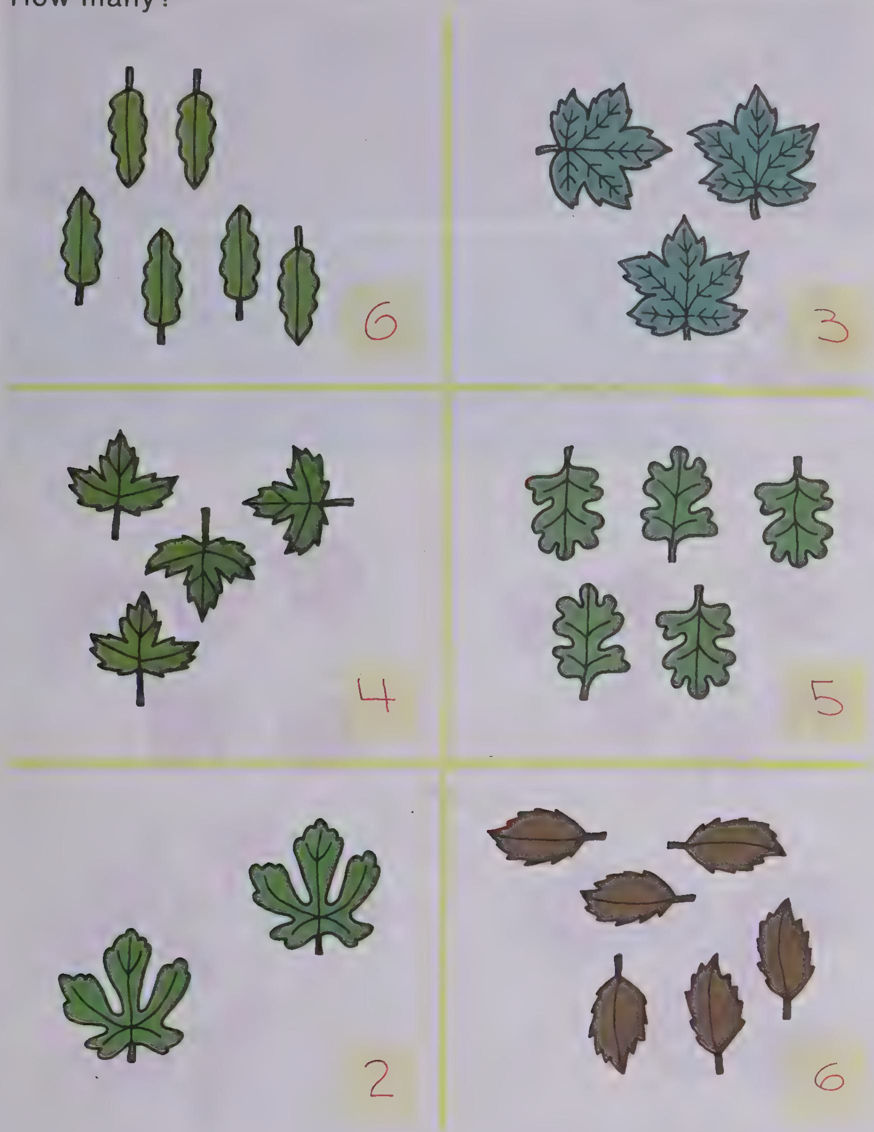
Given a set with six members, the child will be able to identify the number of the set and write the numeral 6.

Just as the number five was presented as *one more than four,* this lesson presents six as *one more than five.* This concept as well as the importance of counting should be stressed.

Pre-Book Activity

Ask the children to count several sets containing five or fewer elements. Use a variety of materials for this: objects on the flannelboard, chalkboard, pegboard, or magnetic board; items in the classroom (chairs, tables, windows); groups of children; and so on. Then display a set of five elements and have the children count them together. When this is completed, and the observation that the set contains five elements has been made, put a sixth element with the set and repeat the counting process. Observe with the children that the set contains one more than five, or six, objects. Display the numeral 6 and give the children an opportunity to count several sets of six around the room.

How many?



Teaching
Page a-30

For page a-30, you might ask one of the children to explain what he thinks they are supposed to do. Since the exercises on this page are similar to the exercises on a-29, he should be able to explain what to do. The counting activity here is more difficult than on the previous pages, so you should move around the class as the children work and help those having difficulty. Suggest to these children that they mark each object as they count.

Follow-up

Make a chart having five rows and four columns on a piece of A-1 (594 mm × 841 mm) tagboard or on the chalkboard. In the first column, draw and color five uncomplicated sets in order from one to five. In the second column, print the word and numeral for the number of each set. In the third column, ask a child to make a set having one more object than the original set. In the fourth column, print the number name of the new set. Ask a child to place the numeral of the set beside the written word. Such an activity brings together the concept of how many in a set, the written word for the number, and the numeral.

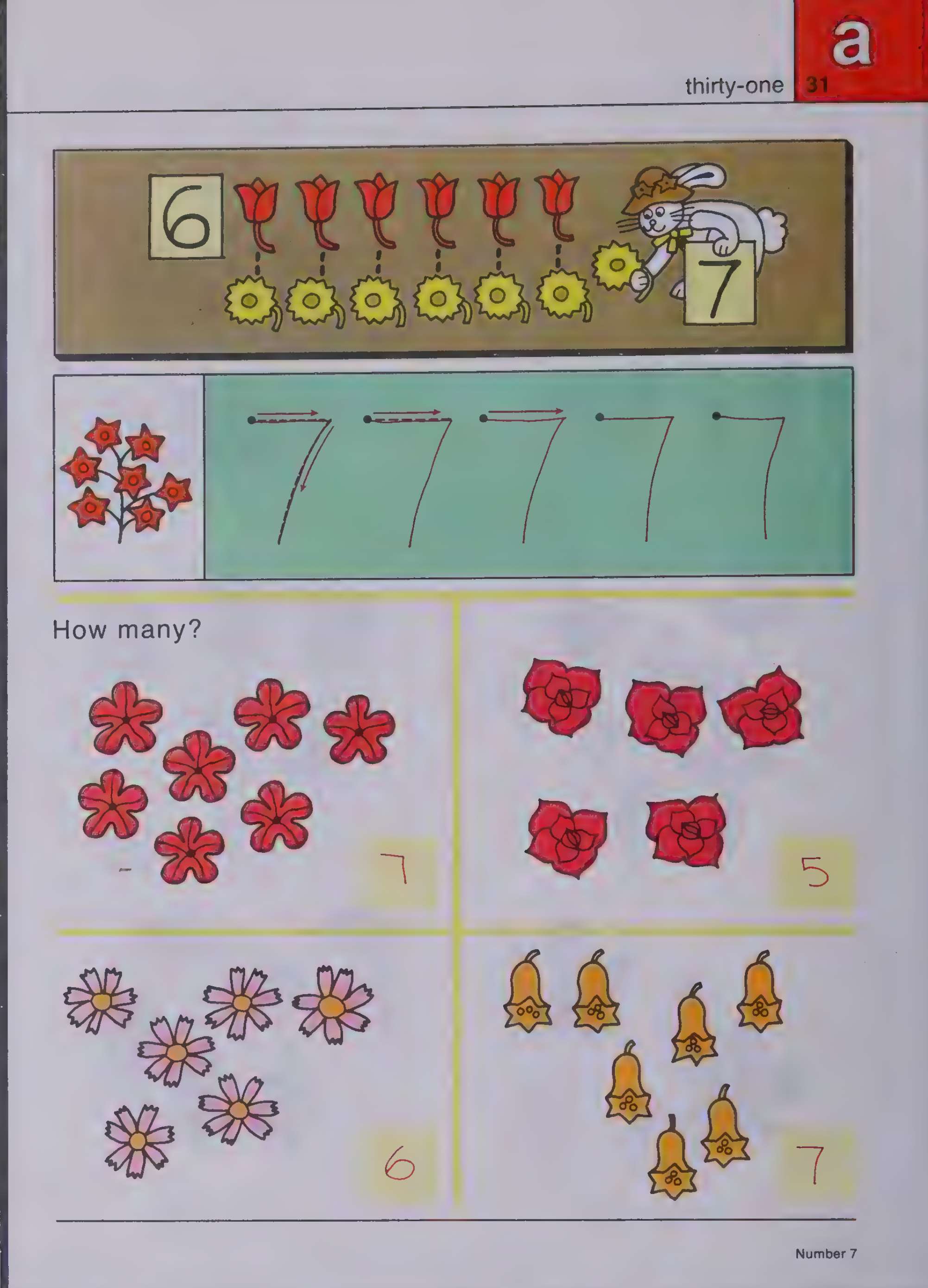
Set	Name	One More	Name
△	one 1	△ △	two 2
	two 2		three 3
	three 3		four 4
□ □ □ □	four 4		five 5

Teaching
Page a-31

Ask the children how many flowers there are in the set at the top. When they determine that there are six, call attention to the numeral 6 beside the set. Tell them to compare the top set with the set of flowers below it by tracing over the dotted matching lines. Observe with the children that there is one more yellow flower than there are orange flowers and that there are seven yellow flowers. Call their attention to the numeral 7 beside the set of seven.

Explain how to write the numeral 7 and guide the children in forming the numeral in the air. Then have them trace the numerals printed on page a-31 and finish the row.

Since the type of exercises in the frames should be familiar to the children, ask one of them to explain what they are to do. Be sure that they understand that the best way to find how many are in these sets is to count. You may want to tell some children to mark each object in a set as they count it. Others might simply point to each object as they count.



Objective

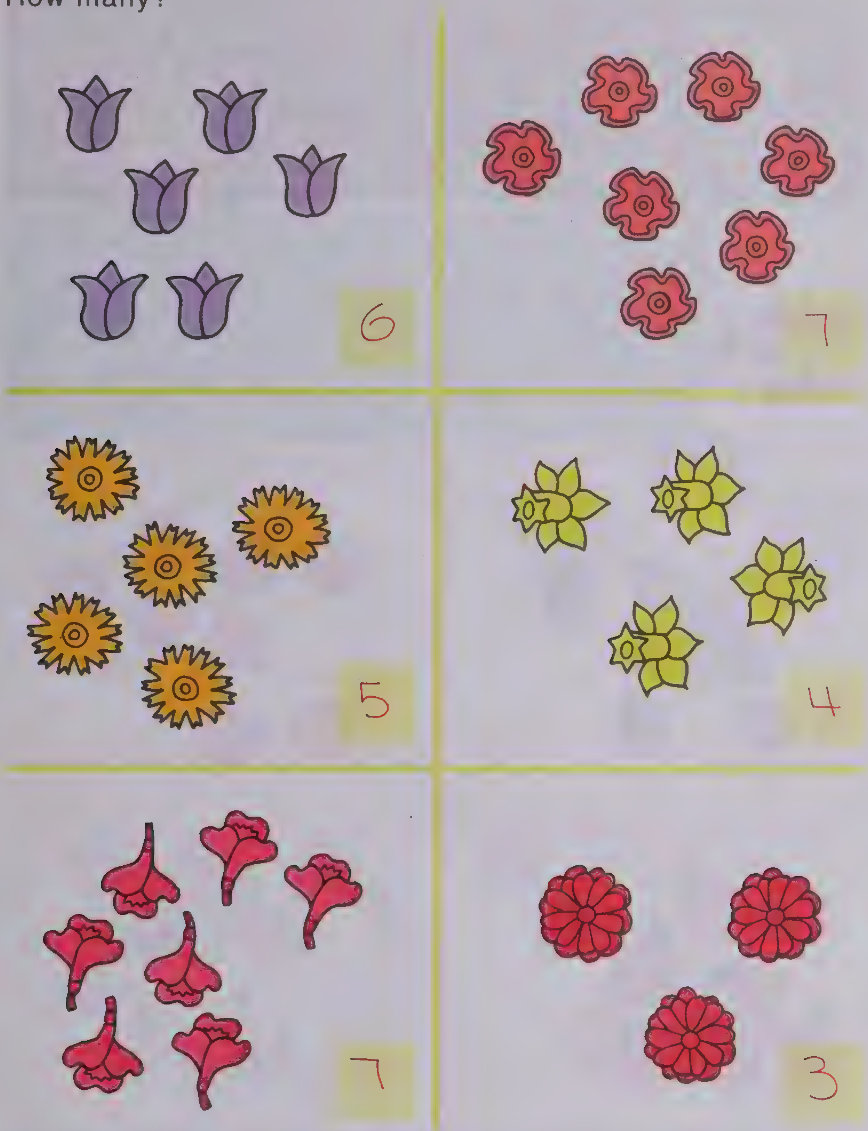
Given a set with seven members, the child will be able to identify the number of the set and write the numeral 7.

Pre-Book Activity

As with the previous lessons, a counting activity is suitable as preparation. Ask the children to look for sets of five and six in the classroom. Then display a set of six objects and have the children count the set together. When they observe that there are six objects in the set, place another object with the set and count again. Observe with the children that there are seven objects in the new set and that seven is one more than six. Display the numeral 7 for the number of elements in this set.

Alternately, you might use an activity such as the following: Prepare felt or paper strips marked in 3-cm squares so that you have at least seven single squares and five strips—one each—of 2, 3, 4, 5, and 6 unit squares. Begin by placing a single unit near the top of the flannelboard. Ask the children, "How many?" After they answer "one," place two single units, one of each color, beneath the original unit and say, "One and one more is how many?" After a child answers "two," place a strip of two units in one color and a single unit in the other color below the first unit, and again ask, "How many?" Continue building up units until you reach six and one, making a stair-step design with each added unit in the contrasting color. (See illustration in the next column.) When you have discussed seven as six and one more, display the numeral 7.

How many?



Number 7

Teaching
Page a-32

Since page a-32 has exercises similar to those on page a-31, children should not have difficulty with them. However, move around the room as they work to guide children when necessary. It is most important that children have the basic number concepts and counting patterns correctly established, so that time spent helping children now will save much more remedial time later.

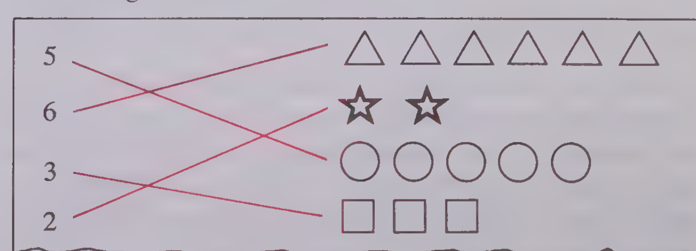
Follow-up

If number books were begun as suggested for page a-24, bring them up to date by adding the numerals 5, 6, and 7 on separate pages and by drawing the corresponding sets. If they were not begun earlier, this would be a good time to start them.

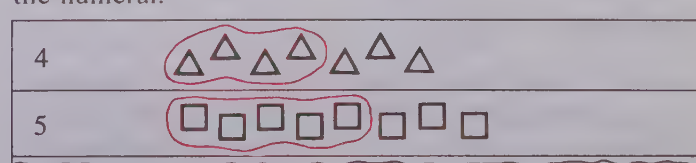
Also add other "number boxes," as suggested for page a-25, for the newly introduced numbers.

Worksheets such as the following might also be helpful. Be sure you give clear directions to the children.

Direct the children to match the numeral with the set containing the same number of items.



Direct the children to ring the number of items shown by the numeral.

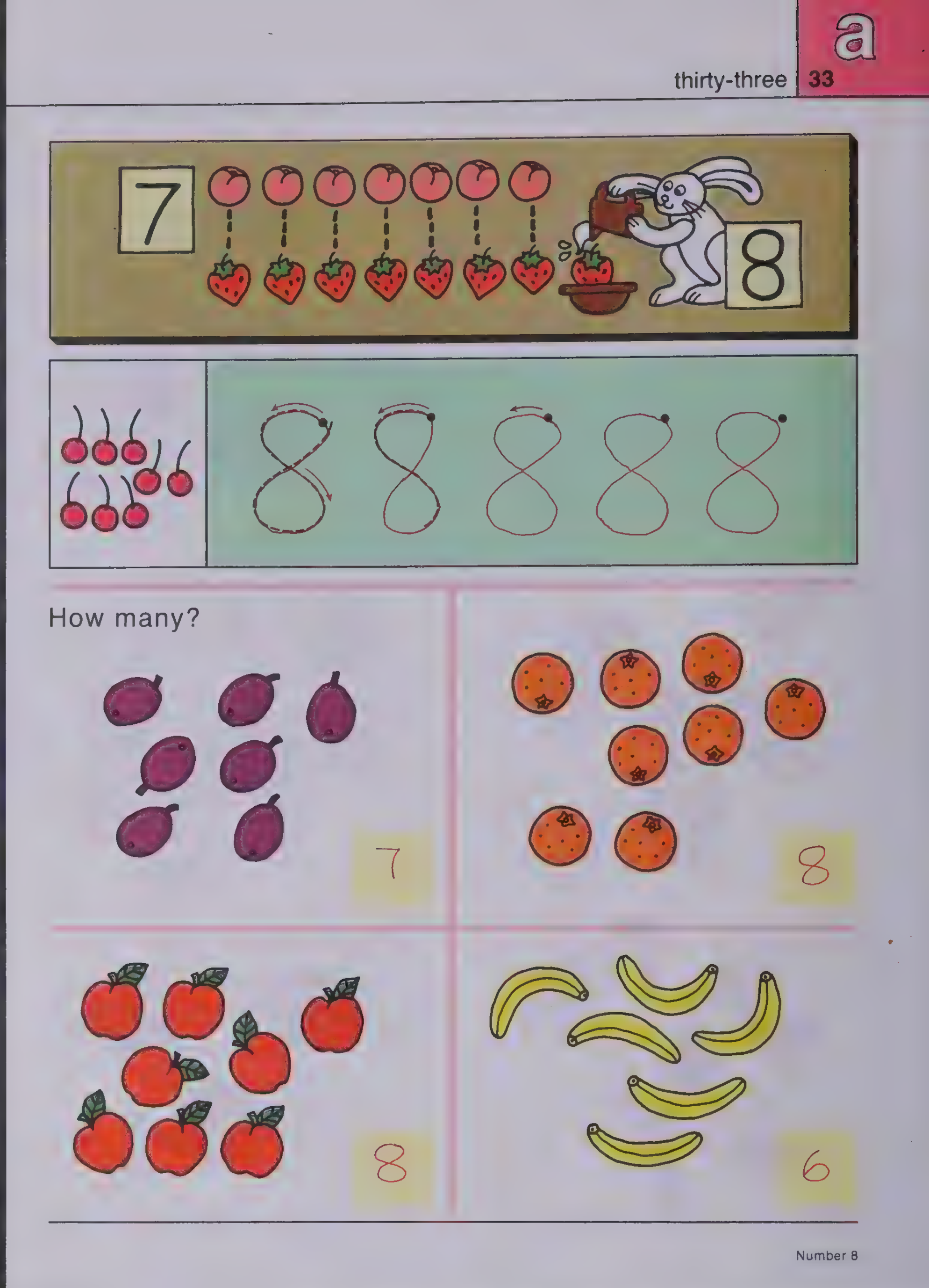


Teaching
Page a-33

Ask the children to find out how many peaches are in the set at the top of the page. When they say "seven," point out the numeral 7 beside the set. Instruct them to compare the set of strawberries with the set of peaches by tracing over the dashed matching lines. Observe that one strawberry is left over. After having a child point out that there are eight strawberries, call attention to the numeral 8 beside the set of squares. Emphasize that eight is one more than seven.

Point out the numeral 8 and guide the children in forming it in the air. You might say: "Over, down, around and up" as you go through the motion of forming the numeral. Then have them trace over the numerals on the page and finish the row of eights.

When the children have made the eights, ask a child to explain what they are to do in the four frames at the bottom of the page. Remind them that they might find it helpful to mark or at least touch each illustrated item as they count.



Objective

Given a set with eight members, the child will be able to identify the number of the set and write the numeral 8.

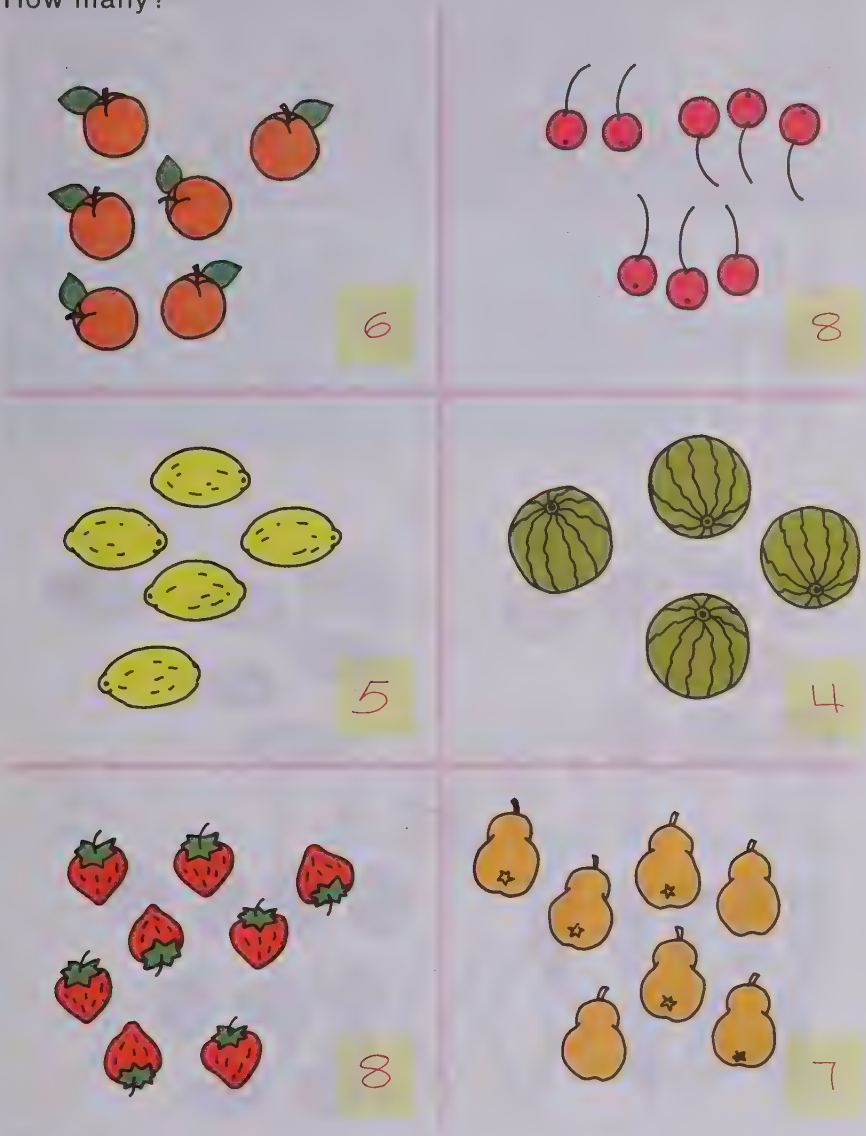
This lesson continues development of the on-going objective of helping children understand the basic number concepts from zero to nine and be able to write the numeral for each number. Children will need continued practice in writing the numerals even though they can correctly identify the number of a set.

Pre-Book Activity

Ask the children to count several sets of five, six, or seven objects. Then display a set of seven and have the children count together as you point to each object. Put another object with the set and count again. When you reach eight, observe with the children that there are eight objects in the new set and that eight is one more than seven. Display the numeral 8 and have the children count several other sets containing eight or fewer objects. You might make 24 A-5 (148 mm × 210 mm) cards out of tagboard (one circle on one card, two on another, three on another, and so on, to eight).

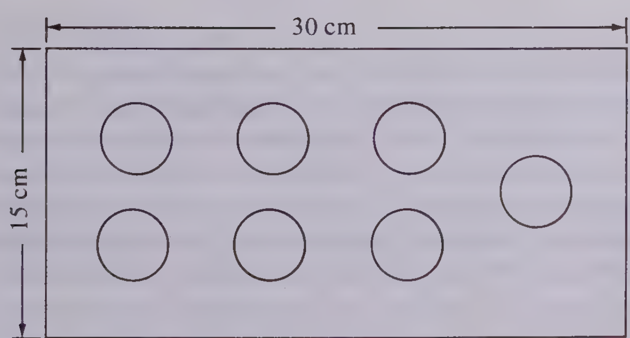
Arrange the cards in random order on the chalk tray in front of the class, and pass felt numerals 1 through 7 out to different children. Ask each child to find a set corresponding to his numeral. Then let the child choose another child and give him the numeral while the teacher reshuffles the stacked cards. Continue until all have participated. (See illustration of a sample card in the next column.)

How many?



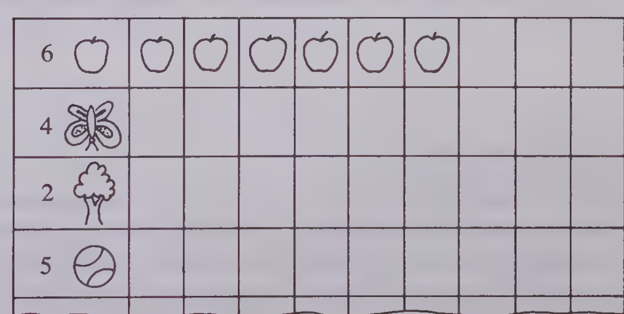
Teaching Page a-34

Instruct the children to continue from page a-33 to page a-34. Since the format of the exercises is the same, they should have no difficulty in understanding what to do. However, move around the room and observe the children carefully in order to help a child correct any misunderstanding he might have.



Follow-up

Give each child a worksheet marked in squares, ten rows down and eight or nine across. Make directions in the form of a numeral and a drawing at the beginning of each row. Alternatively, directions may be placed on the chalkboard. Explain that everyone should draw the number of items indicated by the directions given for each row. (See illustration.)

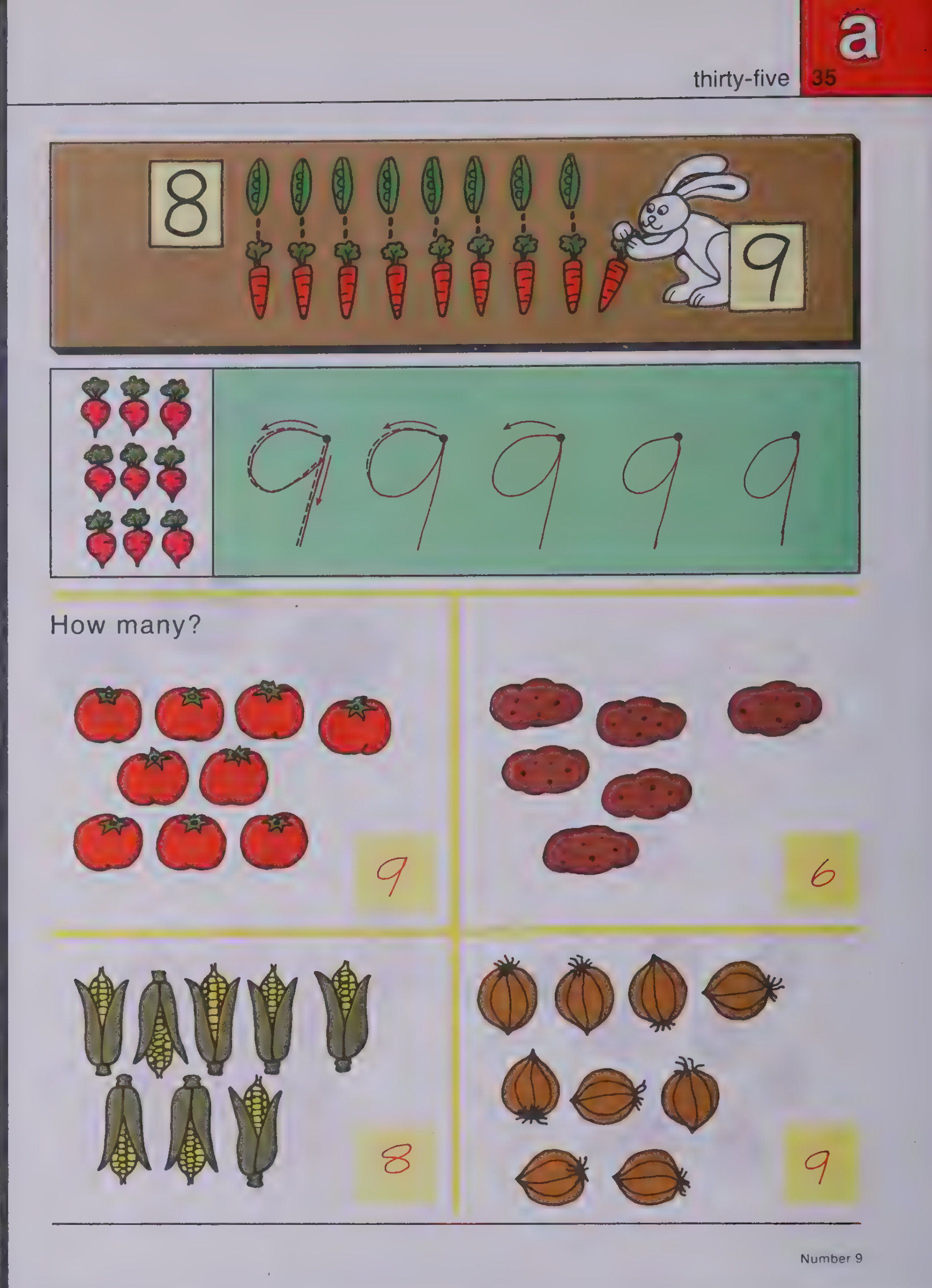


Resources for Active Learning

MATHEX: Numeration No. 2, ". . . Cardinal Number Concept," pp. 6–8, Encyclopaedia Britannica Publications Ltd.

Teaching
Page a-35

Point out the two sets at the top of the page. Ask the children: "How many pea pods are there in the set on the top?" When they respond "eight," call their attention to the numeral 8. Instruct them to compare the set of carrots with the set of pea pods by tracing over the dashed matching lines. Observe that there is one more carrot than there are pea pods, and point out that the number of the set of carrots is nine. Call attention to the numeral 9 beside the set. Then guide the children in forming the numeral in the air before directing them to work on the page. When they have written the 9's on the page, ask a child to explain what to do in the last four frames. Few directions should be needed as most children will understand what to do.



Objective

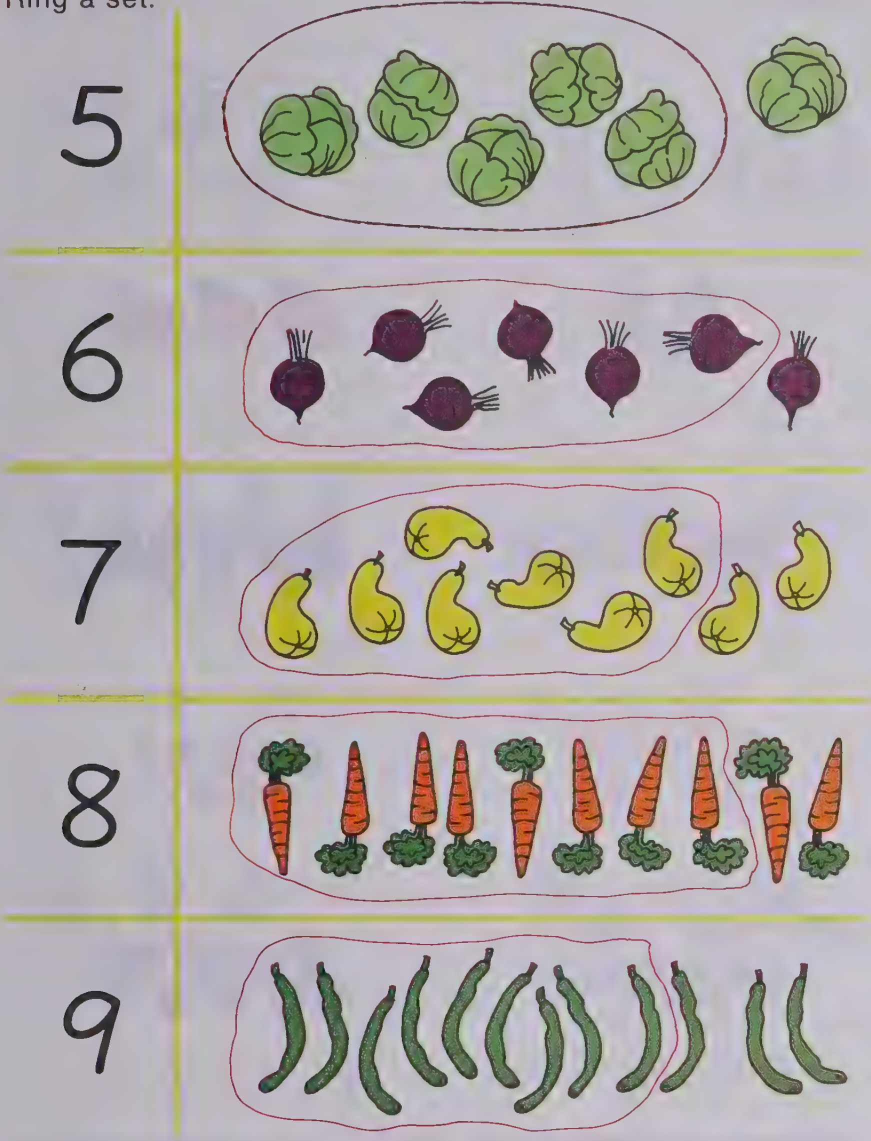
Given a set with nine members, the child will be able to identify the number of the set and write the numeral 9.

Pre-Book Activity

Continue to conduct counting activities as a preparation for this lesson. For example, have the children count several sets of six, seven, or eight elements. Display a set of eight and have children count the elements together. When they determine that there are eight in the set, put another element with the set and have them count again. When you reach the number nine, have one of the children point out that there are nine elements in this new set. Display the numeral 9, and count with the children several other sets containing nine elements or less. Then display a set of eight on the flannelboard or chalkboard, and place a felt numeral 8 beside the set. Ask one of the children to build a matching set by placing another set below or beside the original one. Ask him to add one more to the new set. Call on another child to identify this new set and place the numeral 9 beside it. Continue practicing with other sets containing nine elements or less.

a 36 thirty-six

Ring a set.



Number 9

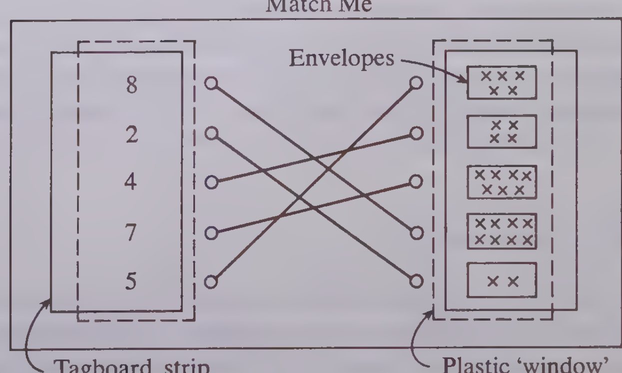
Teaching
Page a-36

Call the children's attention to the numeral 5 and the set of heads of lettuce in the first frame. Instruct them to trace over the dashed ring circling 5 heads of lettuce. When everyone has completed this exercise, be sure they understand that the numeral 5 meant for them to ring five of the items in the frame. Ask them to explain what should be done in the second frame. When a child responds that they should ring the first six turnips, have everyone do this. Some of the children may need to mark each turnip as they count in order to ring the proper number of them. Suggest this to children who seem to be having difficulty. Depending on the ability of the children, have them continue independently with numerals 7, 8, and 9, or work through each of the frames together with them.

Follow-up

A shoestring board is a helpful device to give children practice in matching. Use a piece of heavy cardboard approximately A-2 (420 mm × 594 mm). Punch ten or twelve pairs of holes down the centre of the board about 10 or 12 centimetres apart. Through each of the left-column holes, run a half shoestring and knot it underneath. On the outside column to the right and left of the holes staple a piece of plastic that can be used as a window envelope for prepared strips of tagboard. Then prepare different strips of tagboard to use in "Match Me." For example, on one strip write some numerals (so that they are in line with the holes) and on another strip draw sets matching the numerals. If possible make about ten different sets of these and have the children use them at various times. You can make your strips of tagboard cover a range of topics. In relation to numbers and numerals, you might also have pairs of strips with a numeral on the left matched with a one-more-than set on the right; or numerals matched with number names.

Match Me

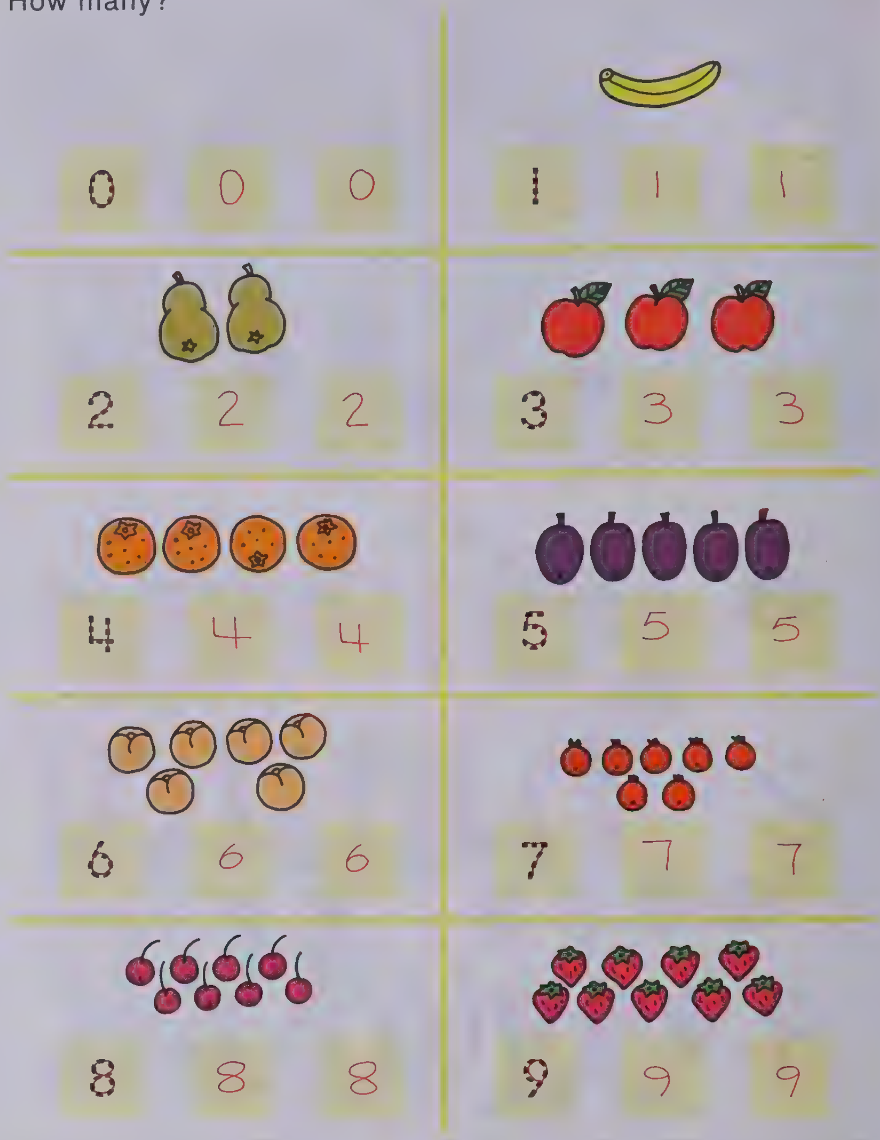


Teaching
Page a-37

Obseve with the children that there are zero objects in the first set. Direct them to start at the top and trace the zero by going left along the dotted line. They should then make two more 0's in the answer screens. If the children continue to answer "none," rather than "zero," when asked how many objects there are in the empty set, ask them to give the number that tells how many are in the set.

Continue by discussing one as the number of the second set in the frame at the top right. Tell the children to trace over the numeral 1 first and then to make two more ones. Ask them to identify the number in the first set of the second frame and then practice writing each numeral. Continue to work through the page. Urge the children to work independently as soon as they understand what to do.

How many?



Objective

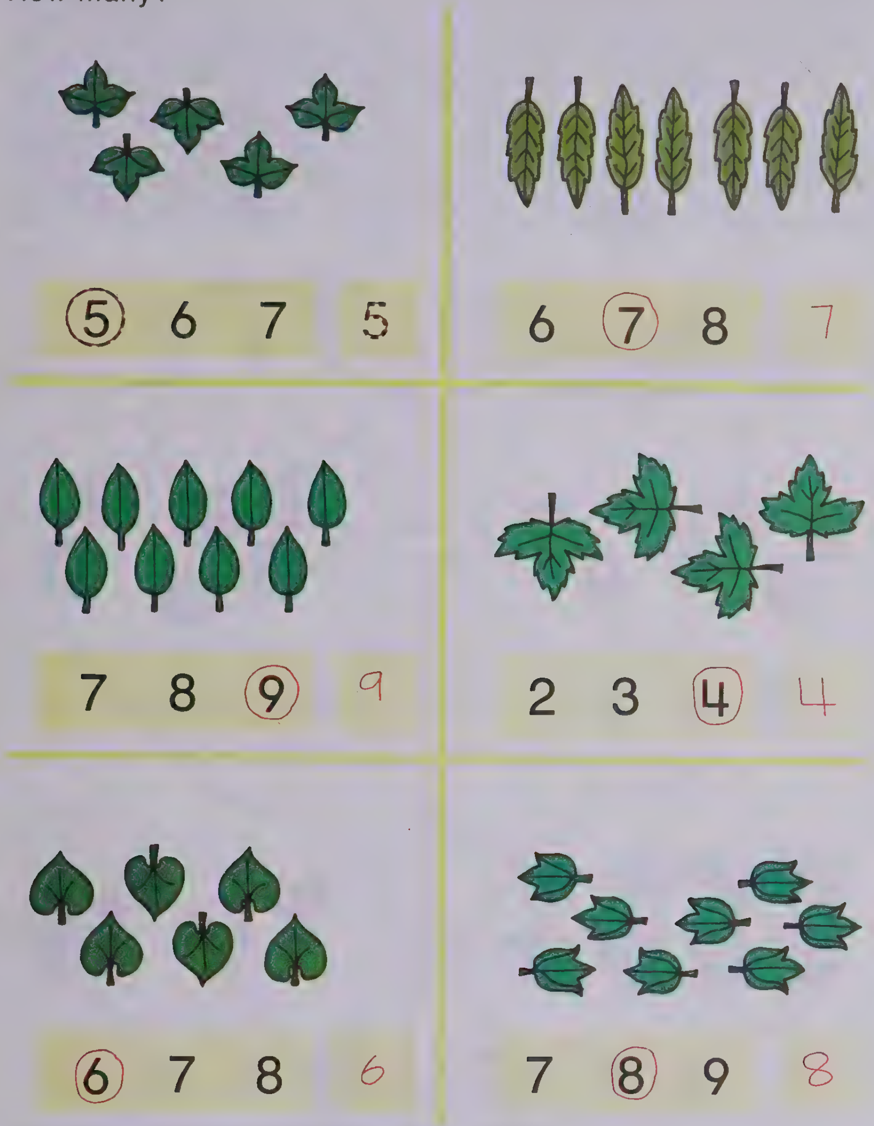
Given sets of zero through nine and corresponding numerals, the child will be able to identify the number of each set and write its numeral.

Both ongoing objectives, identifying numbers of sets zero to nine and writing the numerals 0 through 9 are specifically treated in this lesson. It might be helpful to give other pages similar to these to any children who need further practice.

Pre-Book Activity

Conduct an activity which reviews both numeral writing and identifying sets. For example, show the large cards suggested for page a-33 or display sets around the room on the pegboard, chalkboard, or flannelboard. Display a card or point to a set and ask a child to indicate how many items are in the set by "skywriting" the numeral. Then ask other children to name the number the child has written in the air, or the number of the set you are pointing out.

How many?



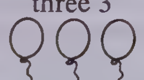
Teaching
Page a-38

Tell the children that for each of the sets on this page they have two things to do. Their first job is to decide the number of objects each set has and to ring the numeral for that number. Their second job is to write the numeral in the shaded box.

Point out the set of leaves in the top row. Ask the children how many leaves there are in this set. When someone says "five," instruct all the children to ring the numeral 5. Then have them write the numeral 5 in the shaded box. Ask the children how many leaves there are in the set in the second frame of the first row. When someone says "seven," have them ring the numeral 7 and then write the numeral 7 in the shaded box. Tell the children to complete the page in this way.

Follow-up

To give the children more practice, duplicate a number worksheet (see below) or draw one on the board. Write the word-name for the number, and a sample numeral, at the top of each section; then direct the children to draw a set having the indicated number of objects and to practice writing the numeral.

three 3  3 3	six 6	eight 8
five 5	seven 7	four 4

Resources for Active Learning

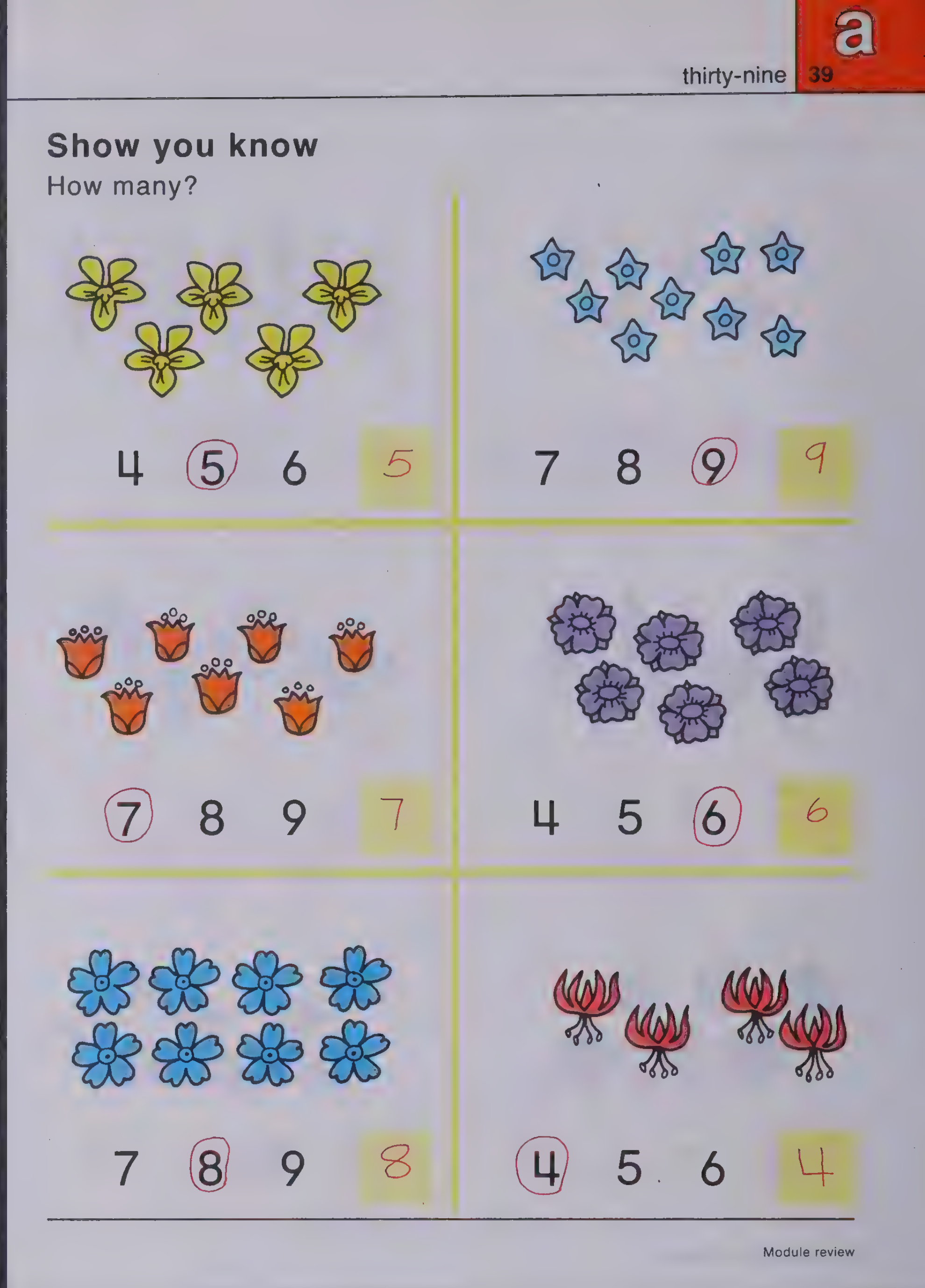
EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Counting," Nos. 12–15, Responsive Environments Corp.

Look to the orange module of Unit A, page a-22, for games that provide practice in recognizing numerals and their number values.

MATHEX: Matching and Graphing No. 1, "Game 5," p. 7, Encyclopaedia Britannica Publications Ltd.

Teaching
Page a-39

Since this page is similar to page a-38, children should not have difficulty in understanding what to do. Ask a child to explain what is expected. Be sure they realize that they should first circle the numeral according to how many are in each set and then neatly write the numeral in the space provided. Encourage the children to work the page independently, but move around the room and give help to any child who needs it.


thirty-nine 39 a

Show you know

How many?

4 5 6 5

7 8 9 9

7 8 9 7

4 5 6 6

7 8 9 8

4 5 6 4

Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

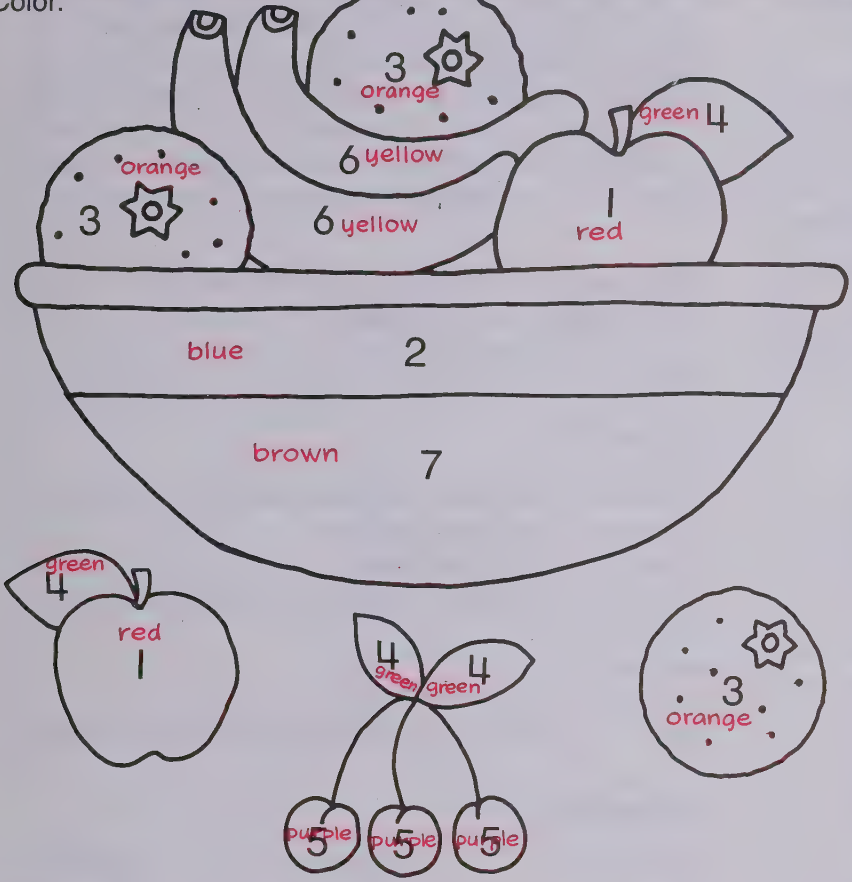
Although this page is presented as a review of the topics treated in this module, evaluation of a child's understanding of them should be based on more than his work on one page. However, any child who cannot identify the number of the sets on page a-39 should be given special help. Many children will need continued practice in numeral writing before they form their numerals correctly.

Pre-Book Activity

Conduct a matching game using pairs of matching sets. If you don't already have cards or paper plates which could be used, use eighteen A-7 (74 mm × 105 mm) tagboard cards to make nine pairs of matching sets. One card of a pair may show one boy, for example, whereas the other card for the pair might show one boat. Another pair might show two trees and two children. Prop all these cards up in random order along the chalk tray. Hold up one card showing, for example, five objects. Ask, "How many?" When someone answers "five," ask if someone else can find the card having the matching set. Place the matched cards side by side on the chalk tray, and ask a third child to write the corresponding numeral on the chalkboard over the pair.

Let's have fun

Color.



Numbers 1–9

TEACHING
Page a-40

Ask children what they think the numeral above each crayon means. Help them understand that here the matching of a number to a crayon means that every time they see that numeral in a region they should color the region with the corresponding crayon. Keep in mind that this page should be handled with a light touch. It simply gives the children an opportunity to match numerals and follow a color code.

FOLLOW-UP

With the addition of a pair of cards showing the empty set, the set cards or plates used in the pre-book activity will provide 20 ready-made sets containing nine or fewer objects. You may wish to write the numeral on the back of each card. Shuffle the cards and display them in random order. Call on a child to give the number represented on each card.

Another activity you might suggest is that pairs of children play a card game like "Fish," using the ace through nine only. The players should start with seven cards apiece, with the rest of the cards in a pile between them. The first player selects one of his own cards and asks his opponent for one to match it. If his opponent holds it he must give it over and then the first player may ask for another card. If his opponent does not have the card requested, the first player then "fishes" the top card off the pile of remaining cards. Every time a player can make a matching pair he places those two cards down in front of him. The player who finishes with the greatest number of matching pairs wins. Help the children adapt these rules as necessary.

Slower children may benefit from matching the cards by themselves, without hurrying.

RESOURCES FOR ACTIVE LEARNING

MATH ACTIVITIES, Materials 2/51, p. 44, Allyn and Bacon

THINK AND COLOR, "Displacements and Rotations," pp. 6–23, Educational Science Consultants

LIGHT GREEN MODULE, UNIT A

More, Less, and Order

Pages a-41 to a-50

General Objectives

To provide further practice in identifying the number of a set containing nine or fewer objects

To introduce the number line

To develop concept of the order of the numbers zero through nine

To extend concepts of more and less

To provide further directed practice with numeral writing

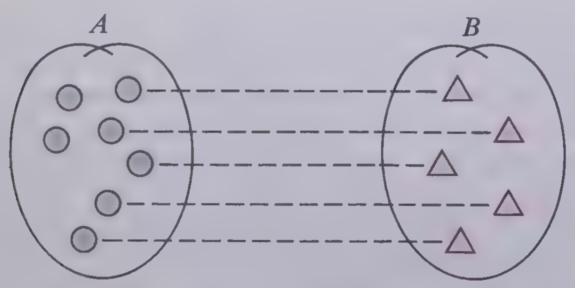
One of the main points of this module is for children to gain a better idea of the relationships between the numbers they have been studying. The first few pages deal with concepts of more and less.

Mathematics

The idea of one number being greater than another is intuitively obvious for the set of whole numbers. A precise mathematical definition of "greater than" becomes vital, however, when one is working with more complicated sets of numbers, such as integers or rational numbers. The definition of "greater than" for cardinal numbers is as follows:

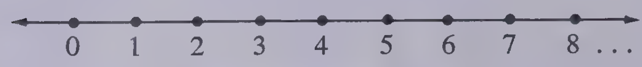
> The cardinal number of set A is greater than the cardinal number of set B if and only if A has a proper subset that is equivalent to B.

The figure, which compares the numbers seven and five, illustrates how this definition agrees with our intuition about one number being greater than another. Consider sets A and B, from cardinal numbers seven and five, respectively.



Notice that the subset of shaded circles is equivalent to set B. Further, the subset of shaded circles is a proper subset of set A because it does not contain all of A. According to the definition, we claim that seven is greater than five. If we were asked to give an intuitive explanation of why seven is greater than five, we would simply point out that a set of seven has more objects than a set of five.

The order of the whole numbers is shown geometrically by establishing a correspondence between the whole numbers and points on a line.



The Whole Number Line

This can be done by selecting a unit and "starting" point on the line. The whole numbers are then associated with points on the line as follows: Zero is associated with the "starting" point, and the whole numbers following in order are associated with points one unit apart on a given side (usually the right) of the zero point. Thus the number of units of length from zero to the point for any whole number is equal to that whole number.

Teaching Light Green Module, Unit A

Approximate Time: 5 to 7 days

Materials

clothespins (9)
counters
flannelboard, felt numerals, and other felt objects for comparing sets and numbers
number line for demonstration
objects for set demonstrations and manipulation by the children
paper bags (9)
twine, about 2 or 3 metres long
numeral and set cards as used in the previous modules

Vocabulary

as few as	less than	numeral
as many as	more than	order
fewer	number line	sequence
greater		
greater than		

The major ideas introduced in this unit centre on the concepts of inequality and order.

The number line will frequently be used as an aid in understanding order and in presenting future ideas. Note that the arrows on either end indicate that the line continues indefinitely in both directions.

The idea of inequality is intuitively obvious to children. The ideas of *greater than* and *less than* should be restricted to numbers and should not be used to compare sets. For example, if the term *greater than* is used to compare sets, certainly one would have to claim that a set containing two elephants is greater than a set containing three marbles. Therefore, keep in mind throughout the work on inequalities that *greater than* and *less than* are relations defined for numbers and not for sets. If necessary, use the matching method with actual objects to

have children verify their choice of one number being greater than another, but continue to stress the idea of comparing numbers of a set, not sets. Most children will have less difficulty with the phrase "more than" or "greater than" so do not belabor the use of the "less than" terminology.

Evaluation of Progress

In judging the children's understanding of inequalities and order, keep in mind that this is an introduction. You should not expect mastery of the ideas or vocabulary at this time; they will be emphasized frequently throughout the book.

Resources for Active Learning

General Activities:

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Natural Numbers," No. 20 (Teacher's Guide, "Application to other activities," pp. 37–40), Responsive Environments Corp.

Information about graphing. Begin now and use the ideas throughout the year:

MATHEX: Matching and Graphing No. 1, pp. 16–25, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: PICTORIAL REPRESENTATION $\boxed{1}$, Wiley

MATH ACTIVITIES, Activity 2/1, p. 25, Allyn and Bacon

Sets, counting, ordering:

WORKJOBS, pp. 129–145, 156–157, 162–179, Addison-Wesley

Manipulative Devices:

Cubical Counting Blocks (Milton Bradley, school supplier)

Number lines (school supplier)

Parquetry pieces (Ideal; school supplier)

Commercial Games:

Hi-Spot (Educational Playsystems)

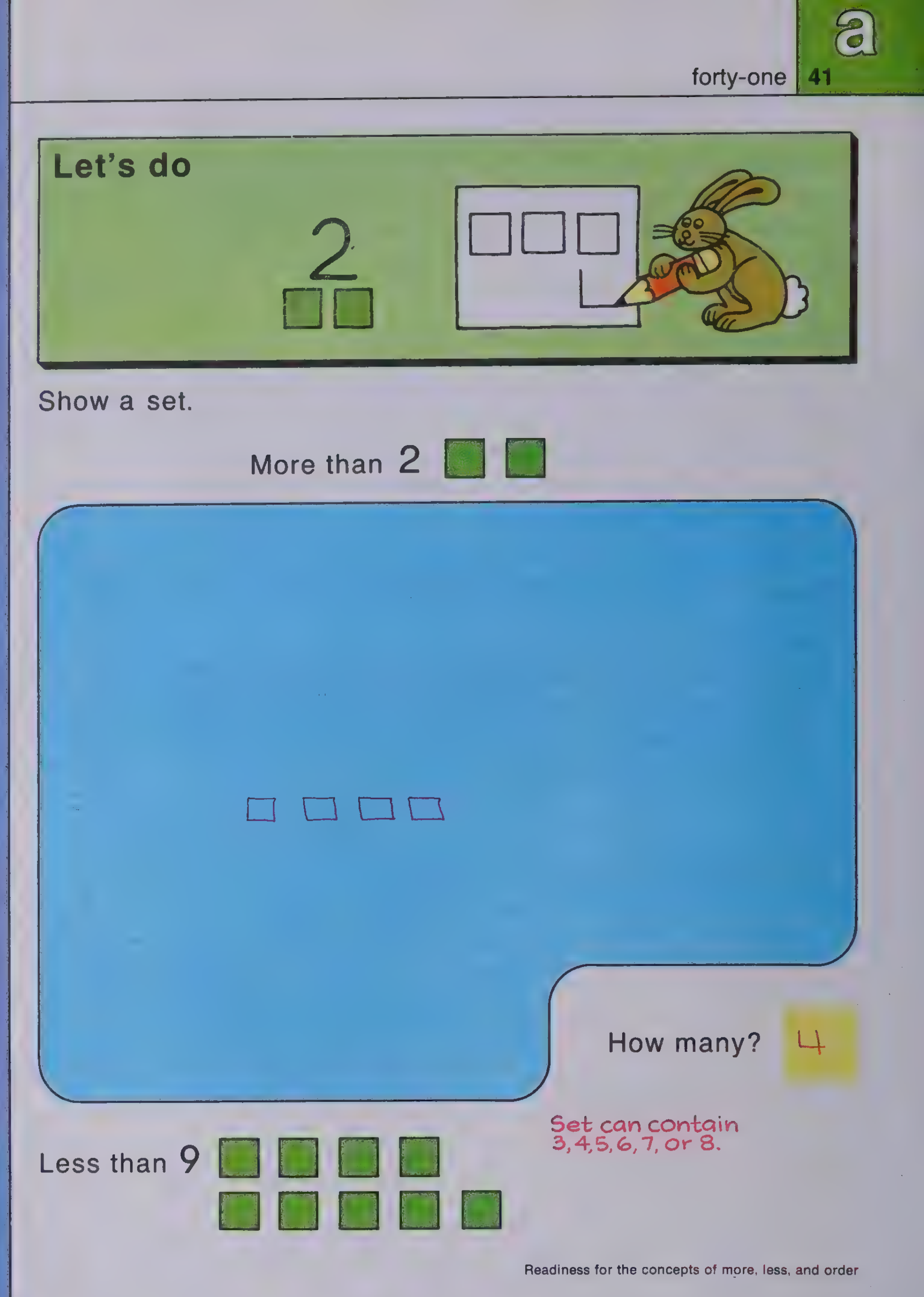
One, Two, Three, Think! (Selective Educational Equipment)

Tri-Ominos (Educational Playsystems)

Investigation
Page a-41

If you have followed the suggestions in the preparation, the child has been building sets according to your directions. Now he is to follow directions from the page. Explain to the children that the phrases "more than 2" at the top of the page and "less than 9" at the bottom are directions for building a set. Help them understand that they are to build some set that is more than 2 and less than 9. The large blue space should serve a twofold purpose. First, explain that it is provided as space on which the child can show his set with counters. Then he should draw his set in the blue space and write in the yellow box the numeral which tells how many he has in his set.

Note the element of choice in this investigation. Have the children share their answers or discuss them generally. With some children, you might have each child build as many different sets as he can. In this case, you might want to provide him with other paper on which to build and record these other sets. Be sure they realize that sets of 3, 4, 5, 6, 7, and 8 are all acceptable but sets of 0, 1, 2, and 9 are not. It would be helpful to list the correct answers on the chalkboard.


a

forty-one 41

Let's do

2

Show a set.

More than 2

How many? 4

Set can contain 3, 4, 5, 6, 7, or 8.

Less than 9

Readiness for the concepts of more, less, and order

Purposes

To introduce the concept of "more than" and "less than" for the numbers zero through nine.

Preparation

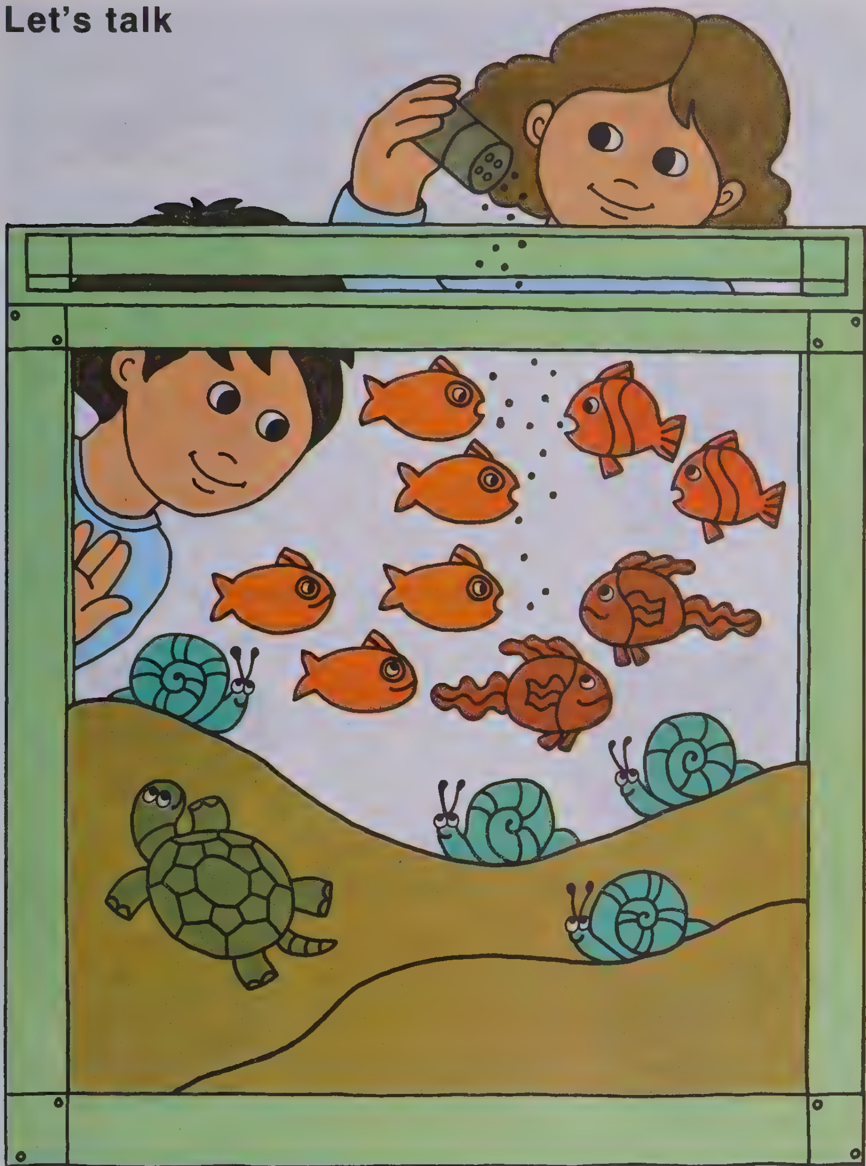
Materials

at least 9 counters for each child

To prepare for this lesson, you might simply have children build sets with their counters according to numbers you give them. For example, say: "Use your counters to make a set of 5," or "Use your counters to make a set of ____," and write the numeral on the chalkboard. Include a few examples wherein you say: "Use your counters to make a set of less than 7," or "Use your counters to make a set of more than 3." Finally distribute the book pages and direct the children in how to use them.

a 42 forty-two

Let's talk



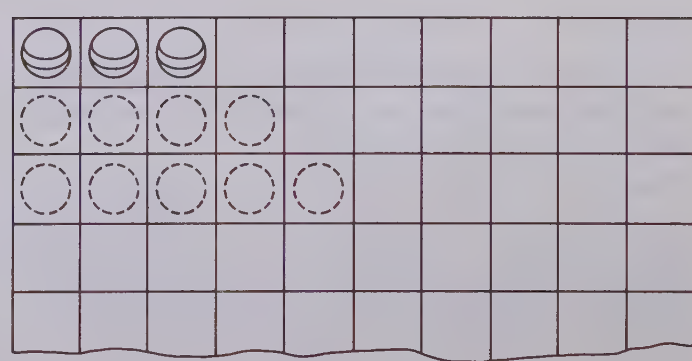
Readiness for the concepts of more, less, and order

Discussion
Page a-42

Ask the children to look at the picture and describe what they see. When they mention a particular item, such as fish, ask them to tell how many fish there are. Then ask questions such as "Are there more fish than snails?" "Are there more children in the picture than turtles?" Emphasize the "more than" concept also using the phrase "greater than" as in "Is the number of snails greater than the number of turtles?" When possible work in the phrases "fewer" or "less." Some children have greater difficulty with the "less than" concept so develop it gradually. You may want to use the phrases "not as many as" or "fewer" along with "less than." You might also review the concept of zero by asking how many pink elephants are in the picture, or if there are birds in the picture.

Follow-up

The following activity, which the children can do at their desks, will help them understand the concept of order. Give them crayons and duplicated sheets showing a grid pattern with the first row of balls drawn in (see illustration). Ask the children to look at the first row carefully and draw as many balls in the second row as there are in the first, and then add one more. Instruct them to continue this pattern to the end, adding a ball to each row and using a different color of crayon for each row.

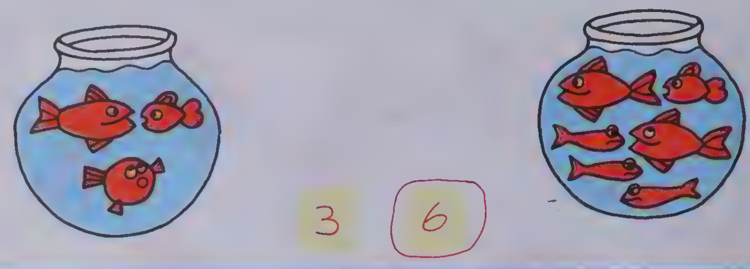


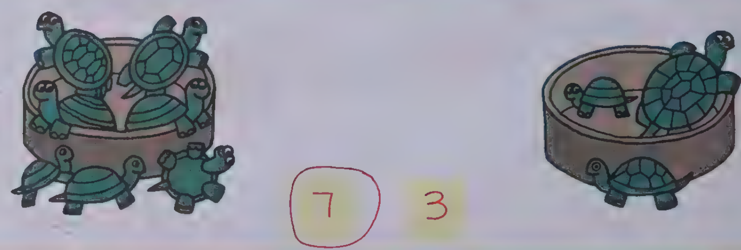
Teaching
Page a-43

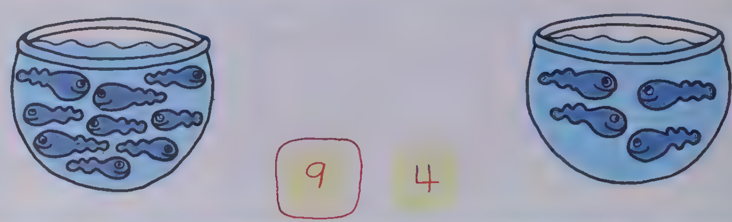
Ask the children to look at the two sets of frogs at the top of the page. Have a child explain why the rabbit has written the numeral 2 in the yellow box on the right and 5 in the yellow box on the left. Then ask: "Which set has more frogs, the one with 2 or the one with 5?" When children agree that the set with 5 has more, point out that the numeral 5 is being ringed. You might say: "Since the set with 5 frogs has more than the set with 2 frogs, we ring the numeral 5 and we say, 5 is greater than 2." You might ask the children to finish the ring around the 5 by tracing and completing the ring begun for them. It would also be helpful to point out that the directions might read "Ring the greater" and we can say: "5 is greater than 2" or "5 is larger than 2."

The amount of direction you give for the remaining pairs of sets will depend on the individual needs of the children. For some it would be best to work through each pair in the manner suggested for the set of frogs. Other children, however, should be encouraged to complete the page on their own.

How many? Ring the larger.







Concept of greater

Objective

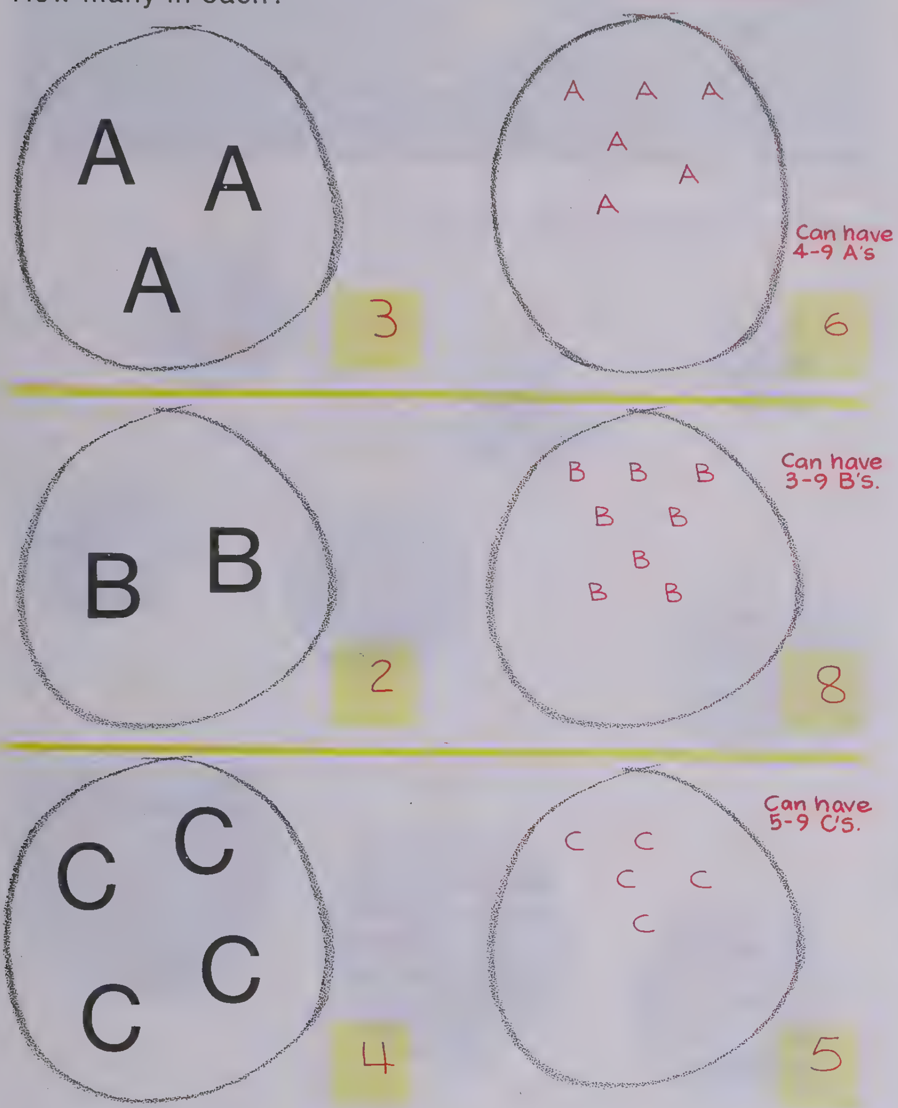
Given two non-equivalent sets, the child will be able to tell the number of each, write the correct numeral, and identify the greater number.

This lesson focusses attention on one number being greater than another. Sets are used to help the child relate comparison of numbers to concrete objects. The mathematics section at the beginning of this module explains the "greater than" concept developed here.

Pre-Book Activity

Give the children many opportunities to compare sets. For example, have available a variety of items. Ask a child to make a set for all to see and write its number on the chalkboard. Then ask another child to make a set which has more (or less, fewer) items in it and write the number for his set on the chalkboard. Ask the children to compare the two sets and have one of them ring the numeral of the larger set. Use phrases such as "The set with 6 has more objects than the set with 3," "6 is greater than 3," "If you have 6 you have more than if you have 3." Include phrases using *less* or *fewer*, such as: "The set with 3 has fewer members than the set with 6." But don't belabor the *less than* phrases. Many children find this concept more difficult than *more than*.

Draw a set that has more.
How many in each?



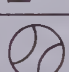
Concept of greater

Teaching
Page a-44

For each of these frames, the children should decide how many letters are in the given set, write the correct numeral, and draw a set that has more. If you wish to guide the children through each frame, do so in a manner which allows the important element of choice to remain. For example, ask the children to look at the set of A's and write the numeral which tells how many A's are in the set. Then ask them to draw a set which has more. Note that the items in their set need not be A's or even other letters, they may be anything the child can draw in the circle. Also the child can choose to have any number greater than three. When they have finished they should write the numeral for the number of items in their sets in the yellow box on the right. When the children have drawn their three sets, have several children show how many are in their set on the chalkboard or flannelboard. If you first build one of the sets in the text on the chalkboard or flannelboard and then have a child build his set next to it, all the children should benefit from discussion of the various choices made by different children. Continue to use phrases such as: "The set with 4 has more members than the set with 3, so we say 4 is greater than 3."

Follow-up

Duplicate a sheet on which the children can draw sets with objects either less than or more than a given set.

Greater than	Set	Less than
	△△	
	○○○	
	□□□□	
		

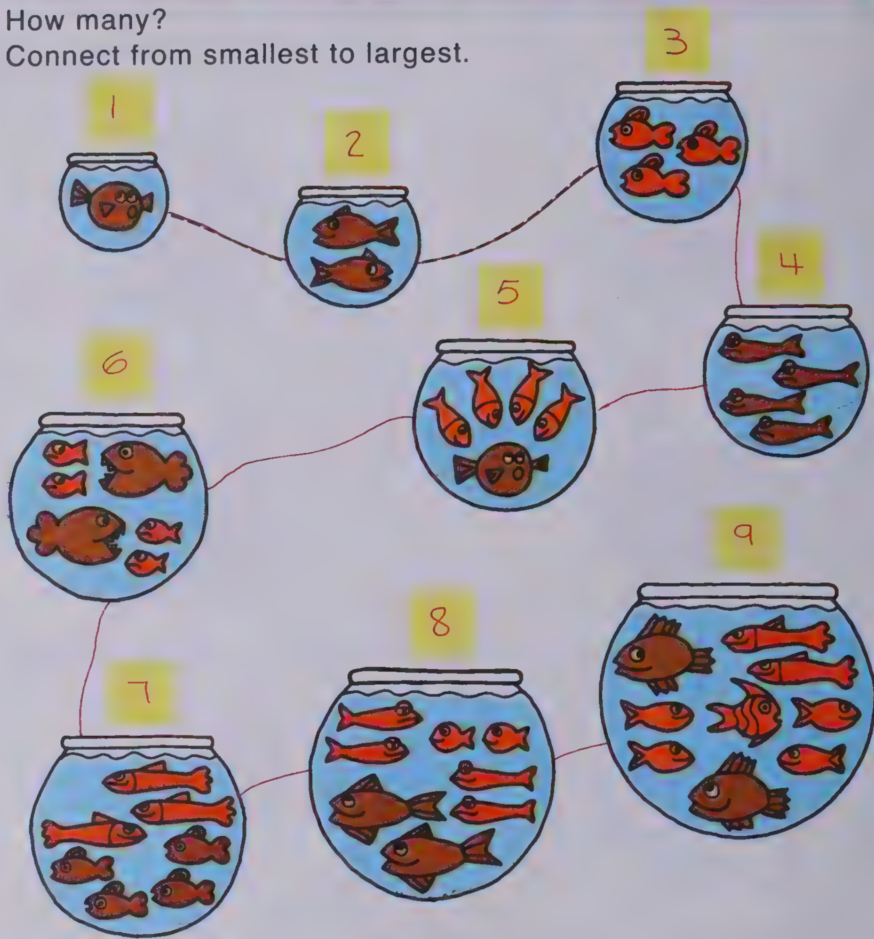
Resources for Active Learning

DEVELOPMENTAL MATH CARDS, A#15, Addison-Wesley

Teaching
Page a-45

Use the demonstration art at the top to explain the directions. Ask the children to look at the fish bowl with one fish and have a child explain why the numeral 1 has been written above it. Do the same for the next bowl with the two fish. Then ask a child how many fish are in the bowl that is connected to the second bowl and have them write 3 above it. Help them realize that after they have identified the number in each bowl and written the correct numeral, they should continue the dotted line so the path is in order from 1 through 9. Supervise this connecting carefully. Point out that they are putting the numbers in order and that the sets along the path get larger by one fish. Also have the children read the numbers in order as they have connected them.

How many?
Connect from smallest to largest.



Order of the numbers 1–9

Objective

Given sets and numerals for the numbers one through nine, the child will be able to order them.

Since children have been counting in order to find how many members a set might have, they are accustomed to putting the numbers one to nine in sequence. In this lesson, their attention is focussed directly on this concept of order. Again sets are used to help the child relate the order of the numbers to sets of concrete and illustrated objects.

Pre-Book Activity

To introduce the concept of order, you might use the following activity. Put different numbers of counters into nine bags so that you have sets from 1 through 9. Arrange these in random order on a demonstration table. Have children find out how many counters are in different bags. For example, point to a bag and ask "Who can tell us how many are in this bag?" The children will first have to count the counters to find out. But when you point to a bag for which someone has already counted the counters, ask the child if he can remember how many counters it held. Even if one or two children can remember how many are in a particular bag, ask "Can anyone think of a way to set up these bags (without labelling them) so someone could easily figure out how many counters are in each?" If children are slow to suggest that they could arrange them in order from 1 through 9, arrange three or four bags yourself, then continue to ask children if they can tell how many are in each. Let those who catch on put the remaining bags in order. Continue to point to the bags, now in order, until children realize that they can

Color the correct number.

Order of the numbers 1–9

Teaching
Page a-46

Ask the children to look at the top of the page. Point out to them how each numeral has a column of squares below it. You might also ask them to count the number of squares below one of the numerals. Then explain that the numerals above each column tell how many of the squares should be colored. They should try to match the colors as much as possible, but do not over emphasize the coloring. When they have finished, ask them if they notice anything interesting about how the columns have been colored. Be sure they realize that if they colored the columns correctly, each column has one more square colored than the column before it, and that the numerals at the top are in order.

simply count from the first bag to the bag which you are pointing to, and the number of their counting will be the same as the number of counters in that bag. Notice that the counting or cardinal numbers are stressed here; it is not necessary to refer to the position of the bags with ordinal numbers unless it comes up naturally.

Follow-up

Distribute numeral cards about A-4 (210 mm × 297 mm) in size so that every child has one of the numerals 1–9. Ask everyone who has a card that names the number that comes right after 4 to stand up and show their card. Or ask anyone who has a number greater than 7 to stand up. Or ask the children to stand up as their number is called and give out various sequences such as 3, 4, 5; or 5, 6, 7; or 7, 8, 9. Develop games of your own. Occasionally have children exchange cards so they have a chance to respond with a variety of numbers.

Resources for Active Learning

DEVELOPMENTAL MATH CARDS, A$^{\underline{1}}$8, 18, A$^{\underline{4}}$20, Addison-Wesley

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, No. 16, Responsive Environments Corp.

MATH ACTIVITIES, Materials 2/6, p. 27, Allyn and Bacon

MATHEX: Numeration No. 2, pp. 4–5, 14–21, Encyclopaedia Britannica Publications Ltd.

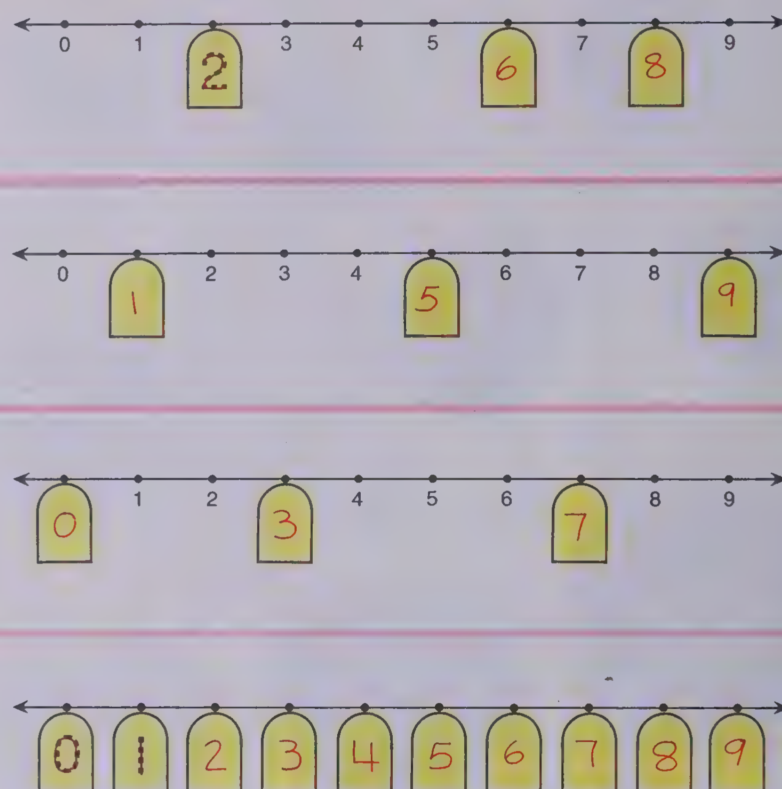
Nuffield Project: MATHEMATICS BEGINS 1, pp. 32–36. Wiley

WORKJOBS, pp. 182–191, Addison-Wesley

Teaching
Page a-47

Discuss the rabbit in the illustration and relate her clothesline to your pre-book introduction to the number line. Then ask the children to look at the number line in the first frame. Explain that it is part of a number line and that the arrows show that we think of the number line as going on and on endlessly. They should now be able to label the number line from 0 to 9. Have them look at the first frame and ask a child to explain why 2 is written on the first yellow label. See if someone can explain what they are to do with the remaining yellow tags. Encourage them to finish the page independently, but help those who need further guidance.

Write the numerals.



Objective

Given a number line showing points for zero through nine, the child will be able to label the points in order from zero through nine.

A number line is one of the most convenient devices for showing the natural order of the whole numbers.

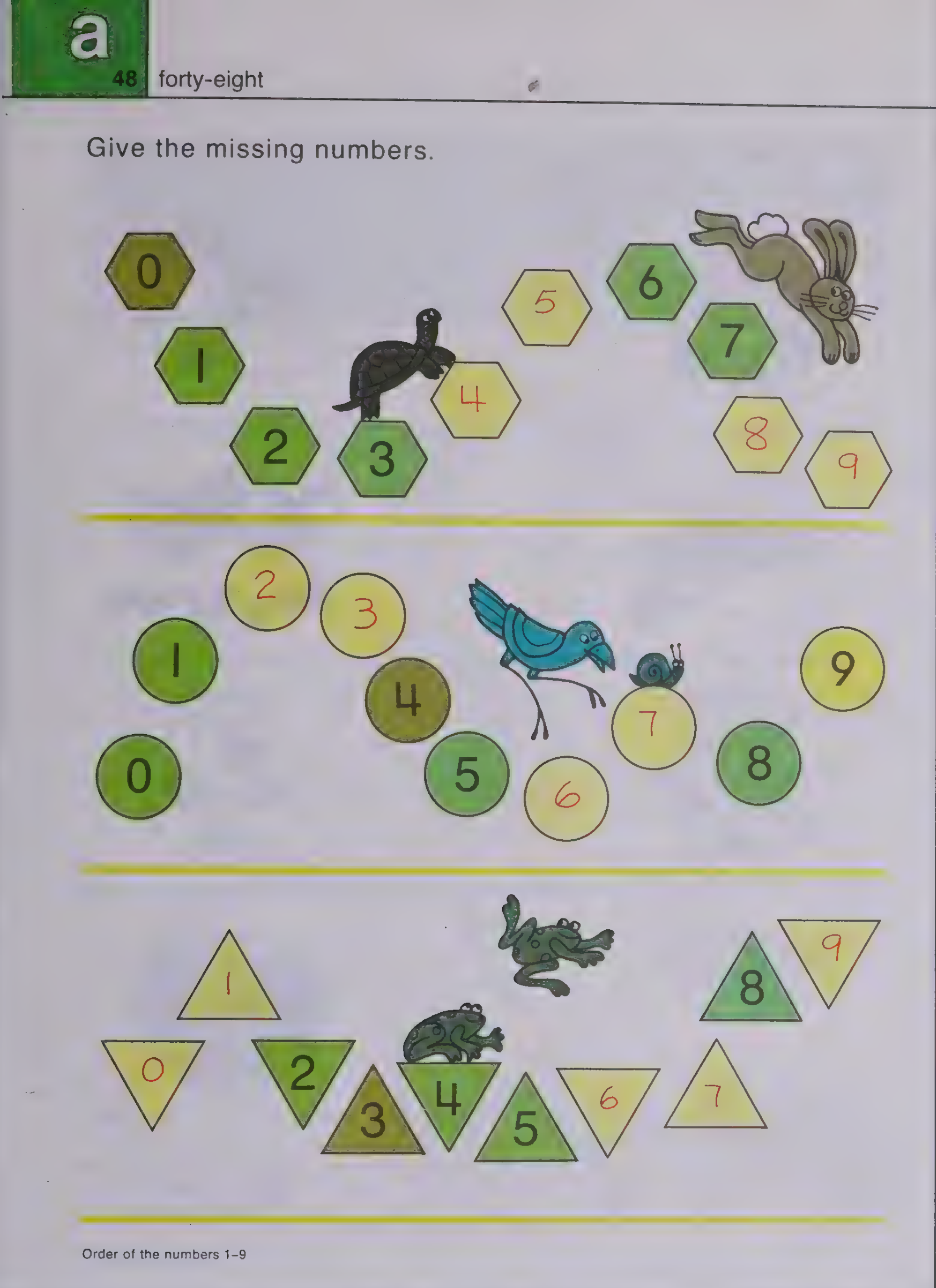
This lesson not only introduces children to the number line, but provides them with practice in ordering the whole numbers from zero through nine.

Pre-Book Activity

To introduce the number line, hang a piece of twine or thin rope at least 2 metres long. Use clothespins to place the numeral cards 0–9 on the line, in order, so that the blank side is facing the class. Tell the children that each card has a numeral on the side they cannot see, but if they can correctly guess the numeral of a card, they may come up and turn that card around. Notice that children may have trouble with their first few guesses until they realize that the numerals start with 0, not 1. If you prefer, this activity may be adapted to a pegboard, but children will enjoy using clothespins on an actual line.

Use the line to talk about order. For example, ask: "If you stand (facing 9) at the pin labelled 4, how is the pin in front of you labelled?" "What numeral is on the card in back of you?" Also point out that 0 is the starting point and the next card is labelled 1.

Finally draw a horizontal line on the chalkboard. (Drawing arrows on both ends indicates that it is a line, and therefore goes on in both directions. If children show curiosity about the arrows, they might think of *zero point*


a

48 forty-eight

Give the missing numbers.

Order of the numbers 1–9

Teaching
Page a-48

Begin by calling the children's attention to the first path. Read with them: "Zero, one, two, three, . . ." and ask children to fill in the missing numbers. Have them complete the remaining paths by filling in the missing numerals. Give them a minimum amount of help and avoid giving the answers. For example, if a child cannot determine the two numerals to put in the first part of the second frame, ask him what comes after 1. When he responds "two," ask him what comes after 2.

When the children have finished, you might use the rows for oral practice with sequences. For example, have everyone put their finger on the 4 they wrote in the first row and count from it, "four, five, six, seven, eight, nine." Use another row to begin with 6 or 7. You might also write sequences on the chalkboard for children to read, such as: 4, 5, 6, 7; 3, 4, 5; 7, 8, 9; 5, 6, 7; 2, 3, 4.

as a centre mark and imagine taking steps in either direction away from zero. For now, they will only be working with some of the numbers shown on the right side of zero.) Put one chalk mark on the line near the left end, and label it 0. Put another mark one unit away and label it 1. Put still another mark one unit from 1 and ask a child to volunteer its name. Label it 2 and continue developing the line. As time permits, erase some of the labels, and ask if someone can fill in the missing numerals.

Follow-up

Make a worksheet having four or five number lines with ten dots marked, but not labelled. On one, label the odd numbers, and ask the children to find and label the missing numbers. On another, label the even numbers, and challenge the children to label the missing ones. On the third number line you could label multiples of three, and on the fourth, only numbers eight and nine.

Resources for Active Learning

MATH ACTIVITIES, "Hot or Cold," Game 2/18, p. 31, Allyn and Bacon

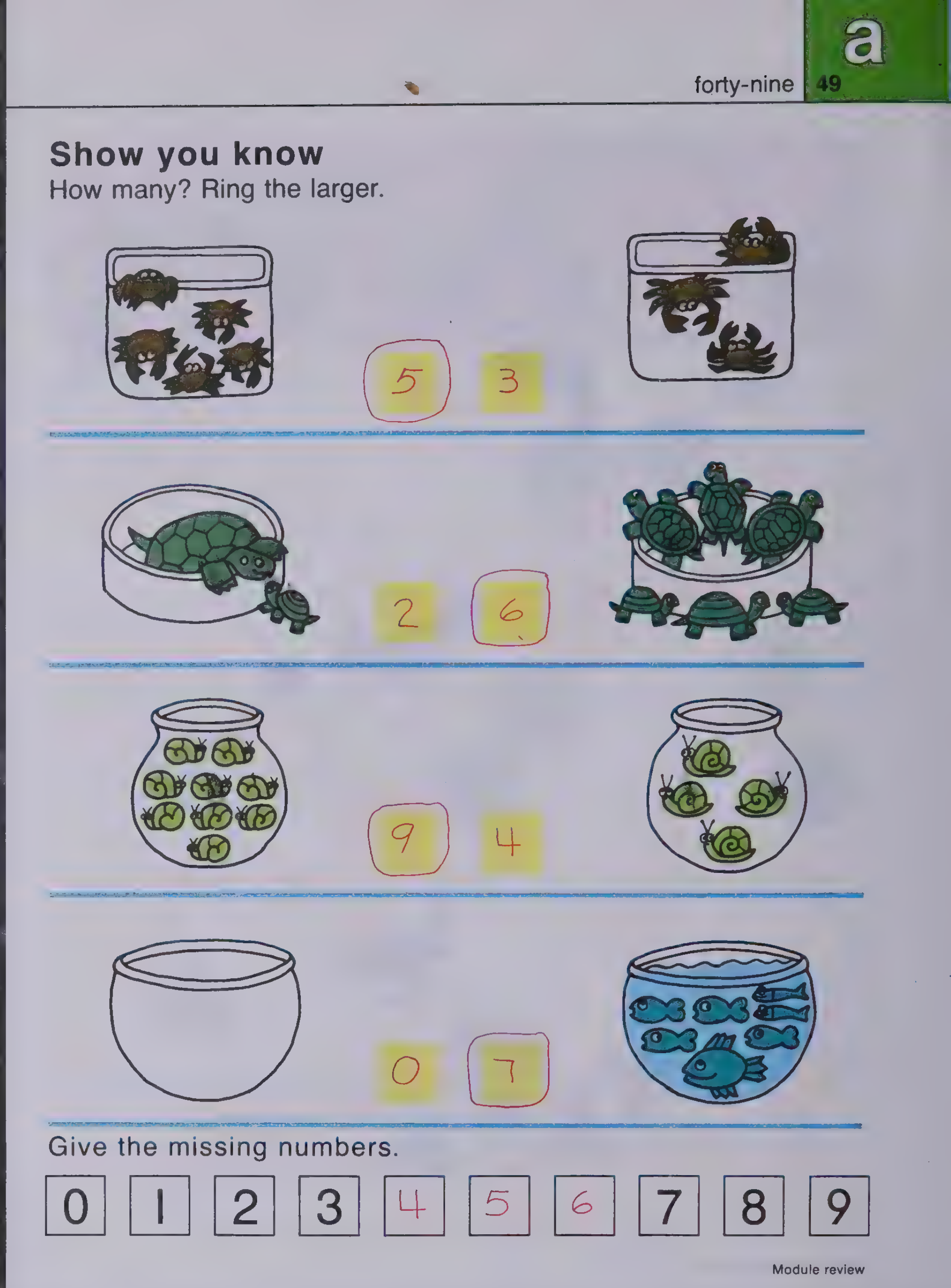
MATHS MINI-LAB, Card 52, Selective Educational Equipment

Nuffield Project: MATHEMATICS BEGINS 1, "Number strips," pp. 40–41, Wiley

WORKJOBS, "Number Lines," pp. 136–137, Addison-Wesley

Teaching
Page a-49

Read the directions at the top of the page with the children. Then direct the children's attention to the first frame. Ask them to write the numeral for the first set in the space on the left and the second set in the space on the right. Then explain that they should put a ring around the numeral of the larger set. When you are sure that they know what to do with these frames, have them look at the bottom of the page and explain that they should write the missing numbers so that all of the numbers are in order. Then encourage them to finish the page on their own, but give help to any who need it, particularly for the last frame of filling in the numbers.



Objective

The child will demonstrate his ability to work with the concepts presented in this module by identifying and circling the larger of two numerals.

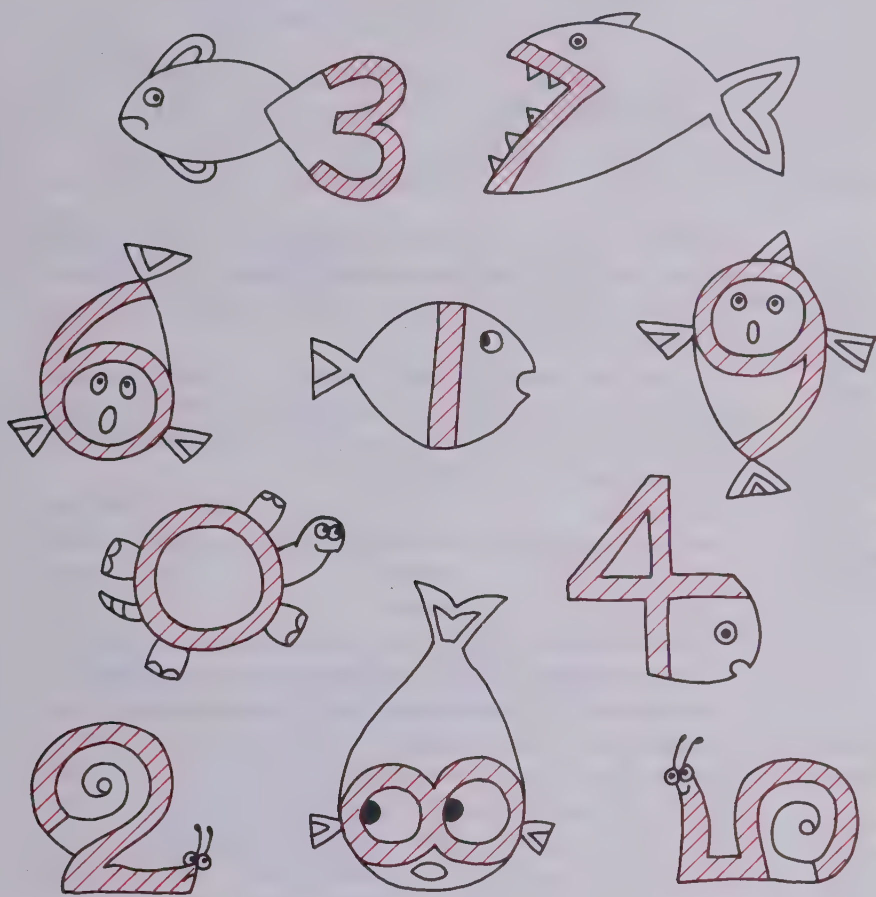
Pages a-49 and a-50 treat both major concepts presented in this module. The exercises on page a-49 treat the more and less than concept and the concept of order. Page a-50 is intended as a change of pace to be treated with a light touch.

Pre-Book Activity

Provide the children with opportunities for comparing sets and their numerals. For example, distribute some of the various cards you have made for matching sets and numerals so that each child has a card. Ask a child to come to the centre of the room and hold up his card. When he does this, all the children that have a card showing a numeral or set less than his should walk to one side of the room, those with a number greater than his to an opposite side of the room and those with a card that matches his to the centre of the room. When they are all in the different areas, ask three or four children to compare their number with the centre number using terms such as: "5 is more than 4," or "4 is less than 5." Then have all of the children return to their places and repeat the game.

Let's have fun

Find and color the numerals.



TEACHING
Page 1-50

This page should be treated with a light touch. Some children will be able to figure out what to do on their own, but for many you will want to explain that each of the numerals 0–9 have been "hidden" in the pictures. They should study the pictures and trace over the hidden numeral. When they have found and colored the numerals, suggest that they finish coloring each picture.

FOLLOW-UP

Many children would benefit from practice with the more and less than concepts on a duplicated worksheet such as the following:

Ring the larger:		Ring the smaller:	
5	8	7	4
7	2	2	1
3	6	0	9
4	9	8	3
8	5	6	5

If the children are capable, you might include an activity sheet such as the following:

Less than		Greater than
4*	5	6*
	2	

*Other answers possible.

RESOURCES FOR ACTIVE LEARNING

Counting games:
MATH ACTIVITIES, Game 2/14, pp. 29–30, Allyn and Bacon
MATHEX: Numeration No. 2, pp. 27–28, Encyclopaedia Britannica Publications Ltd.
DEVELOPMENTAL MATH CARDS, "One More" game, B-12, Addison-Wesley
MATHEX: Numeration No. 2, ". . . Counting Process," pp. 1–3, Encyclpaedia Britannica Publications Ltd.

DARK GREEN MODULE, UNIT A

Geometry

Pages a-51 to a-62

General Objectives

To introduce some basic geometric shapes: circle, triangle, square, rectangle

To introduce simple closed curves

To introduce segments and curved paths

To provide experiences in comparing the relative sizes of the basic geometric figures

In general the concepts treated in this module are as follows: square, circle, triangle, rectangle, open figure, closed figure, inside and outside of a figure, point, segment, path, curved, straight, same size, and shape. All of these concepts are not developed with the same thoroughness.

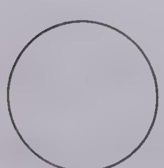
The basic aim of this module is to get the child actively involved in geometric experiences such as: matching cutouts with figures, curves, segments, or paths between dots; and sliding or turning figures to find a figure upon which they will fit exactly.

Mathematics

Only the simplest geometric figures are introduced in this module. The definitions of the basic figures follow:

circle:	The set of all points in a plane which are at a fixed distance from a point in the plane called the centre.
square:	A polygon (closed plane figure made up of segments) having four sides of the same length and four right angles.
triangle:	A three-sided polygon.
rectangle:	A four-sided polygon where opposite sides are parallel and whose angles are right angles.

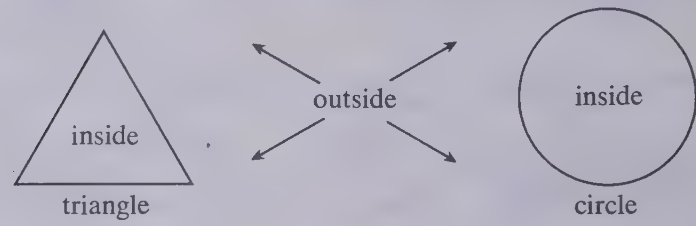
Observe from the definitions that the geometric figures are sets of points on the "boundary" of a region.



a circular region

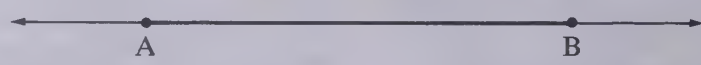
a triangular region

A triangle or circle divides the set of points in a plane into three sets: the points on the inside or *interior* of the figure, the points on the outside or *exterior* of the circle, and the points that make up the figure itself. The set of points in the interior of a circle or on the circle is technically known as a *circular region*. We may think similarly about triangular regions, square regions or rectangular regions.

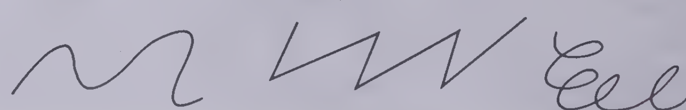


Cardboard models of geometric figures are really models of geometric regions. For young children, it is probably wise not to make a distinction between a geometric figure and its region.

A *line segment*, or simply *segment*, is a subset of the set of points on a line; specifically the line segment $\overline{AB}$ consists of the points A and B and all the points of the line that are between A and B.

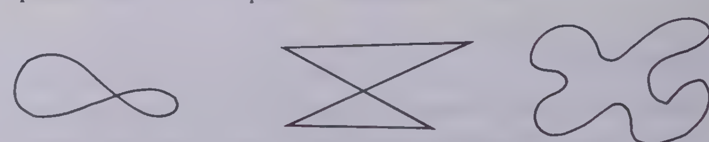


In this module the child is given an introduction to open and closed curves. A child can draw pictures of *plane curves* if he moves his pencil along his paper without lifting it from the paper.



plane curves

Such a curve is a *closed curve* if the child brings his pencil back to the point he started from.



closed curves

If the closed curve does not cross itself, then it is called a *simple closed curve*.

simple closed curves

Notice that the simple geometric figures of this module —circle, triangle, square and rectangle—are all examples of simple closed curves.

Teaching Dark Green Module, Unit A

Approximate Time: 5 to 7 days

Materials

attribute pieces: Each child should have a set of attribute pieces consisting of two large squares, circles and triangles; and two small squares, circles and triangles; in two colors (preferably red and blue).
boxes of the basic geometric shapes of varying sizes: about 1 box for every 4 children
dot paper
envelopes: 1 per child, to store attribute pieces
flannelboard or magnetic board and various sizes of circles, squares, triangles, rectangles
geoboard: 1 per child, if possible
overhead projector
rubber bands: 3 or 4 per child
string or ribbon: about 5 metres long

Vocabulary

attribute pieces	open	segment
circle	outside	shape
closed	path	square
inside	rectangle	triangle

Although this module is basically a single learning unit, it may be handled in two different ways. The lessons may be used consecutively for a week or two, or they may be broken up and used periodically over a greater length of time. For example, you might want to use pages a-51 and a-52 to introduce the basic geometric figures and then plan an on-going class project to which you can occasionally refer before continuing with pages a-53 and a-54.

Keep in mind that children will benefit from an opportunity to feel, manipulate, sort, and simply play with the various shapes studied. For example, a child who is encouraged to cover his area with any combination of the shapes will quickly become aware that the circle has no corners and cannot be used with any of the other shapes studied here to cover a table's surface.

Evaluation of Progress

Since one of the first purposes of this geometry module is to develop intuitive notions in regard to the basic geometric shapes, you should not plan any formal evaluation. Even a child's use of the correct terms to identify each shape should not be considered as an indication that the child's understanding of these shapes is highly developed. Your most helpful method of evaluation will be daily observation of and discussion with each child.

Resources for Active Learning

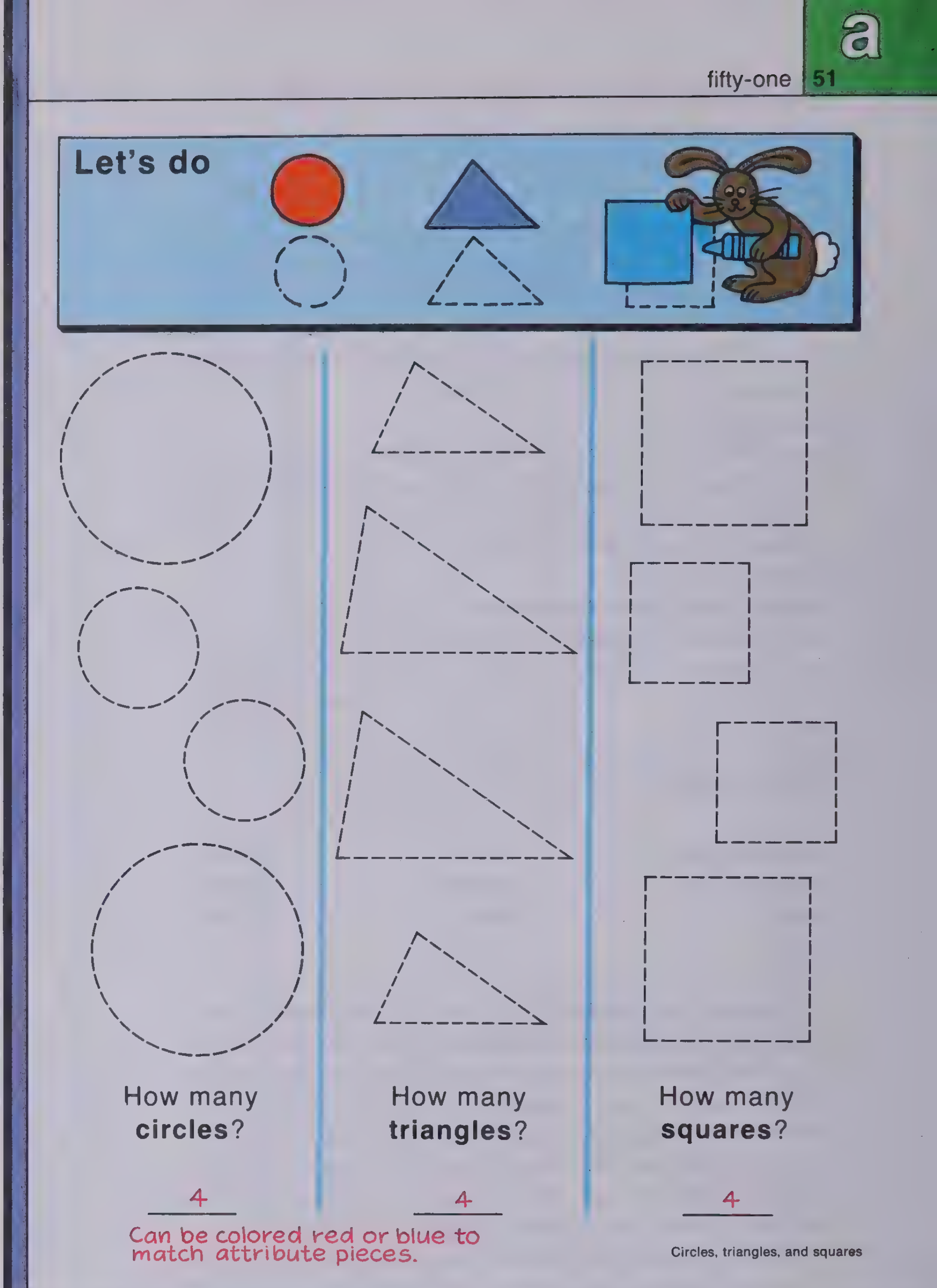
General Activities:
Activities with geometric shapes:
ACTIVITIES IN GEOMETRY FOR PRIMARY PUPILS, Activities 14, 23, 26, 27, 30, Addison-Wesley
EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, "Games . . . Geometry," pp. 35–57, Herder and Herder
MATHEX: Geometry No. 4, pp. 11–19, Encyclopaedia Britannica Publications Ltd.
MATH WORKSHOP: Games and Enrichment Activities, pp. 6–8, Encyclopaedia Britannica Educational Corp.
Attribute games and problems to use now and throughout the year:
ATTRIBUTE GAMES AND PROBLEMS, Webster, McGraw-Hill
DEVELOPING NUMBER EXPERIENCES, Kit A, Holt, Rinehart and Winston
LEARNING LOGIC AND LOGICAL GAMES, Herder and Herder
Symmetry:
MIRROR CARDS, Sets 1–7, Webster, McGraw-Hill
Nuffield Project: BEGINNINGS 1, "Symmetry, Shape and Size," pp. 81–92, Wiley
Tangrams:
TANGRAM CARDS, Sets I and II, Webster, McGraw-Hill
TANGRAMS: 330 PUZZLES, Dover Publications
Working with geometric solids:
DEVELOPMENTAL MATH CARDS, A³5, B³4, 11, 16, Addison-Wesley
ELEMENTARY SCHOOL SCIENCE Primer (TE), "Solid Shapes," T73–76; "Roll-slide," T57–58; Book 1 (TE), "Solid Objects," pp. 100–115, Addison-Wesley
MATHEX: Geometry, No. 4, pp. 1–10, Encyclopaedia Britannica Publications Ltd.
Manipulative Devices:
Attribute games (school supplier; Teaching Resources; Webster, McGraw-Hill)
Basic shapes set (Educational Teaching Aids; Math Media; Responsive Environments Corp.)
Dienes Logical Blocks (Herder and Herder)
Geoboards (Addison-Wesley)
Geometric figures and solids (Milton Bradley; school supplier)
Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)
Commercial Games:
Geometric dominoes games (Childcraft; Selective Educational Equipment)
Geometric Shape Spotting (Selective Educational Equipment)
Shape Analysis Matching Game (Math Media; Responsive Environments Corp.)
Shapes Lotto (CCM School Materials)

Investigation
Page a-51

When each child has his envelope and page a-51 explain that they are to try to match their figures with the figures in the book. To do this they will have to sort the figures by both size and shape. Allow the children freedom in sorting the pieces as they wish; some may put all the circles together, all the squares together and all the triangles together, before matching them onto the page. Others may work haphazardly from one piece to another.

When the children match a figure to a figure on the page, they should color the figure on the page the same color as the piece. Thus after they have matched every figure, they should have colored large and small red circles, squares and triangles, and large and small blue circles, squares and triangles.

Finally, ask them: "How many circles?" "How many triangles?" and "How many squares?" and direct them to write their answers in the appropriate column. Be sure that each child carefully returns his pieces to the envelope and then collect them for future use. You might also begin to refer to them as attribute pieces.


fifty-one 51

a

Let's do

How many **circles**?

4

How many **triangles**?

4

How many **squares**?

4

Can be colored red or blue to match attribute pieces.

Circles, triangles, and squares

Purposes

Given attribute pieces, the child will be able to match them to congruent figures on the booklet page.

Given a set of triangles, circles, and squares, the child will be able to sort them according to shape and name them.

Preparation

Materials

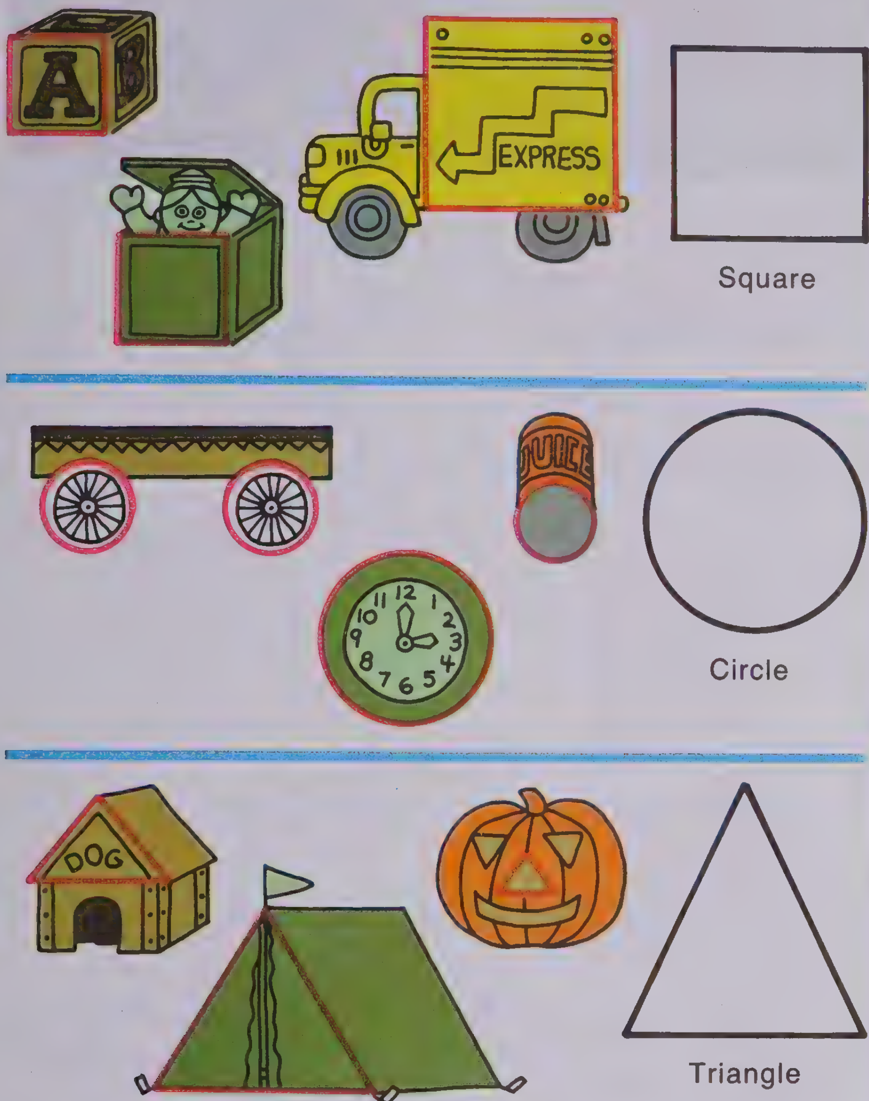
attribute pieces (one set of large and small circles, squares, and triangles in red and blue for each child)
crayons

The materials required for this investigation are red and blue attribute pieces. You might make a set of these figures for each child in advance. Arrange for each child an envelope containing four circles, four triangles, and four squares (two large and two small of each shape). The children also need red and blue crayons for page a-51.

Follow-up

To help children better grasp the idea of a geometric figure, it is often helpful to talk of the inside and outside of the figure. For this purpose, you might give directions such as, "Put a blue "x" inside every square you see on page a-52" or "Put a red dot outside each figure." Also ask them to mark a dot on each figure.

Let's talk

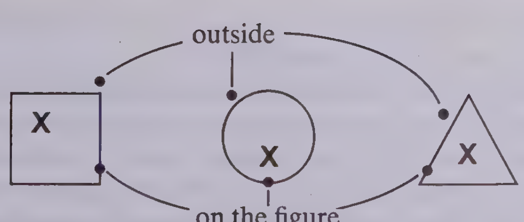


Circles, triangles, and squares

Discussion
Page a-52

Encourage the children to talk about what they see on page a-52. Keep your questions open-ended such as: "What can you tell me about this page?" "What are some other things these shapes make you think of?" Elicit from them that the red outlines shown should help them think of squares, circles, or triangles. Allow the children freedom in expression and you will be helped in evaluating how much children know about geometry and identifying shapes.

After children have freely discussed the page, you might ask them to give examples of other things around them that have square shapes, such as some window frames or tiles on the floor. Also ask how many sides and corners the figures have. Help children realize that a circle has no sides and no corners. As the ideas of square, circle, and triangle become clearer to the children, you might want to have larger cutouts or magazine pictures to hold up and have the children try to recognize the shapes.



the x's are inside each figure

The colored figures may be used in many ways. First ask the children to sort them by color, then by size, then by shape. You might make large models of these figures suitable for demonstration on a magnetic board or flannelboard and use them in various games. For example, hide all the small figures and see if children can name the missing figures. Or hide one figure and see if children can find out which one is missing. You might also show a set of the pieces and see if children can name the set. For example, show all the red figures, all the squares, or all the large figures, and so on.

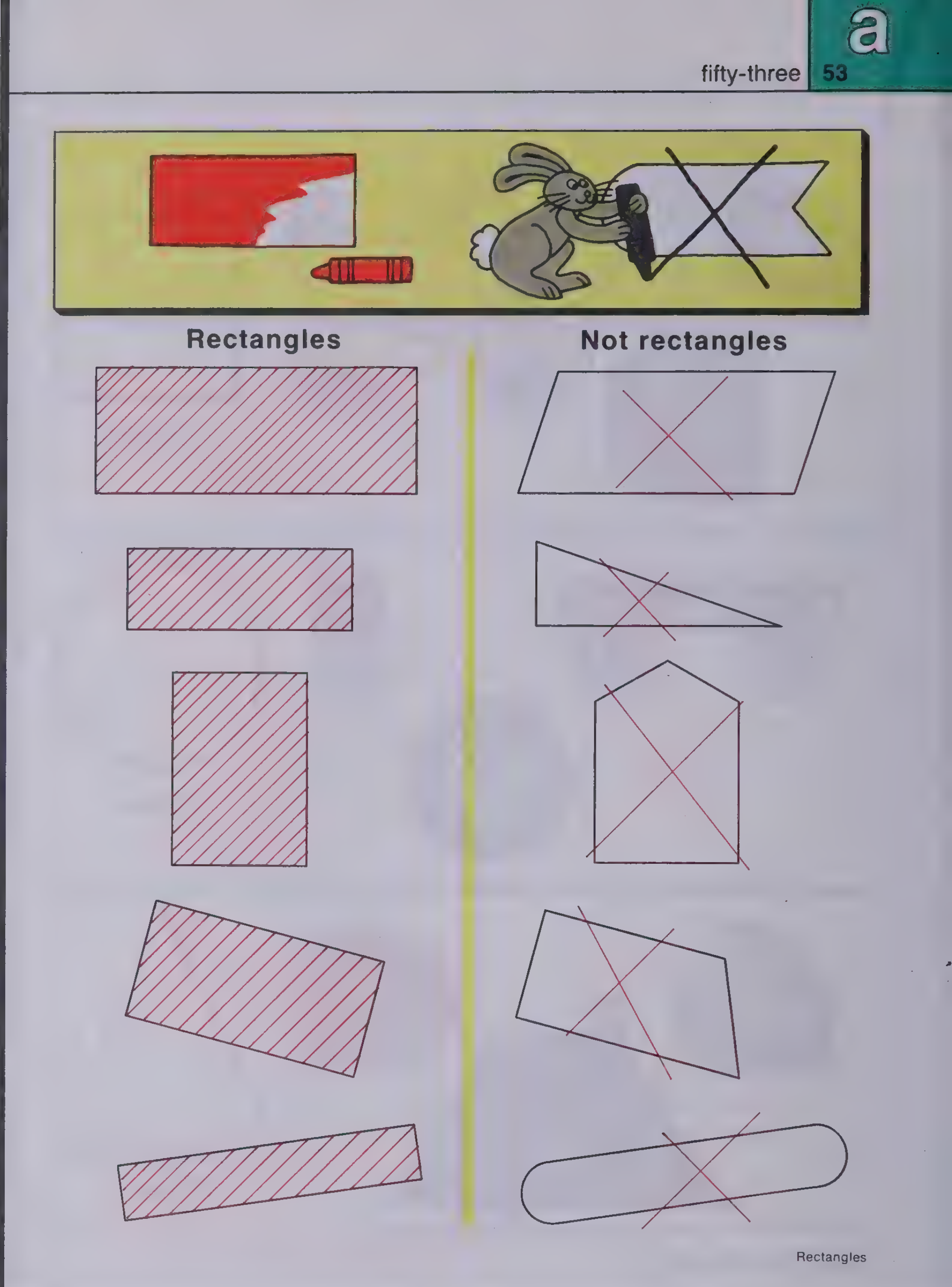
Resources for Active Learning

ACTIVITIES IN GEOMETRY, pp. 44, 46, 47, Addison-Wesley

MATHEMATICS IN MODULES: SK1 • SPATIAL KNOWLEDGE • Faces and Plane Shapes, pp. 4–11, Addison-Wesley

Teaching
Page a-53

Page a-53 may be used to introduce the rectangle. Use the shapes in the two columns to stimulate discussion of what attribute each shape in the left column has that the shapes in the right column do not have. Examples are square corners, straight sides, four sides. Then show cutout models of rectangles and discuss various rectangular shaped objects around the room. Explain the illustrated directions of page a-53, and direct the children to mark all the figures that are not rectangles with a red X and color all rectangles red. As the children work you might walk around the room. Point to various figures that are *not rectangles* and ask a child if he can explain why the figures are not rectangles.



Objective

Given a set of geometric figures, some of which are rectangles, the child will be able to identify the rectangles.

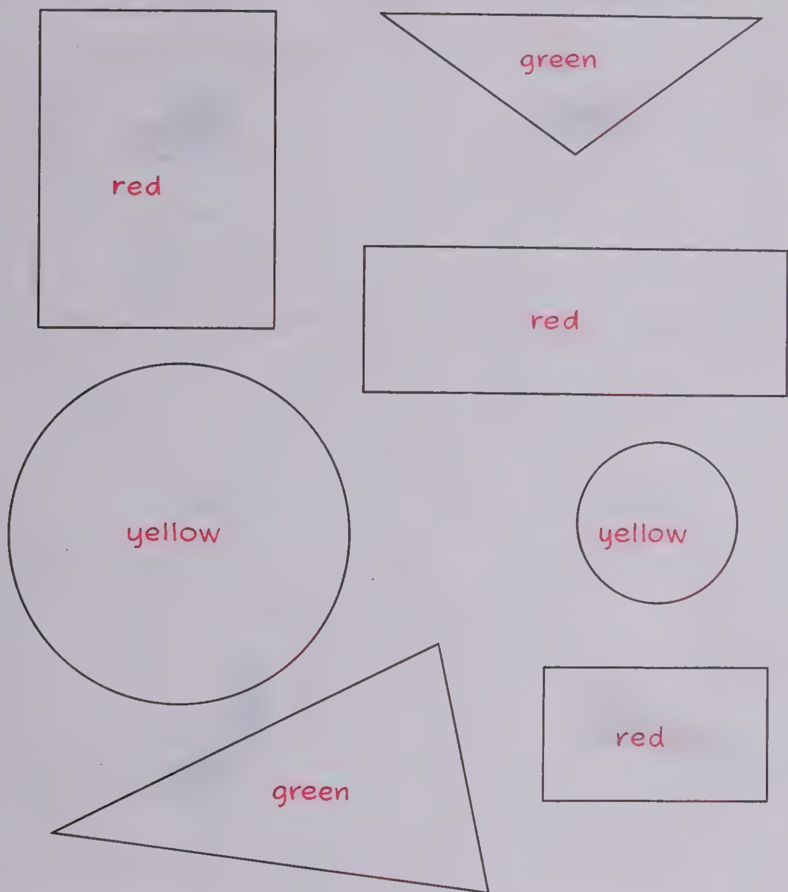
Although a square is a particular kind of rectangle, first graders should not be expected to grasp this idea of inclusion. The main point of this lesson is to help children recognize rectangles. If they call a square a rectangle, fine, but do not belabor the idea. However, for most children, a square is not a rectangle.

Pre-Book Activity

Distribute to each table or group of four or five children a box containing a variety of triangles, squares, circles, and rectangles. Ask the children to sort the figures according to their shapes, that is, to put all the figures with the same shape in one pile.

Observe the children while they do this; note any who have difficulty with the shapes studied in the previous lesson. These children should receive special help in this area. Also notice what the children decide to do with the rectangles. If they think of them as figures that have four sides, they might put them with the squares. Such classification may be considered correct since the rectangles and squares do have four sides. However, if children do this, ask them to look at the shapes of the figures, how long they are, how wide. Elicit from them that even though rectangles and squares both have four sides and four square corners, they do not have the same shape—the rectangle is longer than it is wide, the square is as long as it is wide, that is, all the sides of the square are the same length.

Color.



Triangles, circles, and rectangles

Teaching
Page a-54

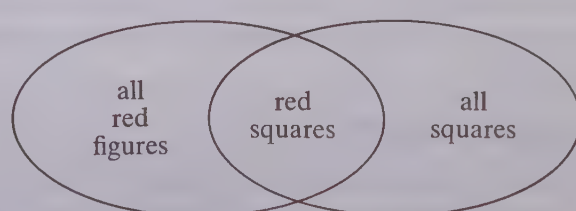
Have the children look at the top of page a-54 and explain that they should color rectangles red, circles yellow, and triangles green. Notice that in giving directions you should instruct the children to color the inside of each figure. If you ask children to color each figure, they would have to color only the outline of the shape.

Follow-up

Children will enjoy labelling various items in the classroom according to their shape. Distribute pieces of construction paper cut in the shapes of circles, triangles, squares, and rectangles or upon which these figures are clearly drawn. Ask the children to name an item and explain which of these geometric figures it makes them think of. If all agree that the item is truly representative of that shape, then let them tape the representative piece of paper on the item. For example: rectangle—door frame, wood around the chalkboard, closets, pictures, etc; circle—top of wastebasket, flower vases; square—floor tiles, decorations; triangle—decorations such as in a corner of a bulletin board.

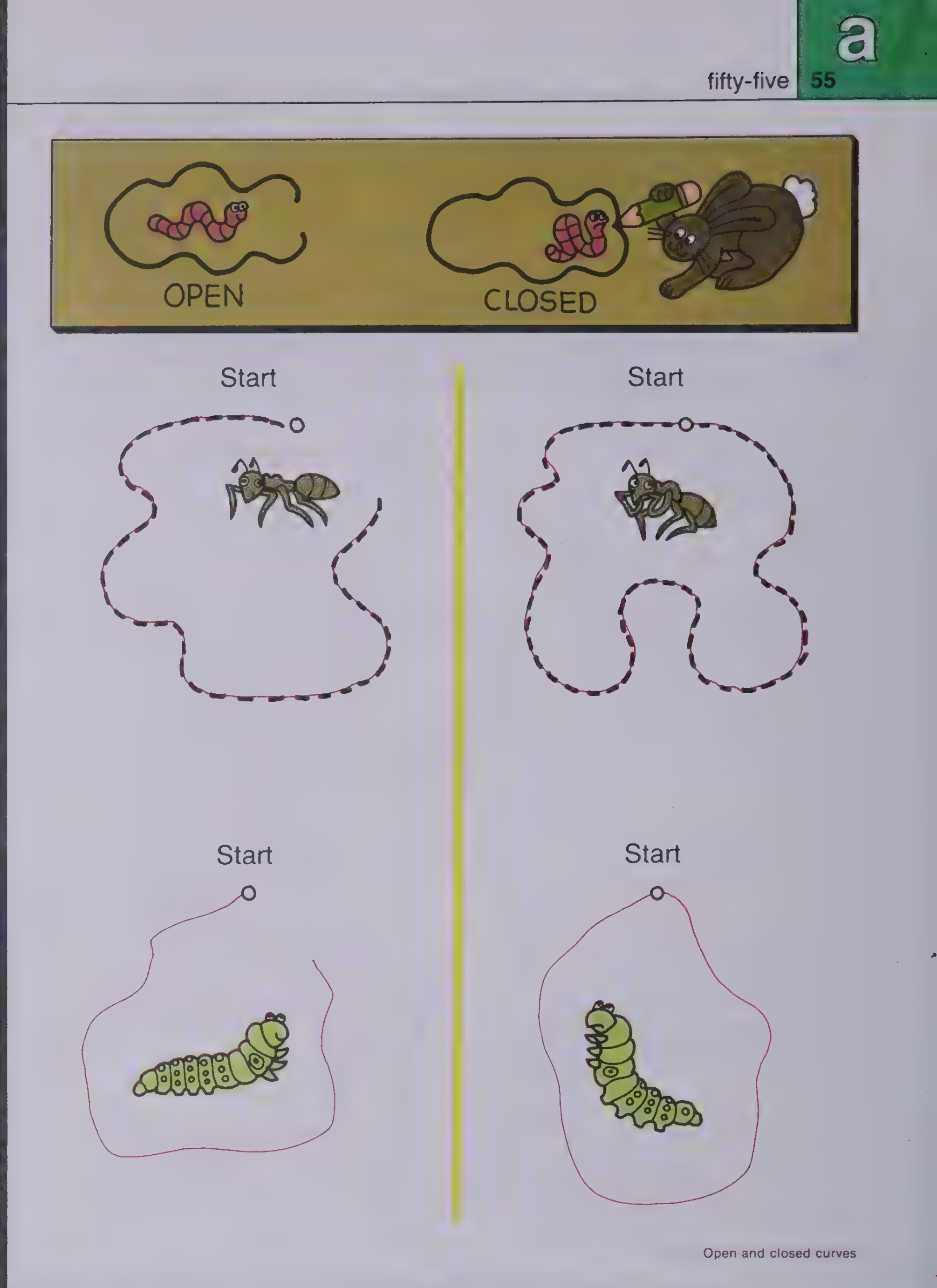
For other games you might say, "I'm thinking of a figure," and the children can then ask questions that can be answered by a yes or no in order to try to find out what figure you are thinking about.

Depending upon the ability of the children, you might also want to use yarn on a flannelboard to form a large red curved outline to use to classify the figures. If you put the red figures in the one outline and squares in the other, see if children can figure out what figures would go in the intersection of the two outlines.



Teaching
Page a-55

Ask the children to look at the top of the page. Teach the words *open* and *closed* as sight words but do not expect mastery; the important thing is that children have an intuitive grasp of the concepts. Discuss the curves in the demonstration art at the top, referring to the worm as being able to come out through the opening in the *open* curve, but locked in by the *closed* curve. Then explain that they are to draw two open curves on the left section of the page and two closed curves on the right section of the page. They should trace over the dashed lines as guides for the first curves they draw.



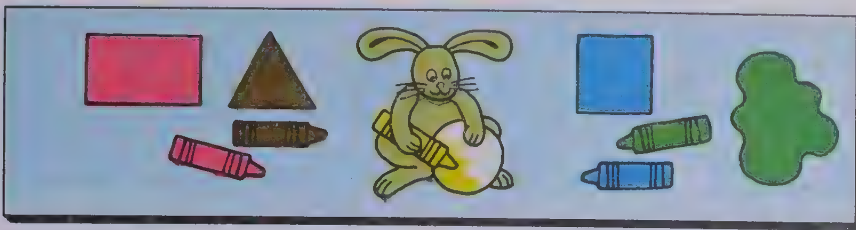
Objective

Given any straight or curved line figure, the child will be able to indicate whether it is open or closed.

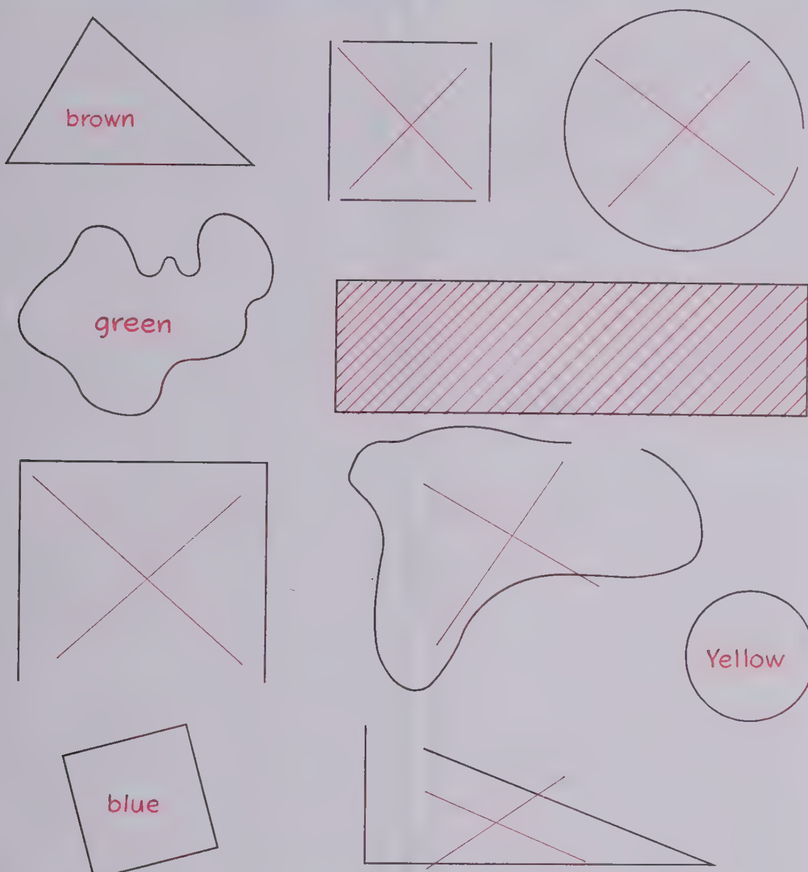
The purpose of this lesson is to help children develop intuitive concepts of open and closed figures and to relate them to the few basic geometric figures they have been studying. Although the idea of an open or closed figure is first presented with curved line figures, the concepts apply as well to straight line figures as shown on page a-56. The stress in this lesson is not on any distinction between curved or straight lines but on open or closed paths.

Pre-Book Activity

To introduce the concepts of open and closed curves, you might place a long piece of string or ribbon on the floor so the children can walk on it. First lay the string out so that its ends do not meet. Tell children to pretend that the string is the top of a fence and they should try to walk on it, following its path. When they get to the end, ask them questions such as: "Why can't you go any further?" "What happened to the string?" "Where is the other end?" Use the term *open* to describe the path of the string and help children realize that since the path is "broken," that is, since they cannot walk along the path and return to their starting point, we call this path *open*. They might think of the fence as having a gate so when the gate is open they can't walk any further. After the children walk along the *open* path, connect the two ends, tying them securely. Then have children discuss the difference between the paths. Use the term *closed* to describe the new path, stressing that it contains no "broken" sections and that one can walk along the string and return



Color the closed figures.



Teaching
Page a-56

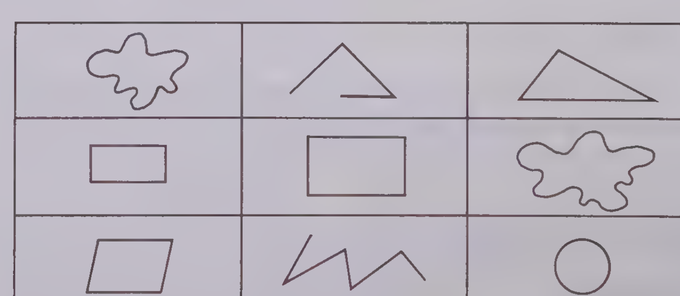
The activity presented on this page not only reinforces the new concept of open and closed curves but also provides practice in recognizing the basic geometric figures. Explain to the children that some of the figures shown are open and some are closed. Since they are to color only the closed figures, suggest that they first cross out with an X all of the open figures. Then explain that they should follow the color codes at the top of the page. They should color any circles yellow, squares blue, triangles brown, and so on. Encourage correct use of the geometric terms as they speak about the figures, but do not over emphasize. Children will begin to use the correct terms by following your example.

to the starting point without ever leaving the path. As children talk about the curves, develop the example using yarn on a flannelboard.

Follow-up

If time permits, it would be helpful to emphasize the ideas of inside and outside. You might distribute a worksheet similar to the one illustrated. Give the directions orally step by step. You might prefer to give similar directions for figures on page a-56.
Directions:

Put a red "X" inside the closed figures.
Put a blue dot on the closed figures.
Color the outside of the closed figures yellow.
Outline the open figures green.



Resources for Active Learning

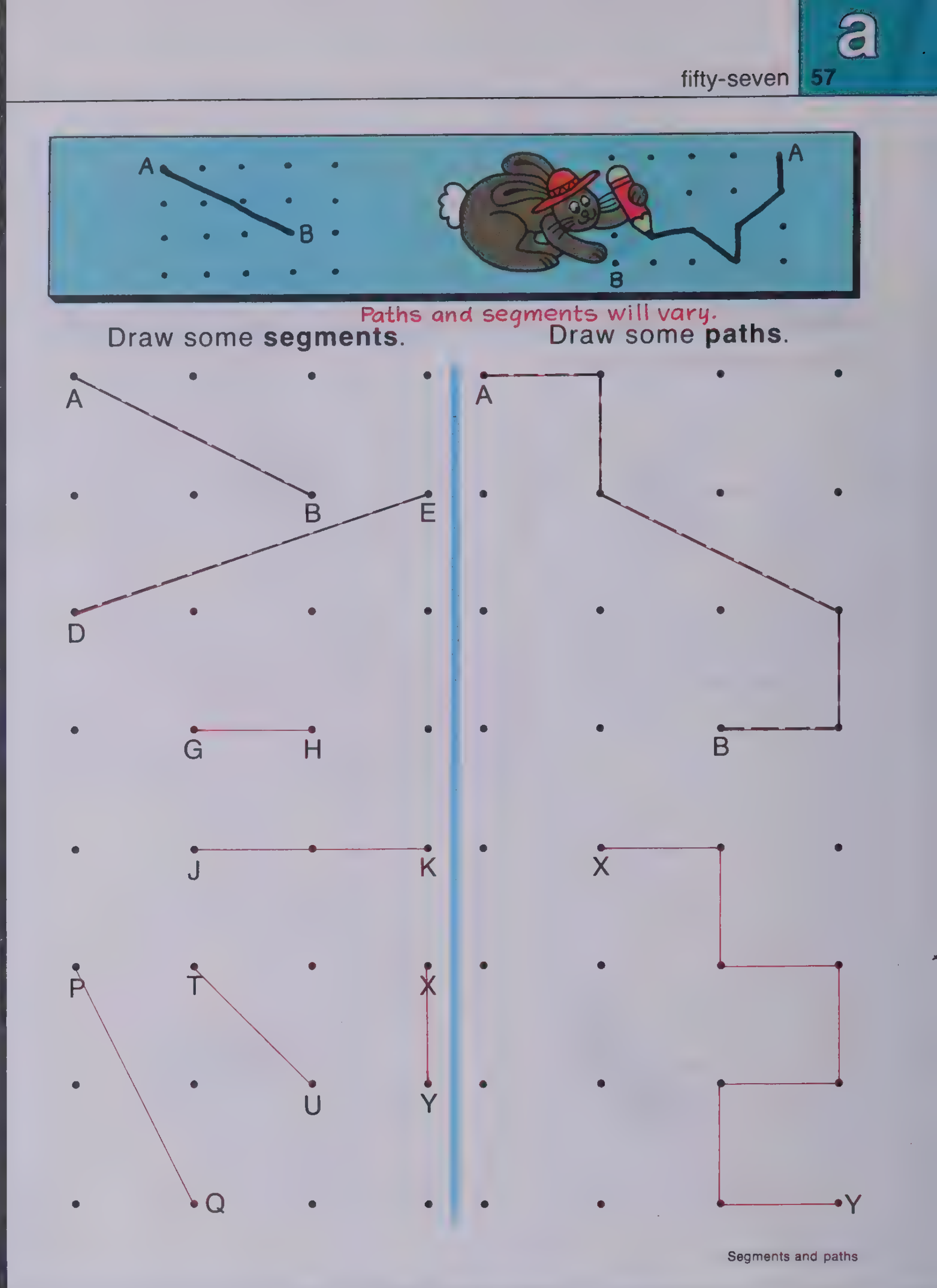
ELEMENTARY SCHOOL SCIENCE Primer (TE), "Straight-curved," T31–32, Addison-Wesley

MATH WORKSHOP: Games and Enrichment Activities, ". . . . Curves," p. 81, Encyclopaedia Britannica Educational Corp.

Teaching
Page a-57

Ask the children to look at the top left section of the page. If possible, use an overhead projector and a plastic transparent geoboard to point out a segment from A to B as illustrated on the text page. Also show other examples of straight line segments. Explain the directions and teach the words *segments* and *paths*. Have the children trace over the dashed segments from A to B and D to E in the text. Then explain that they should make other straight line segments by drawing a straight line from one lettered point to another. The points may be connected in a variety of ways.

For the right side of the page, explain that they should use straight line segments to make paths from one point to another. Have them trace over the dashed line shown for the path from A to B and then make up their own path from X to Y. It would be helpful to have different children show their paths from X to Y and discuss the variety of paths possible. An overhead projector would be convenient, but if necessary, you might use yarn and a pegboard.



Objective

Given a grid, the child will be able to draw a segment between two points and construct a square, a rectangle, and a triangle on the grid.

Pre-Book Activity

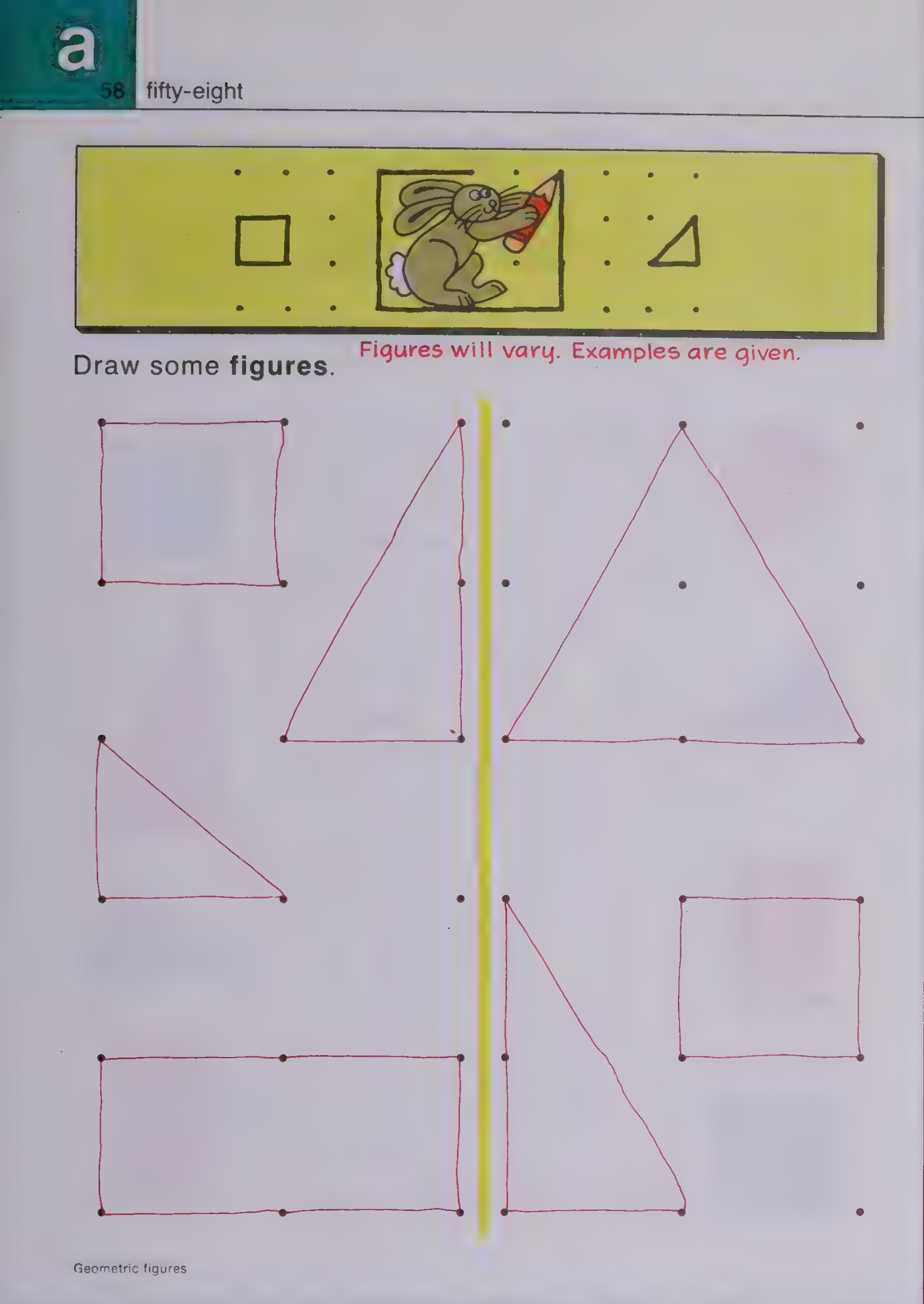
Materials

geoboards or dot paper

If possible, have geoboards available for the children to use before working with dot paper or with the text pages. If necessary, have the children work in pairs. Distribute three or four rubber bands per geoboard. Show the children how a rubber band might be fastened around the nails to form a figure. Observe with them that connecting two nails which are in a straight line is considered forming a line segment. Encourage them to explore and discover what figures they can make. You might ask them "Can you make an interesting figure using one rubber band on the geoboard?. . . two rubber bands? . . . three rubber bands?"

If geoboards are not available, distribute duplicated copies of dot paper (simply prepare a master with dots 2 to 3 cm apart vertically and horizontally). Then suggest that the children experiment drawing designs or figures using only straight lines which connect at least two dots.

This activity should be handled with a light touch. Children should simply be given an opportunity to "play" with the geoboard and if this is not possible, at least to draw pictures of their own on dot paper.



Teaching
Page a-58

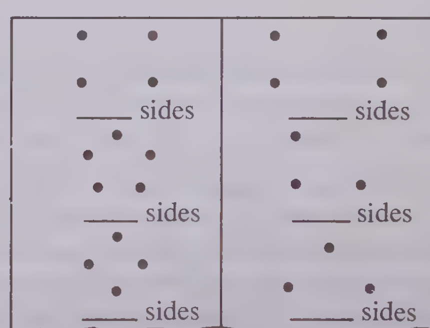
One of the most important points to stress is that the figures the children have been studying (the square, triangle, and rectangle) are made of straight line segments. If possible, use some examples from the designs or figures children made on the geoboard or on dot paper during the pre-book activity to point out that many different figures can be made of straight line segments.

Direct the children to study the illustrated figures at the top of the page and ask them to identify the figures shown. Explain that on the left side of the page they should draw some triangles, squares, and rectangles.

The right section of the page is given for you to have the children use as you choose. You might suggest that they draw any figure or picture they want using straight line segments. Or you might ask a child to draw an open or closed figure that has five or six straight line segments.

Follow-up

If geoboards are available, give children further opportunity to use them. Suggest various questions such as: "Can you make two line segments so that one is longer than the other?" "Can you make two triangles so that one looks bigger than the other?" "Can you make two rectangles so that one rectangle is inside the other?" It would be possible for children to explore these questions using dot paper, but this might cause difficulty for some. If geoboards are not available, you might distribute a worksheet similar to the following. Ask the children what shapes they can make by drawing straight line segments to connect the dots. If their figures are simple figures you might also ask how many sides each figure has.

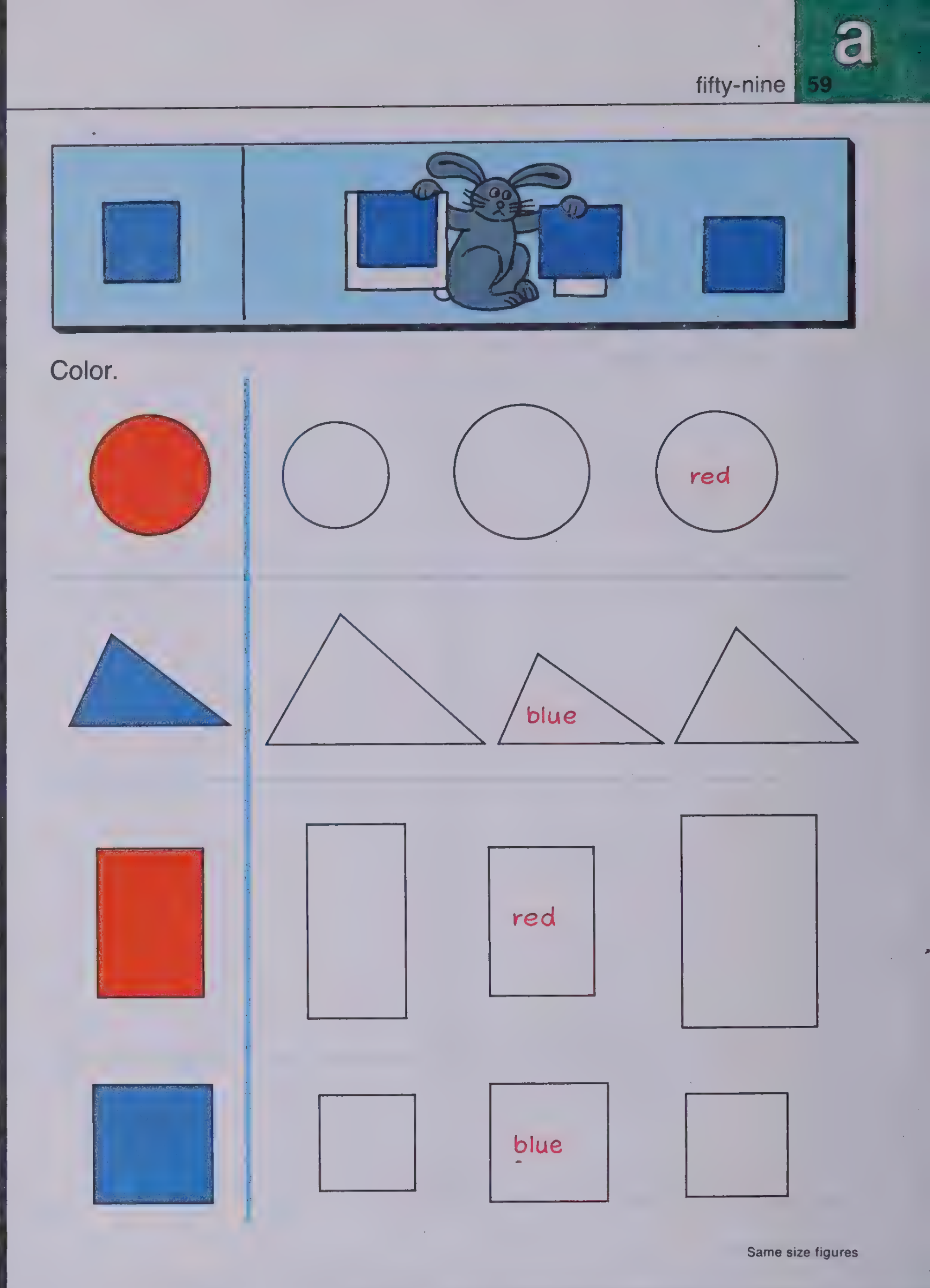


Resources for Active Learning

MATH WORKSHOP: Games and Enrichment Activities, "Line Segments," pp. 83–84, Encyclopaedia Britannica Educational Corp.

Teaching
Page a-59

Have the children first find which of their attribute pieces matches the colored figure in the left column. Then have them use this piece to compare with each of the other figures in the row. The important thing to stress is that the size of the first figure may be matched by fitting the correct attribute piece upon each pictured figure to see which one fits exactly. When they have chosen the figure which is the same size, they should color it the same color as the first figure.



Objective

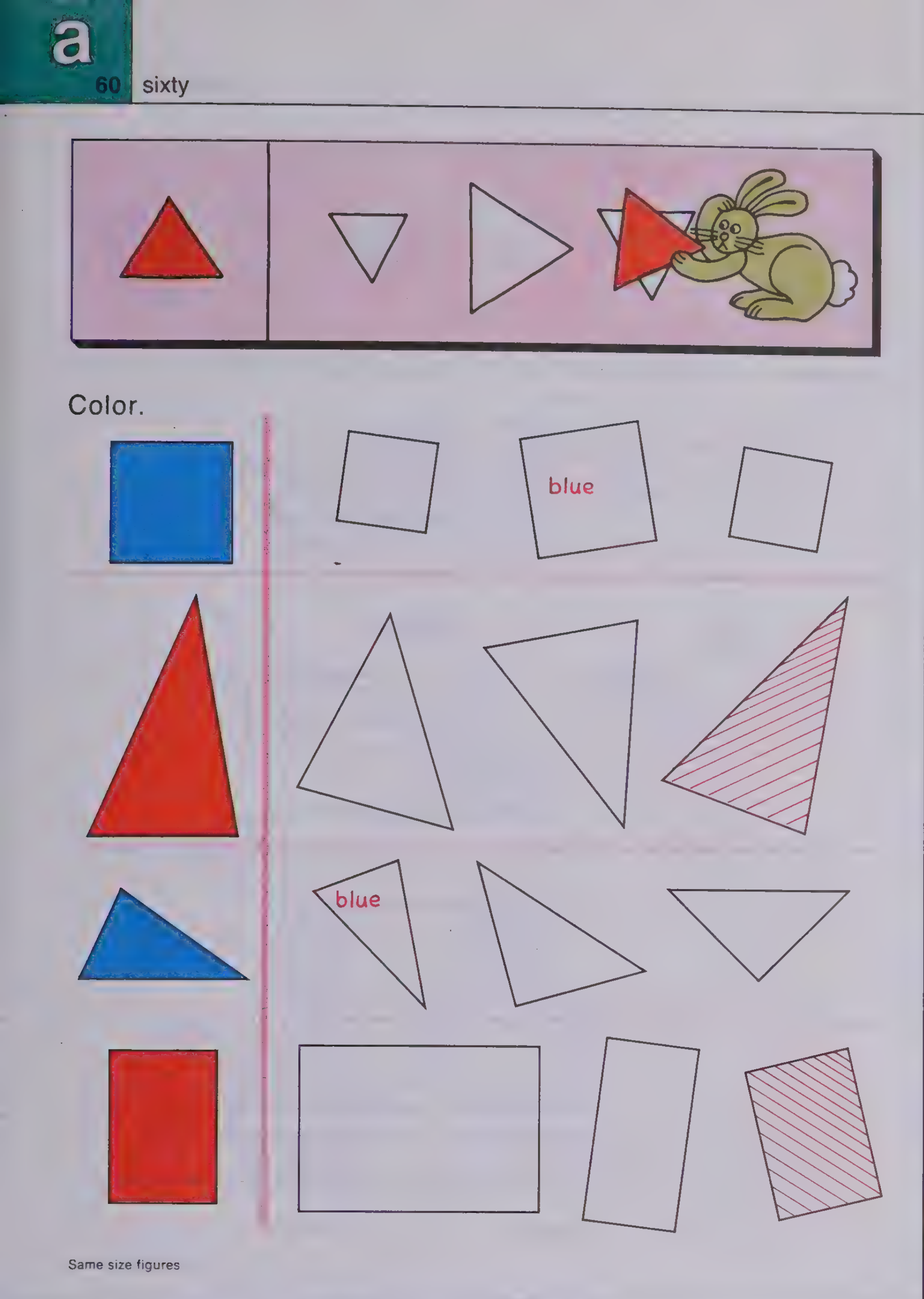
Given a basic figure and a set of three figures in the same relative position or slightly rotated, the child will be able to select the figure that is the same size as the first.

Besides providing extended experiences with basic geometric figures, these pages are intended to introduce the child to intuitive ideas of translations and rotations. The exercises are so designed that the attribute pieces are to be used as aids. With their use the child will be able to match the figures by actually moving and rotating his pieces.

Pre-Book Activities

Distribute the attribute pieces so each child has a set. You might review the basic shapes by asking everyone to hold up a blue triangle, or a red square, and so on. Then ask the children to try to match pieces by size and shape. For example, ask them to take the large red square and find another piece which has the same size and shape. Challenge them to find a partner for each figure so that each piece in the pair has the same size and shape. As the children find pairs, ask them how they know that both pieces have the same size and shape. Most likely the children will indicate that one piece can be fitted exactly upon the other.

As suggested earlier (page a-51), large demonstration models for use on a magnetic board or a flannelboard would facilitate discussion of matching pairs. For example, place the large and small circles in a row and ask children to point out which circles are the same size. Also scatter several of the figures on the flannelboard or magnetic board. Then hold one piece, such as a small square,



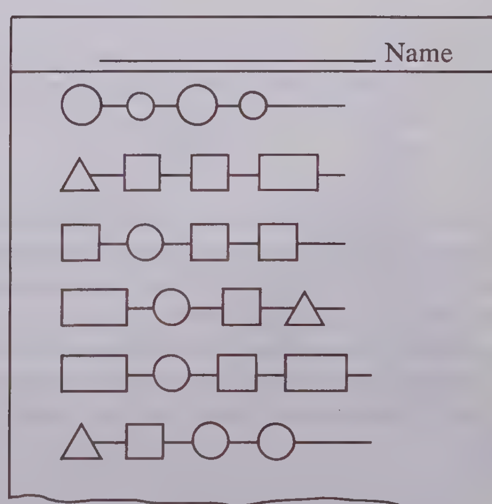
Teaching
Page a-60

Since the procedure for this page is very similar to page a-59, most children will not need many directions. However, work through the illustrated example with them. Make sure they realize that here they must turn or rotate their attribute piece to see which illustrated figure it fits exactly. Also remind them to make sure that the corners of the triangles match exactly before they decide which triangle to color. Continue to stress that they must find the correct attribute piece for each colored figure before they can do any correct matching.

and ask the children to find the partner to it. The purpose of this activity is to prepare the children to recognize figures that have the same shape and size even when the figures have been rotated into different positions. Stress that the pieces may be rotated and placed one upon another to see if they are the same size.

Follow-up

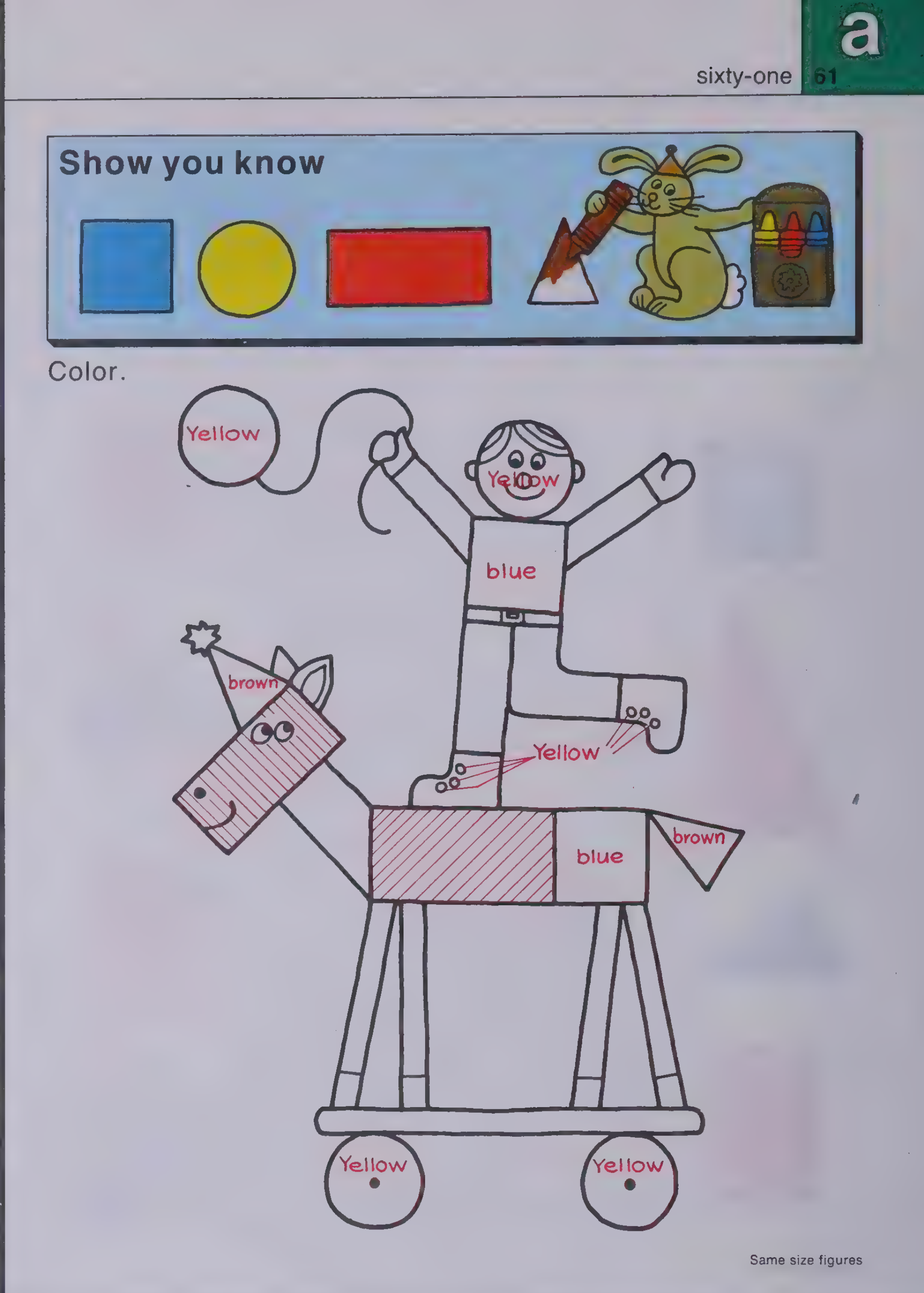
A worksheet in which children can complete patterns would be suitable. Allow children to complete their patterns in any way they choose, as long as they show a pattern.



Teaching
Page a-61

Ask the children to look at the colored figures at the top of the page and have a child tell the color of each. Then explain that they are to use these colors as a code and to color the figures they can find in the picture according to the code. Thus every time they find a square they should color it blue, a circle yellow, a triangle brown, and a rectangle red. You might point out that all of the shapes are not the same size so they should be careful to identify shapes before deciding which color to use.

There are a number of shapes which are not circles, squares, triangles or rectangles; you might ask the children to color these shapes another color.



Objective

The child will demonstrate his ability to work with the concepts presented in this module by matching figures according to shape.

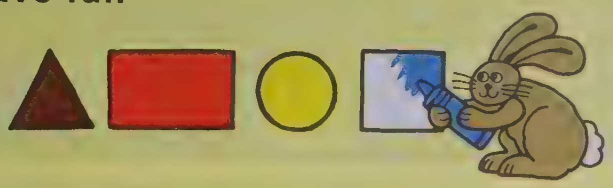
Both pages a-61 and a-62 deal with the basic concept of this module, namely that of identifying figures according to their shapes. These pages may be considered instruments of evaluation of the child's ability to recognize circles, squares, triangles, and rectangles; but they should also be considered as "fun" pages. The children may do them as a fun activity, but you may use them as evaluation.

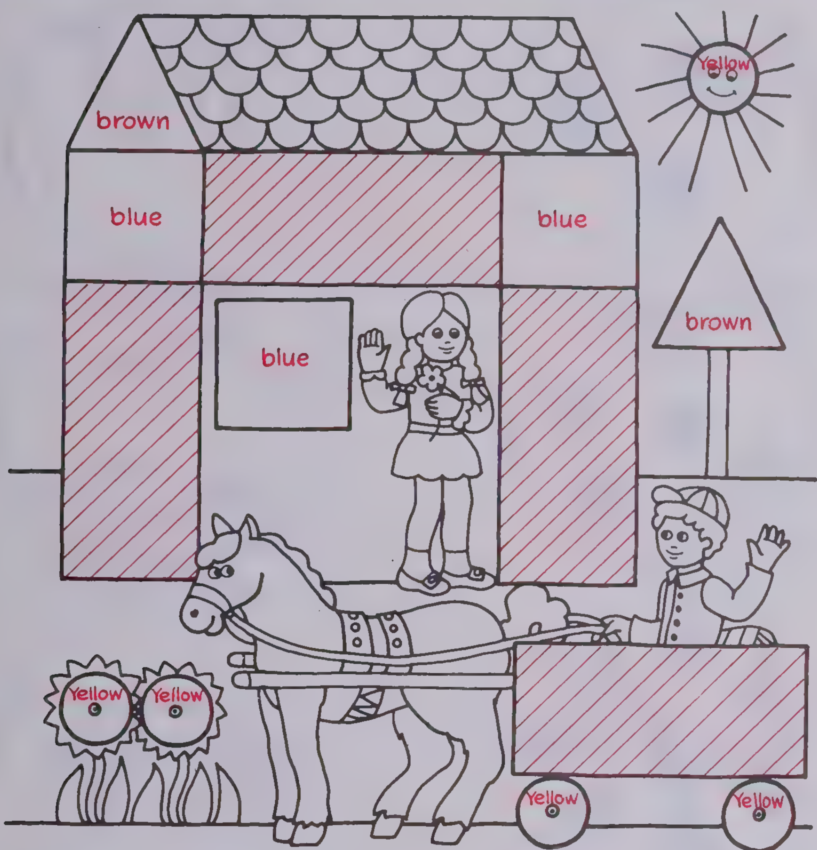
Pre-Book Activity

Distribute the attribute pieces so that each child has a set and can use them in response to an "I'm thinking of a Figure" game. For example, you might say "I'm thinking of a figure that has 3 sides, is blue, and is small." "Hold up the figure I'm thinking of." Children should respond by holding up the small blue triangle. Then ask someone to give the name of the figure. You might refer to the square as "a figure with four equal sides" and a circle as "a figure that has no straight line segments." Use adaptations of these suggestions to review the basic figures and also to provide further experience in comparing size as well as shape.

a 62 sixty-two

Let's have fun





Recognition of figures

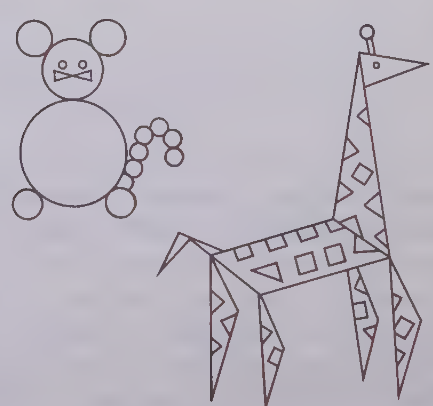
Teaching
Page a-62

Ask the children to look at page a-62 and see if they can figure out what they are expected to do. The procedure is identical with that on page a-61, except that they may have to search harder to find the shapes to color.

When the children have finished, you might take the opportunity to evaluate how well they can find shapes which they think are the same size as the boy's cart. (Example: the sides of the house.) The two flowers are the same size as the sun, and the tree is the same size as the triangle on the house top. You might want to stress that these things appear to be the same size only in the picture.

Follow-up

Assist the children in experimenting with ways to represent a circle by drawing around coins, glasses, bottle caps, and so on. Give each child an envelope of circular regions and other plane figures, paste, and a sheet of construction paper. Suggest that they create an imaginary animal by pasting the geometric shapes together. See the examples.



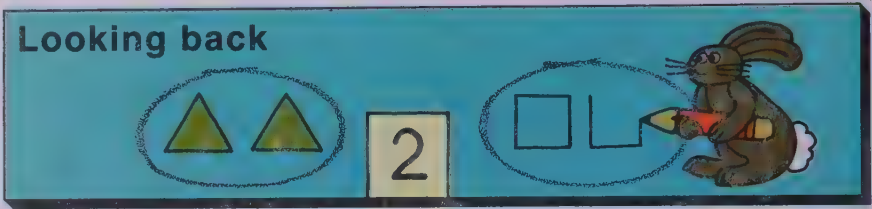
Resources for Active Learning

TEACHING AIDS FOR ELEMENTARY MATHEMATICS, "Polygon City," pp. 110–111, Holt, Rinehart and Winston

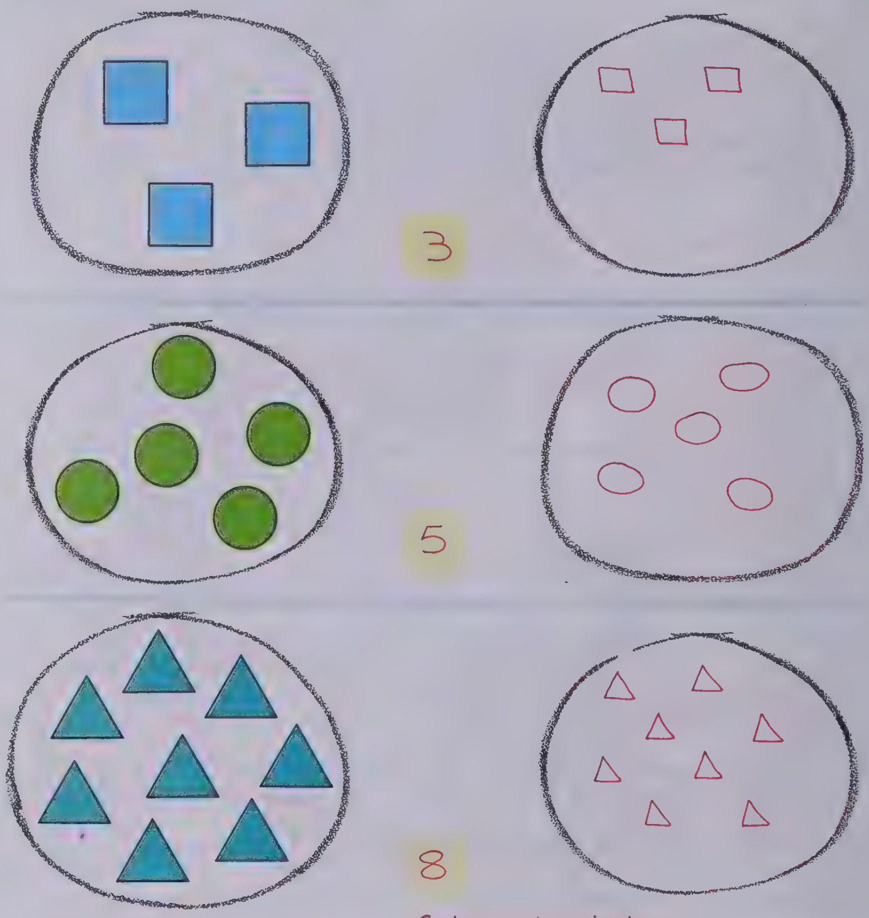
Teaching
Page a-63

Ask children to look at the top of the page and see if they can explain what they are expected to do. Be sure they realize that they should simply draw the same number of objects as in the given set and then write the numeral that names the cardinal number of the set.

Looking back



How many? Draw a set that has the same number.



Cumulative review

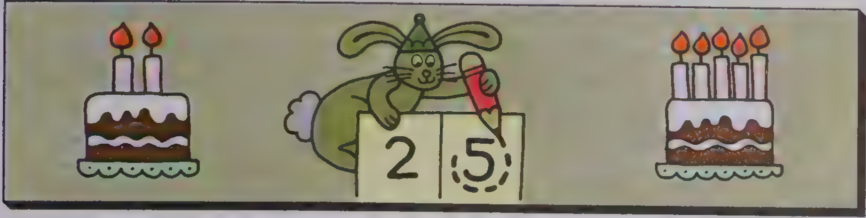
Objective

The child will demonstrate his ability to work with the concepts presented in the first five modules.

While you may use these pages to help you decide whether or not the children are ready to proceed with the next lessons, these pages are intended chiefly to be used as a review of the basic ideas treated in the first five modules.

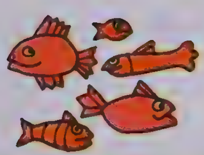
Pre-Book Activity

Use any available materials which are suitable for matching sets and numerals. For example, scatter the paper plates showing sets and those showing numerals along the chalkboard tray or table. Ask a child to pick a set plate and another child to find the matching numeral plate. Do this until all the sets and numerals have been matched. Then use these materials to review larger sets and order. For example, ask two children to each pick a numeral or set plate and then ask a third child to tell which represents the larger number. Finally ask ten children to pick a numeral plate and put themselves in order from 0 to 9.



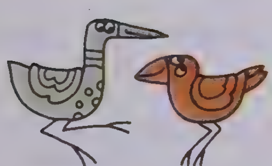
How many? Ring the larger.

4 7

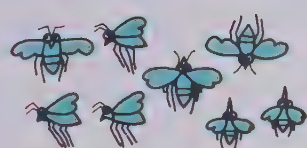


5 2

9 6



2 8



Cumulative review

Teaching
Page a-64

Ask the children to study the pictures at the top of the page and again ask if anyone can explain what to do. On this page children must identify the number of the set, write the numeral, and then ring the numeral of the larger set. You might want to remind some children that they are not expected to tell how many are in a set by merely looking at it, but that they should count the number of objects when the set has more than three or four.

Follow-up

If children need further practice in writing numerals, prepare a worksheet similar to the following.

	0			
△	1			
△△	2			
△△△	3			
△△△△	4			
△△△△△	5			
△△△△△△	6			
△△△△△△△	7			
△△△△△△△△	8			
△△△△△△△△△	9			

Some children might also enjoy a small-group game with a partial deck of cards. You might explain the rules to four more able children and have them be the dealers and teach others to play. Remove from a full deck all tens, jacks, queens, and kings. Deal four cards to each player (six players at the most). The object of the game is to get a run of three or four cards, such as ace, 2, 3 or 5, 6, 7, or 2, 3, 4, 5. A run of three is worth one point and a run of four, worth five points. Each player examines his cards and may discard one or two cards which the dealer then replaces, dealing the cards face up. When everyone has received his new cards, the dealer says "show" and each player shows his cards. The dealer or another player tallies the score. After five rounds the total scores are counted and the player with the highest score wins.

YELLOW MODULE, UNIT B

Sums to 5

Pages b-1 to b-14

General Objectives

To introduce the concept of addition
To introduce the idea of equality
To introduce the idea of solving equations
To begin teaching combinations for sums of five or less
To teach the vertical form of addition

In this module, we introduce the children to addition by having them focus their attention on the union of two sets. The child's first, and most important, encounter with set union is through the physical activity of putting sets together and then relating this action to symbols that "tell the story." In the text the concept of set union is presented to the children by varying the color, size, position, and objects in the sets pictured in the text and by picturing "combining" to form the union of sets. The plus and equality signs are introduced as children learn to relate the union of sets to equations. Special attention is given to the additive property of zero. Once the children understand addition and the idea of solving equations, vertical notation for addition problems is introduced. The module closes with the usual review page and a change of pace page. The latter introduces the strips which accompany this section of the series and will be used throughout the series.

Mathematics

The main mathematical ideas of this module are addition, equality, equations, and solutions of equations.

Addition is approached through the concept of the union of two disjoint sets. *Disjoint* sets are sets that have no common elements. The *union* of two sets A and B is defined to be the set containing those elements that are in A or in B or in both A and B. The following examples may clarify the meaning of this definition.

Example 1.
Consider two sets A and B as follows:

$$A = \{m, n, o, p\}, \qquad B = \{p, q, r\}$$

The union of A and B is denoted by $A \cup B$:

$$A \cup B = \{m, n, o, p, q, r\}.$$

Example 2.
Consider two sets A and B as follows:

$$A = \{1, 2, 3\}, \qquad B = \{15, 16\},$$
$$A \cup B = \{1, 2, 3, 15, 16\}.$$

In Example 1 note that set A contains four elements and set B contains three elements, while the union of sets A and B contains six elements. Clearly, the union of these two sets does not illustrate the concept we call addition, that is, the sum of four and three is seven, not six.

Now consider Example 2, where set A contains three elements and set B contains two elements. This time the union of the two sets contains five elements, which illustrates the idea we call addition.

These examples illustrate the importance of the word *disjoint* in the following definition for the sum of two cardinal numbers.

> Consider two cardinal numbers a and b and sets A and B from these cardinal numbers, respectively, such that A and B are disjoint in relation to each other. The cardinal number of the union of sets A and B is the sum of the cardinal numbers a and b.

Teaching Yellow Module, Unit B

Approximate Time: 7 to 10 days

Materials

yellow triangles, 4 per child
envelopes, 2 per child
flannelboard and felt objects and symbols
individual set collections for the children (beads, counters, checkers)
number line for demonstration
objects for set demonstrations (blocks, pencils, erasers, etc.)
overhead projector, if possible
plastic numeral sets or individual numeral cards, 1 set per child
brown circles, 4 per child
strips which accompany this book (these strips may be replaced with the Cuisenaire rods) – 1 set per child

Vocabulary

add	equals	plus	statement
addition	equation	solve	true
combination	pattern	sum	

Children's understanding of most of the words in this vocabulary will be helpful in communicating the ideas of this module.

They should have many opportunities to work with sets of counters and suitable everyday objects. Proper use of materials plays a vital role in helping children relate their experiences with concrete materials to the abstract ideas of number and the symbolism used to express these ideas.

The strips introduced in the change of pace lesson on page b-14 have a definite mathematical structure. If the white strip is considered the unit, then the strips may be considered to represent the whole numbers from 1 to 10 in the following order

white – 1	dark green – 6
red – 2	black – 7
light green – 3	brown – 8
purple – 4	blue – 9
yellow – 5	orange – 10

It is recommended that you have the children refer to the strips according to their color, postponing the assignment of number names. Children will benefit from discovering relationships among the strips before they are introduced to number names. If a child begins to correctly assign number names to the strips, praise him, but do no overemphasize. (Note that the number names assigned above are correct only if the white strip is considered to represent the number one. That is, if the red strip were considered one, then the white strip would represent $\frac{1}{2}$, and all the other strips would change accordingly.

Evaluation of Progress

Several pages in this unit may serve as evaluation instruments. Observe carefully that the children have a basic grasp of the addition concept and are beginning to master the sums to 5. However do not expect complete mastery at this time. Some children may still need to rely on counting and concrete objects to find even these simple sums.

Resources for Active Learning

General Activities:

MATHEX: Operations No. 3, ". . . Mathematical Operations" and "Writing Equations," pp. 1–8, Encyclopaedia Britannica Publications, Ltd.

Nuffield Project: MATHEMATICS BEGINS 1, "Addition," pp. 43–51, Wiley

SETS, NUMBERS AND POWERS, "Lessons and Games . . . States and Operators," pp. 104–107, Herder and Herder

Manipulative Devices:

Cuisenaire Rods (Cuisenaire Co.)

Sum Stick (Childcraft; Hammett)

Unifix material (Educational Teaching Aids; Math Media; Responsive Environments Corp.)

Investigation
Page b-1

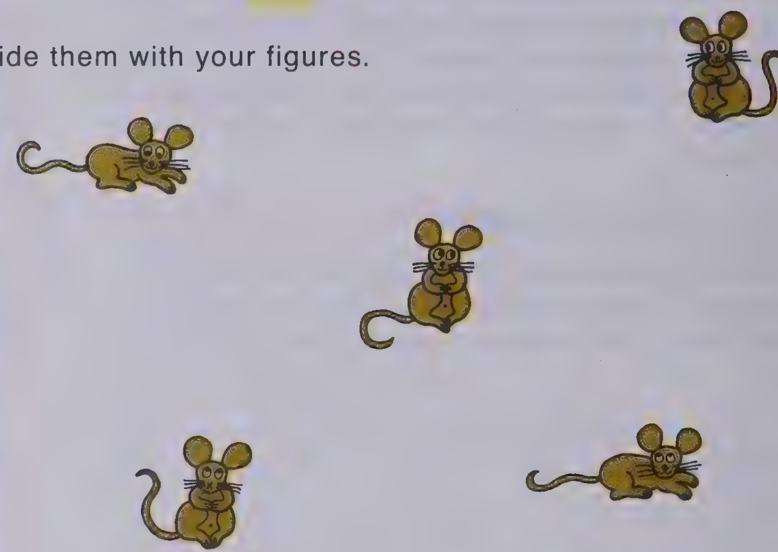
Use the illustration at the top to describe to the children how they will use the circles and triangles to cover the mice. Then direct their attention to the mice in the main section of the page and ask them to count them. They should then write the correct numeral, 5, in the space provided. Ask them to cover the mice using any combination of circles and triangles they want, just as long as each mouse is covered by only one figure. When they have finished, ask them to look at their page and count how many circles they used and how many triangles. Then direct their attention to the questions at the bottom. Read the first two questions for them and tell them to record the number of brown circles and the number of yellow triangles in the appropriate two boxes. Now ask them to cover the mice again, this time with different numbers of brown and yellow shapes. Again when they have finished, help them record how many of each figure they used.

Note that it is not necessary to mention addition during this investigation. The important point is for children to handle the figures and talk about the combinations which result: 4 brown and 1 yellow, 1 brown and 4 yellow; or 2 brown and 3 yellow or 3 brown and 2 yellow.

Let's do

How many mice? 5

Hide them with your figures.



How many circles?		How many triangles?	
How many circles?		How many triangles?	

Answers that are possible: 1 circle and 4 triangles
4 circles and 1 triangle
2 circles and 3 triangles
3 circles and 2 triangles

Readiness for addition

Purposes

To introduce the idea that various combinations can be used to form sets of the same number

To introduce the idea of the union of two sets

To prepare for an understanding of addition

Preparation

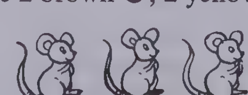
Materials

four brown circles and four yellow triangles

Each child will need four triangles and four circles. Unless you want to begin the math lesson with a warm-up, such as identifying numbers of demonstration numeral cards and set cards, no preparation is necessary. The investigation will be done on the text page.

Follow-up

A worksheet similar to the following would be helpful in directing further experiences with combinations of concrete materials such as counters. Direct the children to use only the number of circles and triangles indicated and cover each mouse as they did in the investigation.

Use 3 brown ○, 3 yellow △ 	____ ○ ____ △
	____ ○ ____ △
Use 2 brown ○, 2 yellow △ 	____ ○ ____ △
	____ ○ ____ △

Let's talk

Readiness for addition

Discussion
Page b-2

After children have discussed the combinations they found with the figures, direct their attention to page b-2. The picture on this page is planned to provide material for discussing the basic additive concept. Ask the children to explain what they see in the picture. Elicit from them that there are three birds on the wire, two birds flying so there are five birds in all; similarly there are two boys and two girls, so there are four children; and there are two dogs playing, one dog sleeping, and three dogs in all. During your discussion, point out how we can think about the same things in different ways. For example, if we think of the birds as perched on the wire or as flying and we think then of 2 sets: one set of 3 birds and another of 2 birds, we can think of birds in general no matter what they are doing and then we count 5 birds.

Mathematics

Addition is approached through the concept of the union of two disjoint sets. For a detailed explanation of the definition of addition, see the mathematics section at the beginning of this module. Note that in this lesson various groupings of sets are used. Brown circles and yellow triangles together become the set of items which cover the mice. Similarly flying birds and perched birds are discussed as a single set of birds. Since addition is so closely related to classifying and grouping things, this introduction to the idea of union of disjoint sets is important. Note, however, that the words *union* and *disjoint* should not be used with the children.

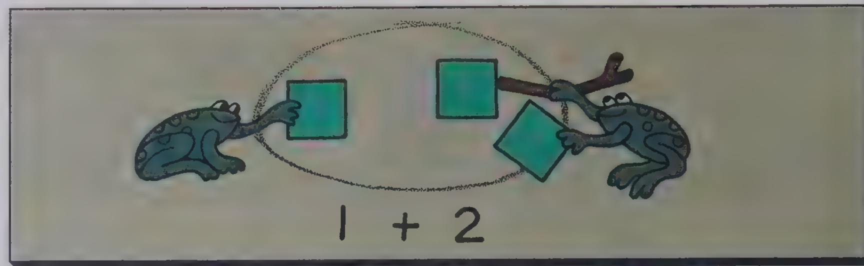
Resources for Active Learning

Combining groups:
WORKJOBS, pp. 148–155, 160–161, 164–165, 168–169, 172–173, 194–197, 202–209, Addison-Wesley

Games providing experience in seeing relationships:
MATH ACTIVITIES, Games 2/22–26, pp. 32–35, Allyn and Bacon

Teaching
Page b-3

If you followed the pre-book activity suggested for this lesson, children should not have trouble interpreting the illustration at the top of the page. You might tell children to think of the ringed section on the page as their string or yarn. Then, as you read the directions for them, they should put their counters in that section as you direct. Be sure children know what the phrase "solve the equation" means, that is, that they should put in the frame the numeral which makes the statement true. Then read the completed equation with them, using the phrase: "one plus two equals three." Have as many children as possible work through the second example independently.



Put in	Put in	How many in all?
1	2 more	3

Solve the equation.

$1 + 2 = \boxed{3}$

Put in	Put in	How many in all?
3	2 more	5

Solve the equation.

$3 + 2 = \boxed{5}$

Objective

Given an addition equation with a sum less than 6, the child will be able to solve the equation using physical objects or appropriate pictures.

It is in this lesson that the child is formally introduced to the addition concept. Children work first with sets, physically joining them together, then they record their action with numerals and the symbols + and =. Note that sets may be joined together, but only numbers may be added and this addition is recorded with numerals and symbols.

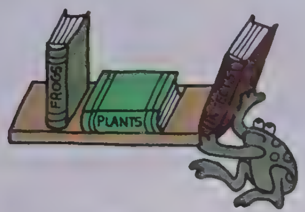
Pre-Book Activity

Materials

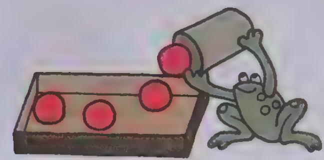
counters

Give each child about five or six counters. It would also be helpful to give each child a piece of string or yarn about 1 metre long to encircle his work. Direct the children in making combinations of three, four, or five. You might speak of the string as a fence, and the figures as barns or buildings, and ask the children to put two barns and two chicken houses inside their fence. Ask: "How many buildings in all?" State the combination numerically by first saying: "2 barns and 2 chicken houses may be thought of as 4 buildings." Then write $2 + 2 = 4$ saying: "Two plus two equals four." When you have done this for several examples, explain that the symbol "+" is used when we add two numbers, and the equality symbol "=" lets us show that different symbols can name the same number. Thus, $2 + 2$ and 4 are both names for the same number so we write $2 + 2 = 4$. Work through several examples of equations with the children.

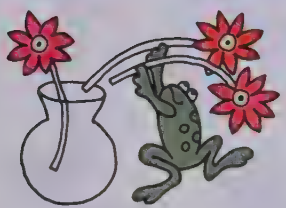
Solve the equations.



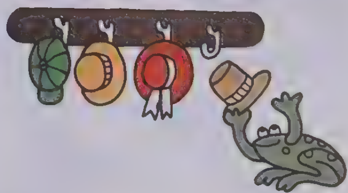
$2 + 1 = \boxed{3}$



$2 + 2 = \boxed{4}$

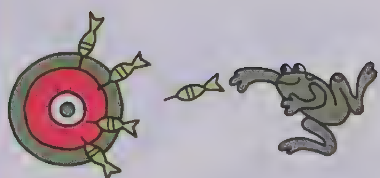


$1 + 2 = \boxed{3}$



$3 + 1 = \boxed{4}$

$2 + 3 = \boxed{5}$



$4 + 1 = \boxed{5}$

Teaching
Page b-4

Call the children's attention to the left top frame. Ask them to explain what is happening in the picture. Relate their description to the equation written below it. You might read the incomplete equation, asking children for the answer "two plus one equals what number?" It is important that children relate the equation for each picture to the action shown. Discuss the action of each picture, giving careful guidance, until the children feel at ease with equation notation. If you think the children are capable, have them finish the last few frames on their own and then have them describe the action in each picture and tell how they completed each equation.

Follow-up

Most children will need much exposure to the equation symbols + and = before they are comfortable with them.

If plastic numerals are available, have the children use them at their desks to form different equations for sums of 5 or less. If they are not available, a triple set of numeral cards marked 0 through 9 will be extremely useful. Cards can be made by cutting small index cards in two, or stiff tagboard can be used. Mark one set of numerals in green, one in blue, and one in red. Also make a set of symbols (the plus, minus, and equal signs) and some placeholder boxes to use throughout the year.

$\boxed{2} \quad \boxed{+} \quad \boxed{3} \quad \boxed{=} \quad \boxed{5}$

Individual numeral cards

Mathematics

Equality is associated with different symbols for the same number. For example, in the first frame on page b-4, the children may think about the number of the set in two different ways: first, as two and one, and then as three. In this lesson, the children are exposed for the first time to equations and the idea of solving an equation. Actually, this step is not likely to cause any difficulty, other than getting used to working with symbols. We do, however, want to introduce the words *equation, solve,* and *solution* at this time. When the children write the correct numeral in the answer frame, we say that they have found the solution or that they have "solved the equation."

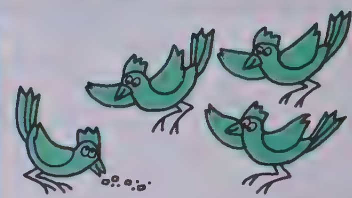
Teaching
Page b-5

Use the illustration at the top to introduce the lesson. Then direct the children to look at the left top frame. Ask a child to describe the two sets and explain that they may be thought of as one set. Ask the children to complete the equation by tracing over the dashed numeral 3. Read the completed equation with them saying "two plus one equals three."

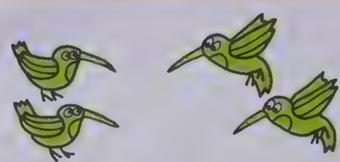
Depending on the children's ability, work through one or two other examples with them. Then ask them to solve the remaining equations. Give help to those having difficulty.

Solve the equations.

$2 + 1 = \boxed{3}$

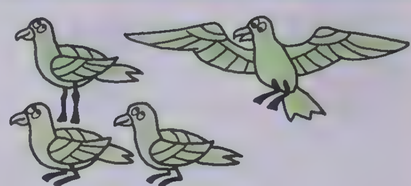


$1 + 3 = \boxed{4}$

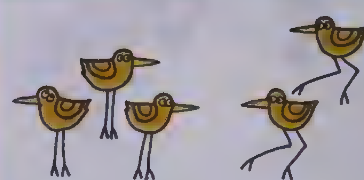


$2 + 2 = \boxed{4}$

$2 + 3 = \boxed{5}$



$3 + 1 = \boxed{4}$



$3 + 2 = \boxed{5}$

Objective

Given addition equations for sums of 5 or less, and related illustrations of sets, the child will be able to solve the equations.

In this lesson, children are again encouraged to think of putting two sets together. Their work with equations continues to be related to pictures of sets. They should be given counters to use as they wish. Many children will be able to solve equations with combinations less than 5 without reliance on counters, but each child should feel free to use counters and to discontinue use of them according to his own choice. Some children would also benefit from manipulating the plastic or tagboard numerals and symbols suggested in the previous lesson.

Pre-Book Activity

Use any items in the classroom suitable for combining sets. Record in equation form every demonstration combination you make and have the children complete it and read it together with you. Groups of children themselves may form sets to be combined. For example, a group of three children might be joined by a group of two children to show $3 + 2 = 5$. If your children are capable, occasionally begin with an expression, such as $1 + 3$ and ask the children to show with counters or on the flannelboard a set combination action to match it and then ask them to build an equation from it, namely, $1 + 3 = 4$. This activity may also be done in story form. For instance, given the phrase $2 + 2$, you might say "Two children on a bench, two more children join them; now there are four in all," and write $2 + 2 = 4$.

Solve the equations.

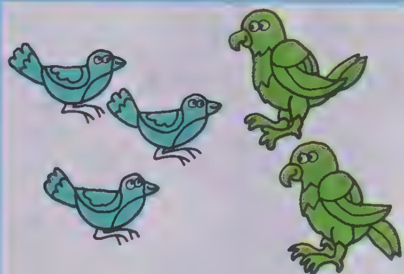
$4 + 1 = \boxed{5}$

$1 + 2 = \boxed{3}$

$2 + 3 = \boxed{5}$

$2 + 2 = \boxed{4}$

$1 + 4 = \boxed{5}$



$3 + 2 = \boxed{5}$

Teaching
Page b-6

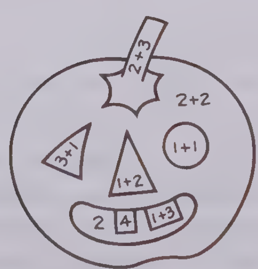
Call the children's attention to the first frame at the top. Ask them to describe the picture. Elicit from them that if we think of the different kinds of birds shown, we can think of one set of four and another set of one. But if we think of how many birds there are all together, we can think of a set of five. Thus we write $4 + 1 = 5$. Ask the children to complete the equation by tracing over the numeral 5; then have them read the equation together with you.

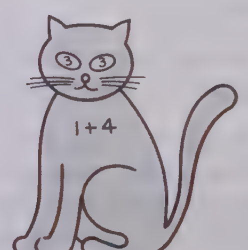
You might want to guide some children carefully through these questions. With others you will want them to work without such direct help.

Follow-up

Make a ditto showing a picture appropriate to the season (e.g., Halloween). Label various parts of the picture with other names for numbers, for example: $2 + 3$, $3 + 2$, $1 + 4$, $4 + 1$ for five; $2 + 2$, $3 + 1$, $1 + 3$ for four; and so on. Then write directions on the board for the children to follow in coloring the picture. For example:

Color the names for *two* yellow.
Color the names for *three* green.
Color the names for *four* orange.
Color the names for *five* black.





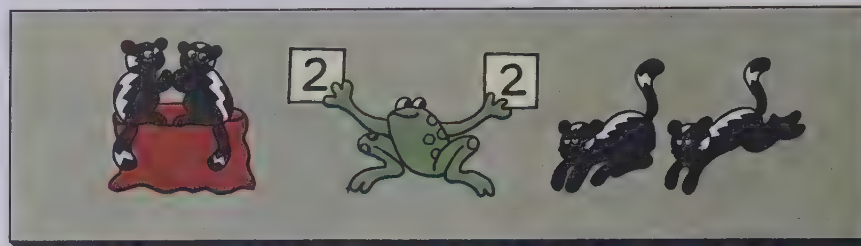
Resources for Active Learning

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Addition," Nos. 28–29, Responsive Environments Corp.

Teaching
Page b-7

Have the children look at the sample problem in the top left frame. Ask them how many beavers are on top of the rock and how many in the water. Finally, ask how many beavers in all. Point out that the first exercise has been completed with a dashed numeral. Direct them to trace over the dashed numeral 3. Then read with them: "Two plus one is three."

Continue through other examples similarly. If possible, have the children work these without your guidance but allow those who want to use counters or other concrete materials to do so.



Find the sums.

$$\begin{array}{r} 2 \\ +1 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 1 \\ +3 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 4 \\ +1 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 3 \\ +2 \\ \hline 5 \end{array}$$

Objective

Given the standard vertical notation for sums of 5 or less, the child will be able to give the sums.

The only thing new in this lesson is the notation for writing sums. Although the vertical form is not an actual equation, children should understand that they can think of it in the same way that they thought of the equations. However, the directions "solve the equation" should only be given with equations; with the vertical form you simply say "add" or "find the sums."

Pre-Book Activity

Give the children an opportunity to solve equations without the aid of demonstration sets. Write equations such as $3 + 1 = \square$, $2 + 2 = \square$, and $1 + 2 = \square$ on the chalkboard, and ask the children to try to complete them without seeing the sets represented. For those having difficulty, suggest that they think about sets when they read an equation like $2 + 2 = \square$. After the children have completed several equations, and while these equations are still on the chalkboard, tell them they are now going to learn a different way to write addition problems. Choose one of the equations, such as $3 + 2 = 5$, and write it again where there is plenty of empty space on the chalkboard. Beside this equation, write the example:

$$\begin{array}{r} 3 \\ +2 \\ \hline \end{array}$$

Instruct the children to think of this in the same way they thought of the equation. They should read it: "Three plus two is . . . ." When the children indicate that the

Find the sums.

$\begin{array}{r} 3 \\ +2 \\ \hline 5 \end{array}$

$3 + 2 = \boxed{5}$

$2 + 2 = \boxed{4}$

$1 + 3 = \boxed{4}$

$1 + 1 = \boxed{2}$

$4 + 1 = \boxed{5}$

$2 + 3 = \boxed{5}$

$1 + 2 = \boxed{3}$

$3 + 1 = \boxed{4}$

$1 + 4 = \boxed{5}$

$\begin{array}{r} 3 \\ +1 \\ \hline 4 \end{array}$ $\begin{array}{r} 1 \\ +1 \\ \hline 2 \end{array}$ $\begin{array}{r} 1 \\ +4 \\ \hline 5 \end{array}$ $\begin{array}{r} 2 \\ +1 \\ \hline 3 \end{array}$ $\begin{array}{r} 3 \\ +2 \\ \hline 5 \end{array}$

$\begin{array}{r} 2 \\ +2 \\ \hline 4 \end{array}$ $\begin{array}{r} 4 \\ +1 \\ \hline 5 \end{array}$ $\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$ $\begin{array}{r} 1 \\ +3 \\ \hline 4 \end{array}$ $\begin{array}{r} 1 \\ +2 \\ \hline 3 \end{array}$

Teaching
Page b-8

Ask the children to look at the pictured set and explain what it has to do with the two addition examples. Read both the vertical and horizontal examples with them and have them write in the answers. Then explain to them that they have just "found the sum" of 3 + 2. Give a few oral examples until they realize that the expression "find the sums" written on their pages is another way of directing them to add.

Have them complete the page on their own. Observe them as they work and give any necessary guidance, particularly to any who have difficulty changing from the horizontal to the vertical form.

sum is five, write the numeral 5 underneath the bar. Do this several times to emphasize the fact that this is simply a different way of writing addition problems. Do not refer to exercises written in this way as equations. Some children may still require concrete objects to count, join, and recount. Allow them to use these concrete objects as long as necessary.

Follow-up

The following activity will give the children more practice in matching number combinations and their sums. Write directions similar to these on the board, or duplicate them on sheets for the class:

Match the sums.	
5	1 + 2
4	2 + 3
3	2 + 2
2	1 + 1

If you have made the "Match Me" boards suggested on page a-36, you might want to adapt them to matching sums by simply making appropriate tagboard strips.

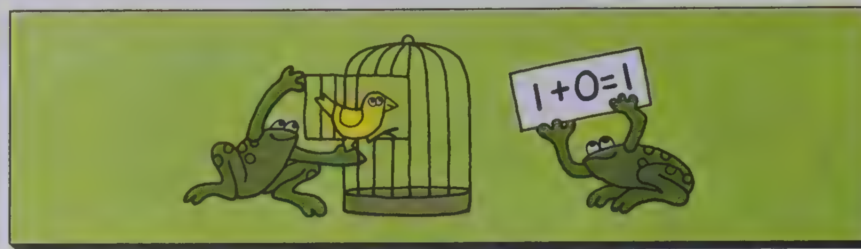
Resources for Active Learning

MATH WORKSHOP: Games and Enrichment Activities, "Domino Cards," pp. 15–17, Encyclopaedia Britannica Educational Corp.

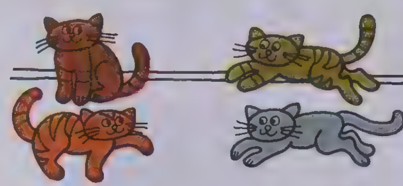
Teaching
Page b-9

The equation $1 + 0 = 1$ in the demonstration art will provide a good introduction. Then call the children's attention to the first exercise at the top of the page. Ask a child to describe how the kittens are related to the equation $2 + 2 = \square$. Direct the children to complete the equation, then read it together with them. Work through the other sample exercise similarly, this time stress the idea that when you add zero to a number, the sum is that same number. Again have them complete the equation and read it together.

The children should finish the remaining exercises on their own. If some children still want to use counters for these simple facts, allow them to do so only after suggesting that they try to do some without them.



Solve the equations.



$2 + 2 = \boxed{4}$

$3 + 0 = \boxed{3}$

$1 + 2 = \boxed{3}$	$1 + 0 = \boxed{1}$
$2 + 0 = \boxed{2}$	$1 + 3 = \boxed{4}$
$3 + 1 = \boxed{4}$	$0 + 3 = \boxed{3}$
$2 + 1 = \boxed{3}$	$4 + 1 = \boxed{5}$
$1 + 1 = \boxed{2}$	$0 + 5 = \boxed{5}$
$4 + 0 = \boxed{4}$	$3 + 2 = \boxed{5}$

Objective

Given addition equations for sums of 5 or less, the child will be able to find the sum.

This lesson serves the dual purpose of providing further practice with combinations of five or less and of introducing the additive property of zero. Children should discover that zero is the identity element for addition, that is, when they add zero to a number, they get that number as the sum.

Pre-Book Activity

Materials

counters

Distribute at least five counters to each child. Ask the children to hold a set of 2 in one hand and a set of 3 in the other hand. Then ask them to combine their two sets, and have a child write an equation to show what they did. Have everyone read the equation together. Direct them through other combinations, each time having a child write the equation. Then ask them to form a set of zero in one hand and a set of three in the other. Ask them to join the two sets on their desk and then ask if anyone can write an equation for this combination $(0 + 3 = 3)$. Continue with examples of combinations where one set is zero. Each time, have a child write an equation to record the combinations. Gradually, start leading the children into oral exercises without use of the counters. Help the chidren respond to phrases such as "Find the sum of two plus two," or "What is the sum of zero and one"?

Find the sums.

$$\begin{array}{r} 2 \\ +0 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 1 \\ +3 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 1 \\ +4 \\ \hline 5 \end{array} \quad \begin{array}{r} 3 \\ +0 \\ \hline 3 \end{array} \quad \begin{array}{r} 1 \\ +3 \\ \hline 4 \end{array} \quad \begin{array}{r} 2 \\ +2 \\ \hline 4 \end{array} \quad \begin{array}{r} 0 \\ +1 \\ \hline 1 \end{array} \quad \begin{array}{r} 2 \\ +1 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 4 \\ +0 \\ \hline 4 \end{array} \quad \begin{array}{r} 3 \\ +1 \\ \hline 4 \end{array} \quad \begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array} \quad \begin{array}{r} 1 \\ +1 \\ \hline 2 \end{array} \quad \begin{array}{r} 4 \\ +1 \\ \hline 5 \end{array} \quad \begin{array}{r} 0 \\ +5 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 1 \\ +2 \\ \hline 3 \end{array} \quad \begin{array}{r} 2 \\ +0 \\ \hline 2 \end{array} \quad \begin{array}{r} 3 \\ +2 \\ \hline 5 \end{array} \quad \begin{array}{r} 1 \\ +3 \\ \hline 4 \end{array} \quad \begin{array}{r} 0 \\ +3 \\ \hline 3 \end{array} \quad \begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$$

Teaching
Page b-10

The exercises on this page differ from those on page b-9 only in form; here children are given an opportunity to become more accustomed to the standard vertical form.

Have children describe the pictured sets of the first sample exercise. Then ask them to find the sum of two plus zero. Be sure all of the children understand that this simply means to add. Instruct them to carefully write the numeral for their answer under the 2 and 0. Read together "two plus zero is two." Work through the next sample exercise similarly. Then ask the children to finish the exercises on their own. If children need counters, allow them, but encourage them to do these simple facts in their heads.

Observe the children carefully. If any child has difficulty, try to diagnose his learning problem as soon as possible and help him with remedial work rather than have him try exercises for which he is not ready.

Follow-up

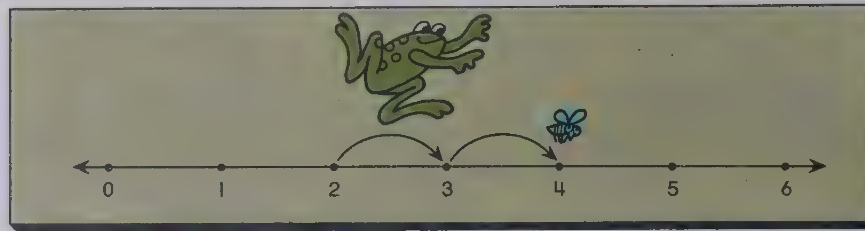
If you think children need further practice with the standard vertical notation, prepare a worksheet similar to page b-10.

You might also distribute copies of the following exercise. Ask the children to choose a bright crayon and check the equations for errors.

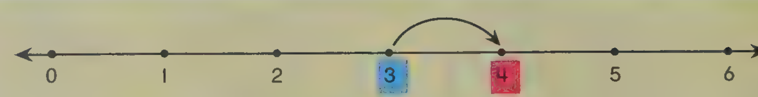
Mark an X beside each incorrect problem.	
3 + 2 = 5	1 + 2 = 2
3 + 2 = 4 X	2 + 3 = 5
0 + 3 = 3	3 + 1 = 4
2 + 2 = 2	3 + 2 = 4
2 + 2 = 4	1 + 4 = 5
3 + 1 = 3	2 + 4 = 5

Teaching
Page b-11

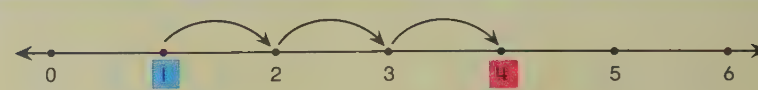
Ask the children to look at the illustration at the top of the page and describe what they see. When children refer to the frog trying to catch the jumping fly you might want to make up some stories to give some personality to the art. Then ask the children to give the number of the fly's starting point. Use a demonstration number line on the overhead projector or the chalkboard to develop the illustration relating it to the equation $2+2=4$. Talk about the different places the fly might land; that is, "Where would he land after 1 jump? . . . after 2 jumps? . . . after 3 jumps?" Then talk about recording his jumps and show the equations $2+1=3$, $2+2=4$, $2+3=5$ as you work through each on the demonstration number line. Next have the children look at the first equation on page 11 and again use the demonstration number line to discuss it. Point out how the starting points match the first number in the equation. Then take the number of jumps as given by the second number in the equation to land on the sum. It is important that children see how the number line may be used as a counting device to aid in solving equations. Therefore work through each of these equations carefully. For the last equation, you might say: "We start at 2; add 3; so we move one, two, three, and we land on 5."



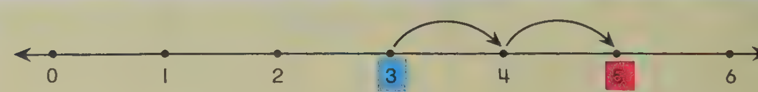
Solve the equations.



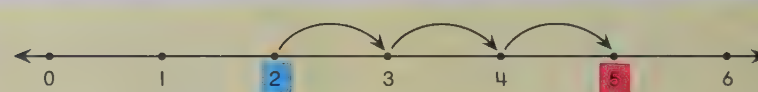
$3 + 1 = \boxed{4}$



$1 + 3 = \boxed{4}$



$3 + 2 = \boxed{5}$



$2 + 3 = \boxed{5}$

Objective

Given a number line showing whole numbers from zero to six, and addition equations to solve, the child will be able to use the number line to solve the equations.

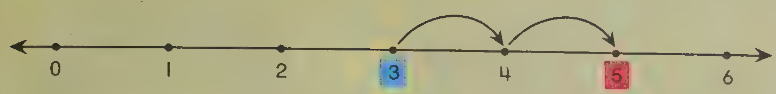
As presented in this module, the number line is a device which children can use to help them find sums. This use of the number line will be even more advantageous to the child with combinations that will be studied later. Thus, the emphasis here is not so much on how the number line might pictorially represent an equation, but rather how it may be used as a counting device for finding sums.

Pre-Book Activity

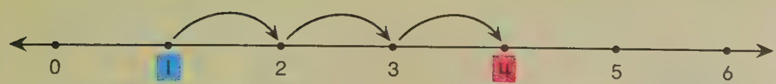
Children would benefit from a review of the number-line and of the order of the counting numbers. Set nine objects along the chalkboard ledge and use them to review counting. As children count have a child point to each object on the ledge while you write the numerals 1, 2, . . . 9, on the chalkboard. Then erase these numerals and show on the flannelboard or on the chalkboard an incomplete sequence, 1, 2, , , 5, 6, , 8, , and have children complete it. Finally, show an incompletely labelled number line (to 6) and see if children can complete the labelling.

Here, even though the labels end at 6, the arrow at the end means the line itself goes on and on.

Find the sums.



$3 + 2 = \boxed{5}$



$1 + 3 = \boxed{4}$

$2 + 1 = \boxed{3}$	$3 + 2 = \boxed{5}$
$3 + 1 = \boxed{4}$	$1 + 4 = \boxed{5}$
$1 + 2 = \boxed{3}$	$2 + 2 = \boxed{4}$
$2 + 3 = \boxed{5}$	$3 + 0 = \boxed{3}$
$0 + 4 = \boxed{4}$	$1 + 1 = \boxed{2}$
$5 + 0 = \boxed{5}$	$1 + 0 = \boxed{1}$

Teaching
Page b-12

Ask the children to study the top equation and number line, and describe what the number line shows. Use the demonstration number line to explain how the 3 in the equation matches the starting point; then they can think: "Start at 3, add 2 (that is, jump one, two), land on 5." Thus 3 + 2 = 5. When they have finished writing 5 in the frame, work through the next equation similarly. You might encourage children to move along their number line on the page with a pencil while you use the demonstration number line.

The remaining equations should be solved by the children independently. Point out the number line given above the equations for their use in solving the equations. If you have other desk size number lines, such as those made of acetate, or plastic, or celluloid, you might want children to use them instead. In either case, stress that even if they know the sums from memory they should check their answers by using the number line. They need not actually draw the arrows on the number line; they should be encouraged simply to use the number line as a counting device.

Follow-up

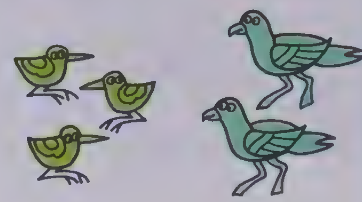
It would be helpful for many children to be able to act out addition equations on a number line placed on the floor. Stretch a roll of paper about 2 metres long and 30 centimetres wide along the floor. Label it from 0 to 5, or 0 to 6. Give children equations such as 3+2=□ and ask them to show the sum by walking on the number line. For this particular equation a child should start at 3, walk or jump 2 spaces, and land on 5.

Teaching
Page b-13

Explain to the children that all of the exercises on this page are similar to some they have already done; they are simply to add or find the sum for each exercise. It would be helpful to work through the first exercise together, but encourage children to do most of the exercises independently. This page might be treated as an evaluation instrument, although children's work on many of the previous pages will have already indicated to you the various levels of understanding and skill the children have achieved.

Show you know

Find the sums.



$3 + 2 = \boxed{5}$

$1 + 3 = \boxed{4}$

$\begin{array}{r} 2 \\ +2 \\ \hline 4 \end{array}$

$\begin{array}{r} 4 \\ +1 \\ \hline 5 \end{array}$

$2 + 1 = \boxed{3}$

$1 + 4 = \boxed{5}$

$5 + 0 = \boxed{5}$

$2 + 3 = \boxed{5}$

$1 + 1 = \boxed{2}$

$3 + 1 = \boxed{4}$

$\begin{array}{r} 2 \\ +2 \\ \hline 4 \end{array}$ $\begin{array}{r} 3 \\ +1 \\ \hline 4 \end{array}$ $\begin{array}{r} 0 \\ +2 \\ \hline 2 \end{array}$ $\begin{array}{r} 2 \\ +3 \\ \hline 5 \end{array}$ $\begin{array}{r} 1 \\ +4 \\ \hline 5 \end{array}$ $\begin{array}{r} 3 \\ +2 \\ \hline 5 \end{array}$

Objectives

Page b-13: The child will demonstrate his ability to work with the concepts presented in this module by finding sums of 5 or less.

Page b-14: The child will organize the colored strips to fit in areas given in the text.

Page b-13 may be used as an evaluation of children's understanding of addition. Be sure that they understand that they may use counters or a number line if they want to.

Page b-14 introduces the centimetre strips. You might choose to spend a full day's lesson distributing the strips, allowing the children free play with them, and then having them complete page b-14.

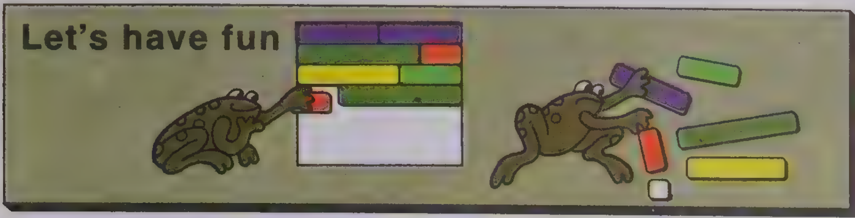
Pre-Book Activity

Materials

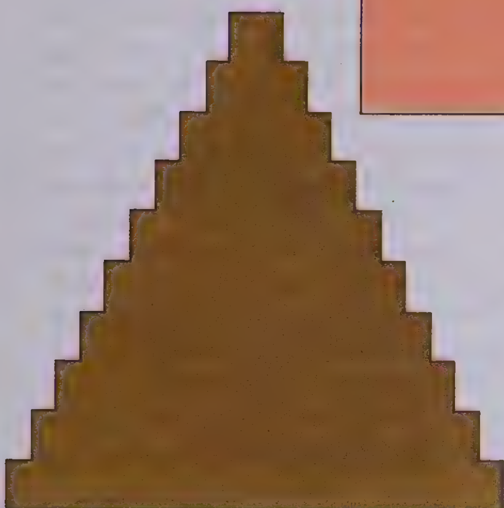
numeral and symbol cards

set of centimetre strips which accompany this series, 1 set for each child

Distribute individual numeral and symbol cards so that each child has at least one each of 0, 1, 2, 3, 4, 5, + and =. If possible, each should have a second card for the numerals 1, 2, 3, 4, 5. Explain to the children that these cards can be used to build equations. Ask them to show $3 + 2 = 5$ on their desks. Give a few other similar equations. Then explain that you will write on the board three numerals and that they should put them together in an



Can you fill the colored spaces with your strips?



Informal introduction to the centimetre strips

Teaching
Page b-14

Distribute an envelope containing a set of strips to each child. Explain to the children that they will be using these strips frequently so you would like them to simply play with them. Suggest that they use them to make the shape of a house, or of an insect, or a butterfly, or a horse, or a man, and so on.

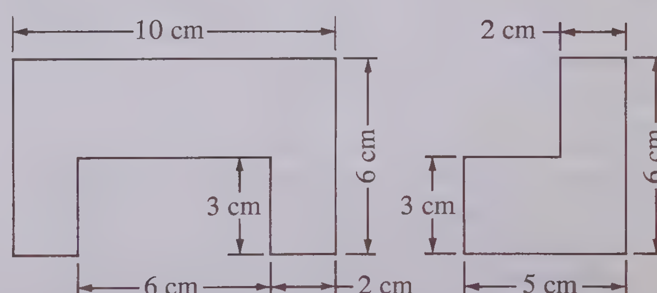
After children have played for a while distribute page b-14 and explain the directions to the children. They may use any strips and place them in any direction (horizontal or vertical) to fill the colored areas. They should discover that the area at the bottom of the page can be filled correctly only by placing the strips horizontally.

As mentioned in the introduction to this module the strips should be referred to by their color names; only later will number names be assigned. Thus, you might talk about the orange strip on the bottom of the staircase or the white strip on top. As children cover the areas have them make comparisons by asking questions such as: "Which strip is the longest? the shortest"? but do not overemphasize these relationships now; this introduction to the strips should be handled with a light touch.

order which will form an equation. Write, for example, 3, 1, 2 on the chalkboard. Children should then build either $1 + 2 = 3$ or $2 + 1 = 3$. You might let one of the first children finished write his equation on the chalkboard and then ask if someone formed another equation whose sum is 3, and have that equation also written on the chalkboard. Be sure children understand that you are writing the numerals in no special order and that they must put them in the correct order to build their equation.

Follow-up

You might encourage children to continue free play with the strips, again suggesting varying shapes they might make. Or you may want to distribute duplicated copies of other areas for them to cover.



Resources for Active Learning

Cuisenaire Rod activities for now and later:

MATHS MINI-LAB, Cards 14–18, 21–34, Selective Educational Equipment.

MATH WORKSHOP: Games and Enrichment Activities, pp. 31–37, Encyclopaedia Britannica Educational Corp.

ORANGLE MODULE, UNIT B

Differences to 5

Pages b-15 to b-24

General Objectives

To introduce the concept of subtraction
To introduce the symbol for subtraction
To begin development of skill with the subtraction combinations related to sums of 5 or less

The concept of subtraction is introduced in the initial stages by using the idea of "take away." This is the most natural method to use in introducing subtraction because the children understand the idea of removing certain objects from a set. They can see that removing certain objects is associated with subtraction just as they saw that putting sets together is associated with addition. In the first grade, great emphasis is placed on the children gaining a firm understanding of subtraction from the "take-away" point of view. The idea of comparing two sets containing different numbers will be taken up later in the program, and the inverse relationship between addition and subtraction will be introduced and developed in an optional module.

After the children have been introduced to subtraction, and the symbol is presented, they begin to develop skill by working with both subtraction equations and the standard vertical form for subtraction.

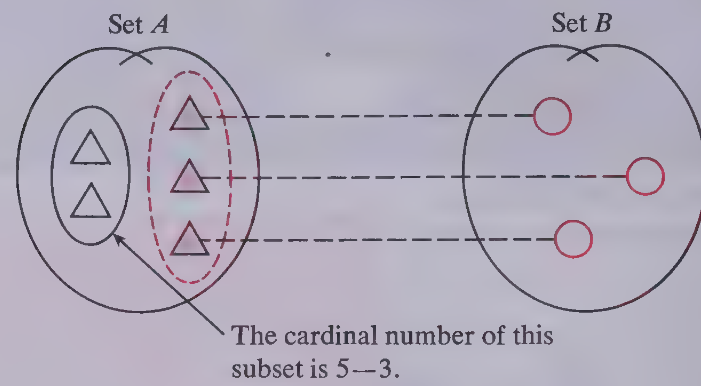
Special emphasis is placed on the subtractive properties of zero.

Mathematics

In this unit, subtraction is introduced by removing objects from a given set. Although this is the clearest, most concrete method of introducing subtraction to children, it is quite difficult to show just how this idea relates to a set definition of subtraction. The following definition of subtraction encompasses both the idea of "take away" and the idea of comparing two sets.

> Let a and b be any two cardinal numbers such that $a > b$ or $a = b$. Choose sets A and B from a and b respectively. Since $a > b$ or $a = b$, there is a subset of A that is equivalent to set B. The difference of a and b, written $a - b$, is the cardinal number of the set of objects in set A other than those in a subset of A that is equivalent to B.

The diagram below illustrates this definition for cardinal numbers 5 and 3.



Of course, the definition and illustration show clearly why subtraction is used for comparing two sets. On the other hand, it is not quite so clear how the idea of removing objects from a given set relates to this definition. Using the diagram above, we can point out that the objects to be taken away from set A to illustrate subtraction would be the subset that we show with the dashed ring. In the diagram, this subset was determined by matching each object in set B with one object in set A. When using the "take-away" method in the exercise $5 - 3 = \square$, we would simply note that set B consists of the counting numbers 1, 2, and 3; we would count 3 objects in set A and take them away. If we wished to consider the problem $53 - 27$ using the idea of "take away," we would merely count out 27 objects from a set of 53. In this case, then, set B could be considered the set of counting numbers 1 through 27. The important point is that the idea of "take away" is merely a special case of the more general concept of subtraction, that of comparing two sets.

Teaching Orange Module, Unit B

Approximate Time: 5 to 7 days

Materials

counters
flannelboard and felt sets and symbols
flash cards showing combinations of 5 and under
individual set collections for the children
number line for demonstration
objects for set demonstrations
overhead projector (if available)
plastic numeral sets for each child or individual numeral cards
strips—one set for each child

Vocabulary

difference	minus	"take away"
	subtract	

Although the phrase "take away" is not a part of the technical vocabulary of arithmetic, it may help if you say "five take away two" rather than "five minus two," at first. "Take away" is precisely applied to removing objects from a set. As soon as the children thoroughly understand the idea of subtraction, use "take away" only when referring to sets and "minus" or "subtract" when referring to numbers. Help the children read a phrase such as "5 − 2," as "five minus two," or "five subtract two." The children should still be given many opportunities to manipulate sets and to write the corresponding equations to help them bridge the gap between concrete and abstract concepts.

Evaluation of Progress

Many of the text pages can aid you in evaluating the children's progress in understanding subtraction and in mastering subtraction facts. You should also help the children to see that for any picture showing a pair of subsets, such as the pictures on page b-18, they can think in terms of either addition or subtraction. Keep in mind that some children may still be dependent on concrete objects to find sums and differences. *Do not expect mastery of the subtraction facts from the children at this time.* Allow them to use the concrete objects as long as they find it necessary.

Resources for Active Learning

General Activities:

Nuffield Project: MATHEMATICS BEGINS 1, "Pre-subtraction experience," pp. 51–54, Wiley

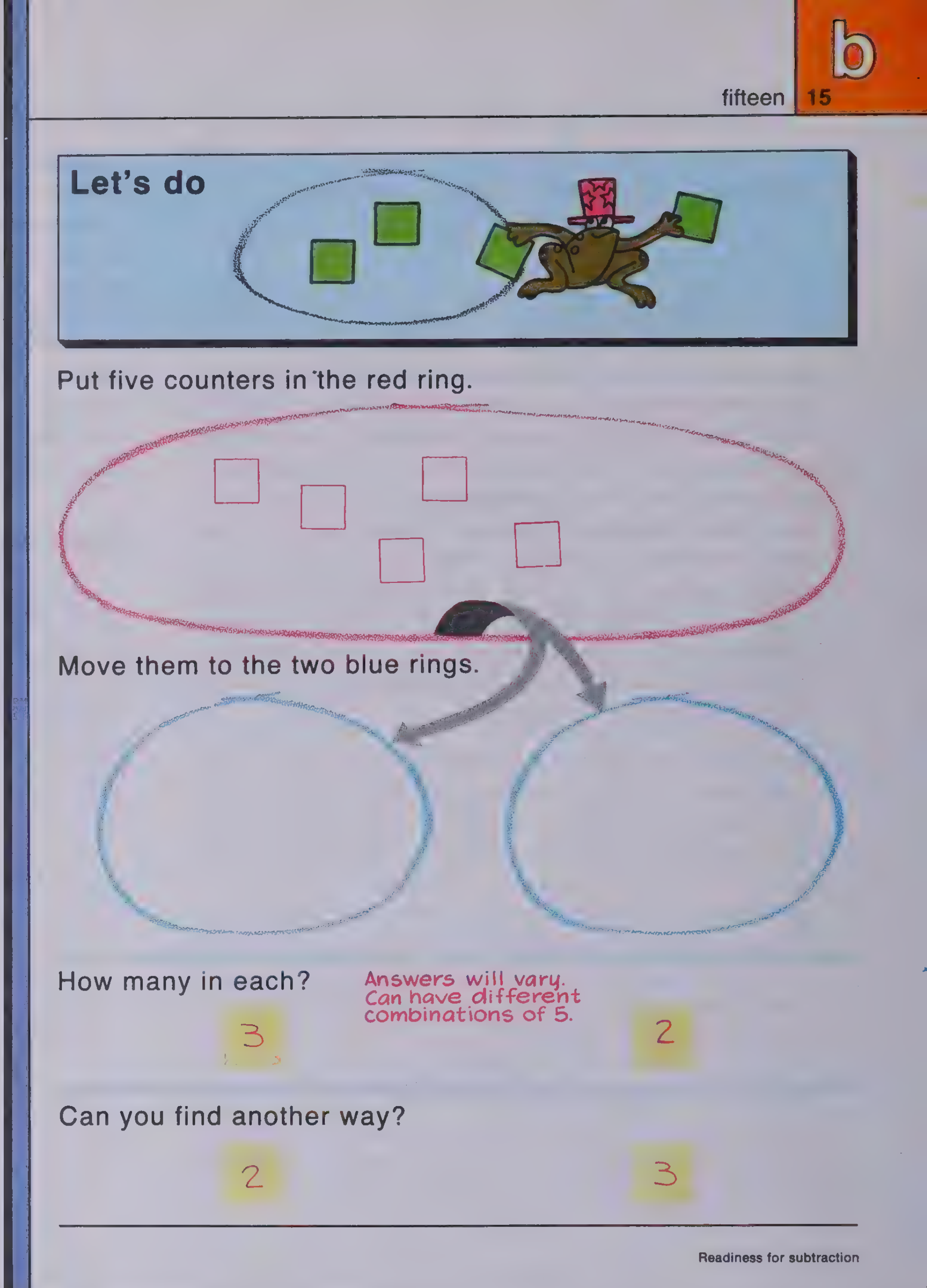
Manipulative Devices:

Sigma Chips (Scott Scientific)

Investigation
Page b-15

Page b-15 provides both an area in which children can work and a space for recording. Direct them to begin by placing five counters in the red ring. Then explain that they should now split up this set of five and move the counters into the two blue rings. When they have done this they should write how many counters they now have in each of the blue rings. Then, again starting with 5 counters, ask if they can find another way to move the counters into the two blue rings. Again they should write how many counters they have put in each blue ring. The possibilities are 2, 3; 3, 2; 1, 4; 4, 1; 0, 5; 5, 0. The 5 and 0 "split" will probably not occur, but there should be no problem if it does.

When children have recorded the two "splits" of five, suggest that they start with another number, such as four, in the red ring and split this set up also. You might have a child record on the chalkboard the numbers of his sets in the two blue rings. Do this again until children have worked with starting sets of 2, 3, 4, and 5. Note that since this "splitting up" of a number is in preparation for "take away," you should not try to teach the "take away" concept, but simply provide children with this concrete experience.


b

fifteen 15

Let's do

Put five counters in the red ring.

Move them to the two blue rings.

How many in each?

Answers will vary. Can have different combinations of 5.

3 2

Can you find another way?

2 3

Readiness for subtraction

Purposes

To provide an introduction to the idea of subdividing a set into two subsets thus creating the "take away" situation

To provide action type take away situations for discussion

Preparation

Materials

counters, at least 5 per child

For this investigation, each child will need five counters. After you distribute them, you might ask the children to explain how they have used the counters so far. That is, review how they have used the counters in combining sets. You might work through an example such as telling them to take one counter in one hand and two in another. Then ask how many counters will they have in all if they combine these two sets. Explain that in this investigation they will again be using counters but in a slightly different way.

Let's talk

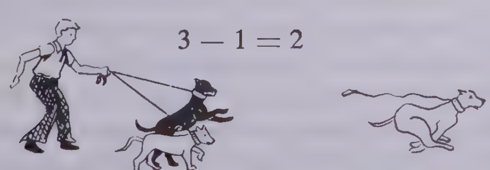
Discussion
Page b-16

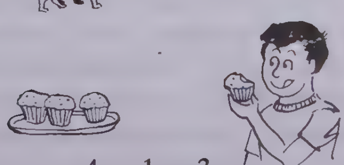
The variety of illustrated sets in the picture on this page should provide a basis of discussion during which you can mention concepts related to subtraction. Ask the children to describe the picture. Use questions such as: "How many children in all?" "How many are walking away?" "How many will be left?" "How many birds in all?" "How many flying away?" "How many will be left?" Use the items in the picture to make up stories which stress the subtractive concept. For example, make up a story about the children, saying: "John had 5 pennies. He gave 2 to the children walking away to get something from the store. How many pennies does John have now?" Encourage children to make up similar stories of their own.

Follow-up

Give the children large sheets of newsprint and crayons. Allow them free rein in creating their own sets showing "take away." You might ask the children to explain their pictures to the class.

$3 - 1 = 2$





$4 - 1 = 3$

Resources for Active Learning

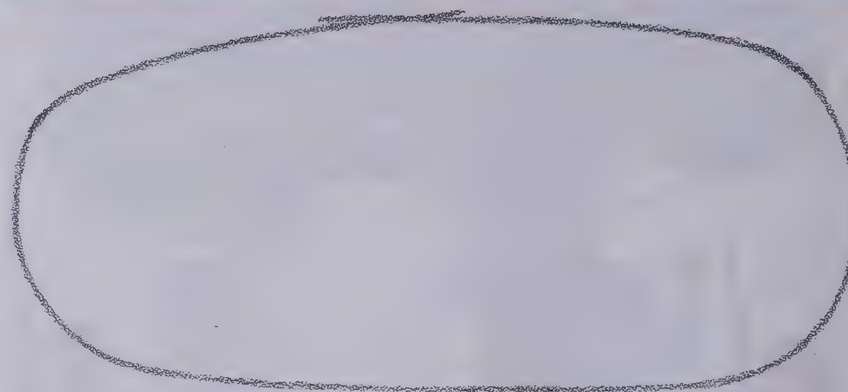
MATH ACTIVITIES, "Trick-or-Treat," Game 2/17, p. 31, Allyn and Bacon

SETS, NUMBERS AND POWERS, "Lessons and Games . . . States and Operators," pp. 104–107, Herder and Herder

Teaching
Page b-17

You will want to carefully direct the children through the activity of this page. Read the directions for the children, directing them to first put 4 counters into the ring. Ask them: "How many are in the ring?" When they respond "Four." record the 4 on the chalkboard. Then ask them to take away one counter. See if a child can tell you what to write to record this action and continue to build the phrase 4 − 1. Finally ask how many counters are now in the ring and ask children to trace the 3 in the first space provided. Then complete your equation, 4 − 1 = 3, and direct children to complete the equation on their page. Read it together with them, saying: "four take away one is three."

Be sure each child has at least five counters. Work through the next problem in a similar manner and, if time permits, work through other equations also. In each case, have the children work first with the counters and record how many are left. Then have them relate the equation to their take away action.



Put in	Take away	How many left?
4	1	3

Solve the equation.

4 − 1 = $\boxed{3}$

Put in	Take away	How many left?
5	2	3

Solve the equation.

5 − 2 = $\boxed{3}$

Objective

Given a subtraction equation involving combinations of 5 or less, the child will be able to solve the equation through use of physical objects or appropriate pictures.

It is in this lesson that the child is formally introduced to the subtraction concept. Children work first with sets, acting out the "take away" situation. Then they record the "take away" action with numerals and the symbols − and =. Keep in mind that although we "take away" a subset of a set, we subtract *numbers*. Note that although a subtraction equation, such as 4 − 1 = 3, would be introduced as "four take away one is three," an on-going objective for the year is to help children eventually understand and use the phrase "four minus one is three," or "four subtract one is three."

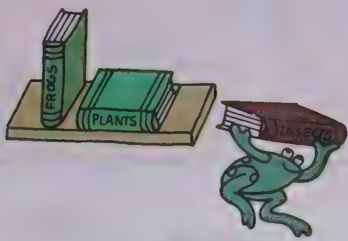
Pre-Book Activity

Materials

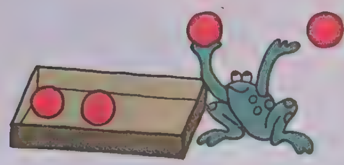
counters, at least 5 per child
objects for set demonstrations

Since the text page itself is designed for work with counters, you might display a demonstration set of five objects and have a child remove three. Write on the chalkboard 5 − 3 as he does this. Explain that 5 − 3 records this take away action and may be read "five take away three." Then ask the class how many objects are left. When they respond "two," complete the phrase 5 − 3 as the equation 5 − 3 = 2 and explain that we write the equal sign because 5 − 3 and 2 name the same number. Teach them to read the equation as "five take away three is two." Work through one or two other demon-

Solve the equations.

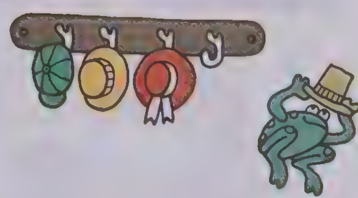


$3 - 1 = \boxed{2}$



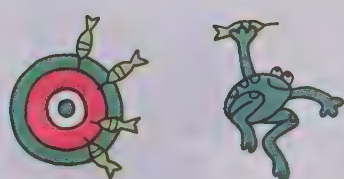
$4 - 2 = \boxed{2}$

$3 - 2 = \boxed{1}$



$4 - 1 = \boxed{3}$

$5 - 2 = \boxed{3}$



$5 - 1 = \boxed{4}$

Teaching
Page b-18

It is important that children relate each equation to the take away action shown in the picture above it. Call their attention to the first illustration of books. Ask them to describe what action they think is taking place. Be sure they include how many books there are, and ask how many books will be left after one is taken away. Have them complete the equation and then read it together with them "three take away one is two." Work through each of the pictures and equations similarly. You might want some children to complete this page on their own and then tell stories to explain their completed equations.

stration sets similarly, but do not spend a great deal of time on this pre-book activity. Children will benefit more from individual work with counters when they use the text page.

Follow-up

Give each child a set of plastic numerals and symbols, or individual sets of cards showing numerals and symbols. Make up subtraction stories and ask the children to build equations for each story. Examples of stories you might use:

1) Four children went to the park. Three got on the swings. How many were left to go down the slide?
2) Billy invited four boys to his birthday party. One boy didn't come. How many came to the party?
3) I have five pennies. If I give two pennies to Tony for a cookie, how many pennies will I have left?
4) Patty wants a big lollipop. It costs two cents. Patty has three cents. How much will she have left if she buys the lollipop?

Resources for Active Learning

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Subtraction," Nos. 50–53, Responsive Environments Corp.

MATHEX: Operations No. 3 "Subtraction by Matching" and "Writing Equations," pp. 4–8, Encyclopaedia Britannica Publications Ltd.

Separating groups:

WORKJOBS, pp. 150–151, 198–201, 210–211, Addison-Wesley

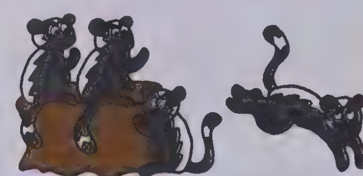
Teaching
Page b-19

Call the children's attention to the first example. Ask them to explain the picture of the sets; note particularly the departing action in the illustration. Help them relate the illustration to the equation 4 − 1 = □ and write the answer in the frame. You might read the equation as "four minus one *equals* three." Be sure children understand that this means the same as "four take away one *is* three." Since the second top frame relates to the subtractive property of zero, it would be helpful to work similarly through it. Remind the children that the correct response for an equation such as 2 − 2 = □ is "zero," but if a child says "none," accept this verbal response as correct.

Ask the children to finish the remaining exercises on their own. When they have finished, you might work through each frame asking the children to describe each equation in terms of the pictured sets.

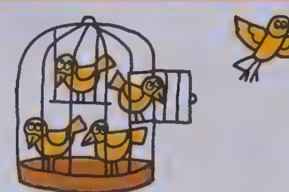
$3 - 2 = 1$

Solve the equations.



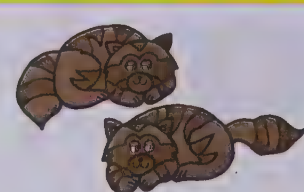
$4 - 1 = \boxed{3}$

$2 - 2 = \boxed{0}$

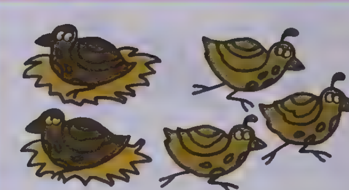


$5 - 1 = \boxed{4}$

$4 - 2 = \boxed{2}$



$2 - 0 = \boxed{2}$



$5 - 3 = \boxed{2}$

Objective

Given subtraction problems for combinations of 5 or less, the child will be able to find the differences using appropriate set pictures or physical objects.

By use of illustrations, the child can relate the subtraction equations to take away situations with sets. Introduced in this lesson is the subtractive property of zero. Examples with concrete materials might better help children come to the realization that zero subtracted from any number is that same number.

Pre-Book Activity

Use subtraction stories with demonstrations to fit classroom situations. For example, begin with such stories as: "There are five erasers at the chalkboard." "If I take one away, how many will be left?" Show five erasers at the chalkboard and take one away so the children can see the sets. Gradually begin to tell stories so they cannot see the sets involved. For example: "Suppose four children are playing a game in the park and two of them have to go home. How many children will be left playing in the park"? Casually introduce a story involving a problem such as 4 − 4 to see how the children respond. If they answer "none," remind them that "zero" is the number we use when talking about a set containing no objects. Continue interspersing exercises such as 2 − 2 until the children become familiar with "zero."

Next, try an exercise like 4 − 0, or "four take away zero." Help the children see that if they have a collection of four and none are removed, they still have four. If you think work on the flannelboard or chalkboard is not suf-

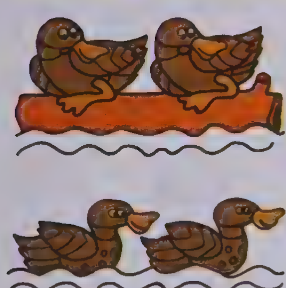
Subtract.

$$\begin{array}{r} 3 \\ -1 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 4 \\ -0 \\ \hline 4 \end{array}$$

$$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 3 \\ -3 \\ \hline 0 \end{array}$$



$$\begin{array}{r} 4 \\ -2 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array}$$

Vertical notation for subtraction

Teaching Page b-20

This page is different from page b-19 only in that the exercises are presented in vertical form. Help the children see that this is merely a different way to write a subtraction problem. Read the first problem with them as "three minus one is two" and ask them to trace over the dashed numeral. Work through the next exercise "four minus zero" in a similar manner, pointing out the relation between the pictured sets and the written problem. Have children complete the remaining exercises on their own and describe each exercise when all of the children have finished them. As the children work, move around the class to observe and help any children who are having difficulty.

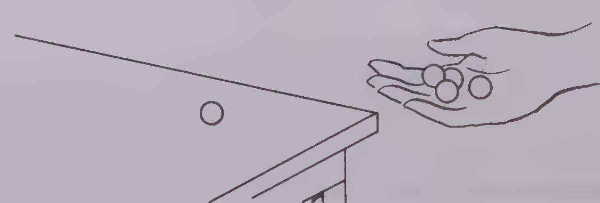
ficient, give children objects to handle, and have them work out the equations at their places.

After working on problems orally, encourage some children to make up plausible number stories and ask others to write the equations on the board. Then, rewrite the problems in vertical form near the equations, and emphasize that this is another way of writing the same problem.

Follow-up

Some children will benefit from further experience with counters. For example, distribute five counters and, if possible, plastic numerals or individual numeral cards and symbols to each child. Tell them that as you read and write an equation on the chalkboard, they should show the action with their counters. For example, write $5 - 4 = \square$. Children should put five counters on their desk and pick up four. If plastic numerals are available, they can form the equation themselves and complete it with the correct answer.

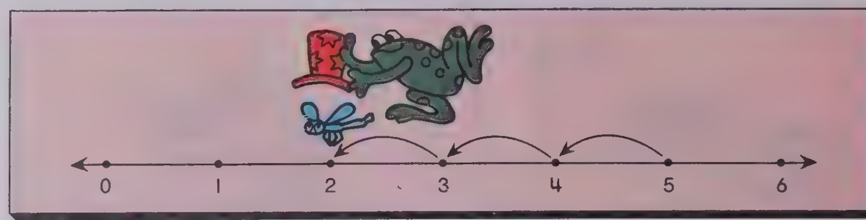
$\boxed{5}\ \boxed{-}\ \boxed{4}\ \boxed{=}\ \boxed{1}$



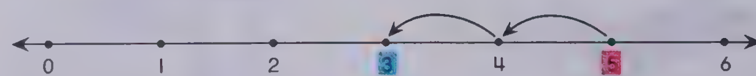
Teaching
Page b-21

Use the frog and insect to introduce the number line. Write $5 - 3 = 2$ on the chalkboard and help children relate the equation to the illustration. Ask the children to explain how the subtraction can be shown on a number line. Then direct their attention to the equation $5 - 2 = \square$ and the number line above it. Ask a child to show on the demonstration number line or the overhead projector how to use the number line to help solve this equation. Although it is not necessary at this time to refer to the color coding, you might point out how 5, the number in red, is greater than both 2, the number taken away, and 3, the answer.

Point out the blank number line and encourage children to use it to solve the equations. You might want to work through the first few examples with some children, but encourage most children to do them by themselves. First note with them that some of the exercises at the bottom of the page are written in the vertical notation. Move around the room while the children work and help any who seem to be having difficulty.



Find the differences.



$5 - 2 = \boxed{3}$

$3 - 1 = \boxed{2}$ $\quad 2 - 0 = \boxed{2}$

$5 - 3 = \boxed{2}$ $\quad 5 - 1 = \boxed{4}$

$4 - 2 = \boxed{2}$ $\quad 4 - 3 = \boxed{1}$

$3 - 3 = \boxed{0}$ $\quad 3 - 2 = \boxed{1}$

4	5	3	2	4	3
− 1	− 4	− 0	− 2	− 1	− 1
3	1	3	0	3	2

Finding differences

Objective

Given a number line showing whole numbers from zero to six and subtraction equations to solve, the child will be able to use the number line to help him find the differences.

The number line can be used as a counting device in subtraction just as it was in addition. If children recall that addition is shown by arrows to the right, they should have no difficulty in showing subtraction with arrows to the left.

Pre-Book Activity

Materials

demonstration number line

Use a number line on the chalkboard or overhead projector to demonstrate subtraction. Label it from zero through six. Beneath the number line write an equation such as $5 - 3 = \square$. You might refer to the frog that is used throughout the art demonstrations of Unit B. For example, speak of him chasing flies or insects and jumping along the number line. Show children how they begin with their pencil on 5, move backward 3 jumps, and read their answer from the number line.

$5 - 3 = 2$

Find the differences.

$3 - 2 = \boxed{1}$ $\quad$ $4 - 3 = \boxed{1}$

$5 - 2 = \boxed{3}$ $\quad$ $5 - 3 = \boxed{2}$

$4 - 1 = \boxed{3}$ $\quad$ $4 - 0 = \boxed{4}$

$3 - 3 = \boxed{0}$ $\quad$ $3 - 1 = \boxed{2}$

$$\begin{array}{r} 5 \\ -1 \\ \hline 4 \end{array} \quad \begin{array}{r} 4 \\ -2 \\ \hline 2 \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline 1 \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline 1 \end{array} \quad \begin{array}{r} 3 \\ -0 \\ \hline 3 \end{array} \quad \begin{array}{r} 4 \\ -3 \\ \hline 1 \end{array}$$

$$\begin{array}{r} 4 \\ -4 \\ \hline 0 \end{array} \quad \begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array} \quad \begin{array}{r} 3 \\ -3 \\ \hline 0 \end{array} \quad \begin{array}{r} 4 \\ -1 \\ \hline 3 \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline 4 \end{array} \quad \begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 3 \\ -1 \\ \hline 2 \end{array} \quad \begin{array}{r} 4 \\ -2 \\ \hline 2 \end{array} \quad \begin{array}{r} 5 \\ -5 \\ \hline 0 \end{array} \quad \begin{array}{r} 2 \\ -1 \\ \hline 1 \end{array} \quad \begin{array}{r} 4 \\ -0 \\ \hline 4 \end{array} \quad \begin{array}{r} 5 \\ -0 \\ \hline 5 \end{array}$$

Finding differences

Teaching
Page b-22

This page might be considered a continuation of the exercises on page b-21. Point out to the children that they might want to use the number line on the other side of this page, that is on page b-21, to solve the problems. However, do not overemphasize the use of the number line as many may want to do the exercises without using it. Some children who may still benefit from the use of counters should be allowed to use them. A careful checking of this page coupled with observation of the methods children use to find the answers should be an aid in your evaluation of children's understanding of subtraction. Provide special help to those who have difficulty.

Work through several similar examples. If children have individual number lines at their tables, have them follow your examples on these. Alternatively, you might stretch a labelled roll of paper along the classroom floor and have children jump out the subtraction equations which you show on the overhead projector. Throughout all of this preparatory session, stress the idea that subtraction is shown on the number line by arrows to the left.

Follow-up

All or part of a worksheet like the following might be helpful in giving the children more practice. Encourage the use of counters and number lines as the children use such a page.

$$\begin{array}{r} 5 \\ -5 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 5 \\ -0 \\ \hline \end{array}$$

$$\begin{array}{r} 1 \\ +4 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +3 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ +0 \\ \hline \end{array}$$

$$\begin{array}{r} 3 \\ -3 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -2 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 3 \\ -0 \\ \hline \end{array}$$

$$\begin{array}{r} 2 \\ +2 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +1 \\ \hline \end{array} \quad \begin{array}{r} 2 \\ +0 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ -1 \\ \hline \end{array} \quad \begin{array}{r} 1 \\ -0 \\ \hline \end{array} \quad \begin{array}{r} 0 \\ -0 \\ \hline \end{array}$$

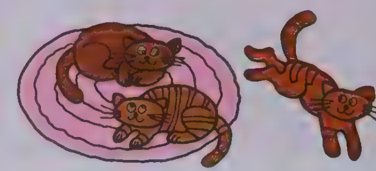
Teaching
Page b-23

Work the first equation with the children. Encourage them to explain how the picture shows what is recorded in the equation. Most children should be able to do these exercises on their own since they have previously worked similar exercises. Move around the room as they work and help any child having difficulty. Those children whose work shows a lack of understanding or of ability to solve subtraction equations should be given extra help. Such children may still need extensive experience with concrete materials.

Show you know

Find the differences.

$5 - 2 = \boxed{3}$



$3 - 1 = \boxed{2}$

$\begin{array}{r} 2 \\ -2 \\ \hline 0 \end{array}$

$\begin{array}{r} 4 \\ -3 \\ \hline 1 \end{array}$

$4 - 2 = \boxed{2}$

$5 - 3 = \boxed{2}$

$3 - 1 = \boxed{2}$

$3 - 2 = \boxed{1}$

$4 - 4 = \boxed{0}$

$5 - 4 = \boxed{1}$

$\begin{array}{r} 4 \\ -1 \\ \hline 3 \end{array}$ $\quad \begin{array}{r} 3 \\ -0 \\ \hline 3 \end{array}$ $\quad \begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array}$ $\quad \begin{array}{r} 4 \\ -2 \\ \hline 2 \end{array}$ $\quad \begin{array}{r} 5 \\ -1 \\ \hline 4 \end{array}$ $\quad \begin{array}{r} 4 \\ -3 \\ \hline 1 \end{array}$

Objective

Page b-23: The child will demonstrate his understanding of the concepts presented in this module by solving subtraction equations.

Note that the change of pace on page b-24 makes use of the strips. Encourage children to explore the strips in various open-ended ways since they will be used more seriously in subsequent lessons.

Pre-Book Activity

Materials

centimetre strips, 1 set per child

Write several subtraction equations on the chalkboard. Provide space for a child to write each answer. Make up a subtraction story as you refer to each equation and then ask a child to complete the equation.

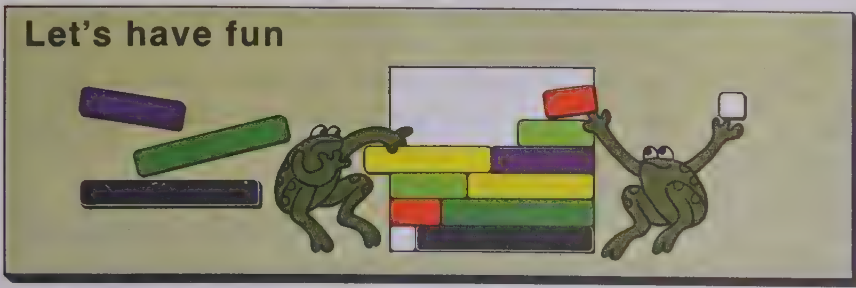
Sample stories are as follows:

$4 - 2 = \square$ Four children were on the swings. Two had to go home. How many were left?

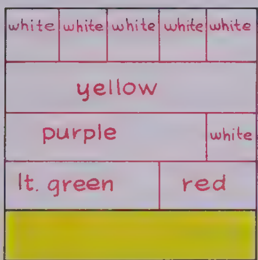
$3 - 1 = \square$ Bill bought three licorice sticks. He ate one. How many does he have left?

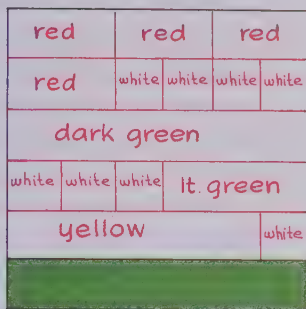
$5 - 3 = \square$ Suzy's cat had five kittens. Her mother gave three away. How many are left?

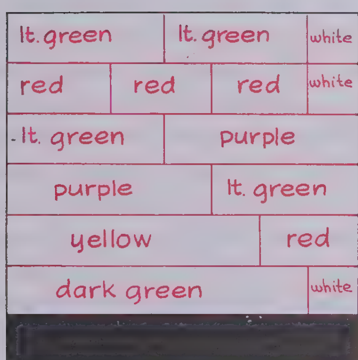
Let's have fun



Fill in each square with your strips. Answers will vary. Examples are given.







Readiness for sums of 6 and 7

Teaching
Page b-24

Distribute a set of strips to each child. Before explaining the page, you might ask the children to build something such as a bird, or a tree, or an automobile with their strips. Then explain that they should use as many different colored strips as they can and completely cover each square. No mention should be made of the fact that this activity may be considered a treatment of the combinations of 5, 6, and 7. The importance of this activity is that children informally experience some of the relationships which exist between various strips. Note that children need not have assigned number names to the strips to use them effectively in this lesson.

Encourage the children to talk with each other about which strips they use to cover the squares. As they work, suggest questions for them to consider and other activities they might do. For example, ask: "Which strip is longest?" (orange strip); "Which strip is shortest?" (white strip); "How many red strips match a purple strip?" (two); "Which single strip can be matched with a train of three red strips?" (dark green); "Can you make a staircase with the shortest strip on top and the longest strip at the bottom?" Such questions also serve as a preparation for the next lesson.

Follow-up

Use this activity to evaluate the children's understanding of ordinal numbers and their ability to follow directions. Each child will need a standard piece of manila paper and crayons. One version of the game might include these oral directions: "Pick up a dark-colored crayon in your writing hand. Lay your other hand on the paper. Hold your fingers apart and carefully trace your hand using the crayon." Vary the remaining directions according to the maturity of your group. Sample directions are: "Draw a ring on the third finger from your thumb." "Make a watch where you wear it." "Put long fingernails on the second and fourth fingers." "Put a bandage on your first finger."

RED MODULE, UNIT B

Sums and Differences of 6 and 7

Pages b-25 to b-38

General Objectives

To increase understanding of addition and subtraction

To introduce the addition and subtraction combinations for 6 and 7

To increase understanding of equations and skill in using them

To begin development of skill in adding and subtracting combinations for sums equal to, or less than, 7

To develop a feeling for number combinations in story problem situations

No new mathematical concepts are introduced in this module. Rather, the purpose of this unit is to maintain and strengthen the mathematical ideas and skills developed so far and extend them to addition and subtraction combinations up to 7.

The first few lessons of the module present and develop the addition combinations for six and seven. Then the subtraction combinations are studied. Finally on page b-36, equations for both addition and subtraction are presented on the same page. The module concludes with a review page of addition and subtraction and a change of pace page which gives children further opportunity to use the strips informally.

Mathematics

As stated earlier, no new mathematical concepts are introduced in this unit. It is important, however, to maintain and strengthen the children's understanding of the use of equations and of the previously developed concepts of addition and subtraction.

Teaching Red Module, Unit B

Approximate Time: 7 to 9 days

Materials

counters for each child

felt-tipped pens or markers

flannelboard and sets of felt objects, letters, numerals, and other symbols

individual set collections for the children

number line for demonstration

overhead projector, if available

plastic numerals (or individual numeral cards) for each child

sets of objects (balls, pencils, paper cups, plastic jugs, paper sacks, etc.) for use in class demonstrations

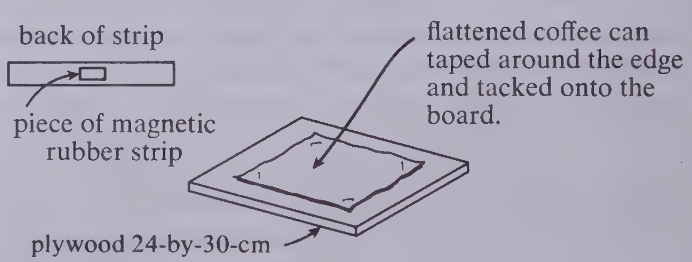
set of strips for each child

tagboard (5-x-15-cm cards)

In this module the strips are used more specifically in relation to the combinations. Many children will benefit from careful use of the strips due to their basic mathematical structure. Set illustrations are again provided to assist the child in working with the number symbols. Counters and the number line are also suggested as helpful aids for working with the combinations. Children will benefit from different learning devices so all of these methods should be developed sufficiently so that a child may have the opportunity to learn from one device what he was not able to grasp using another.

Note that suggestions for oral drill are provided because oral work is an important phase of the children's experience in arithmetic, and should not be neglected. Flash cards and games are useful in presenting oral activities because they encourage the children to respond promptly. Be sure to vary the routine of oral drill since it can become quite boring to children who understand the ideas and know the combinations.

To facilitate manipulation of the strips, each child would benefit from a magnetic strip board. Such a homemade device may be made from a piece of plywood (24-by-30-centimetres) on which has been fastened an opened flattened-out coffee can (or comparable piece of metal). Then, on the uncolored back side of each centimetre strip, glue a small piece of magnetic rubber strip (such magnetic strips with adhesive backs may be purchased from most school supply companies). Thus, when the children build trains and explore combinations, the strips will adhere to the board and remain in position. This board can also be used in back of the text pages themselves. That is, the text page may be placed on the board and the strips will adhere to the page due to the magnetism of the board underneath.



Evaluation of Progress

You can evaluate whether or not the children have attained skills with addition and subtraction combinations from the way they solve problems on b-32, b-33 and b-36, b-37.

Resources for Active Learning

General Activities:

Combining groups:

WORKJOBS, pp. 148–155, 160–161, 164–156, 168–169, 172–173, 194–197, 202–209, Addison-Wesley

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Subtraction," Nos. 54–55, Responsive Environments Corp.

Materials to study facts:

MATH ACTIVITIES, 3/3–26, pp. 88–95, Allyn and Bacon

Nuffield Project: COMPUTATION AND STRUCTURE ②, "The Operation of Addition," pp. 58–69, Wiley

Manipulative Devices:

Hundred peg board and cylinders (Educational Teaching Aids; Responsive Environments Corp.)

"Invicta" Math Balance (Math Media; Selective Educational Equipment)

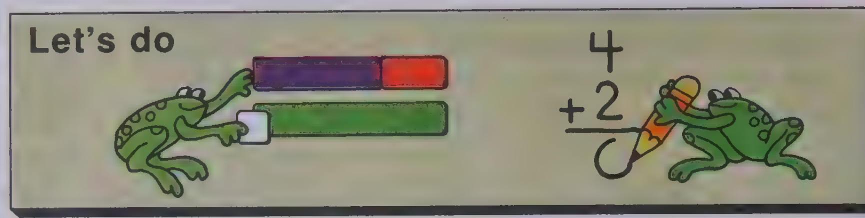
Investigation
Page b-25

In this investigation children will begin to relate the concept of number to the strips. In any development with the strips, emphasis should be placed on the fact that once a strip is designated as the unit strip, all of the other strips may be thought of as representing specific numbers. In the first year's work, the white strip is the only strip chosen as a unit strip; thus the strips will represent the numbers from one to ten.

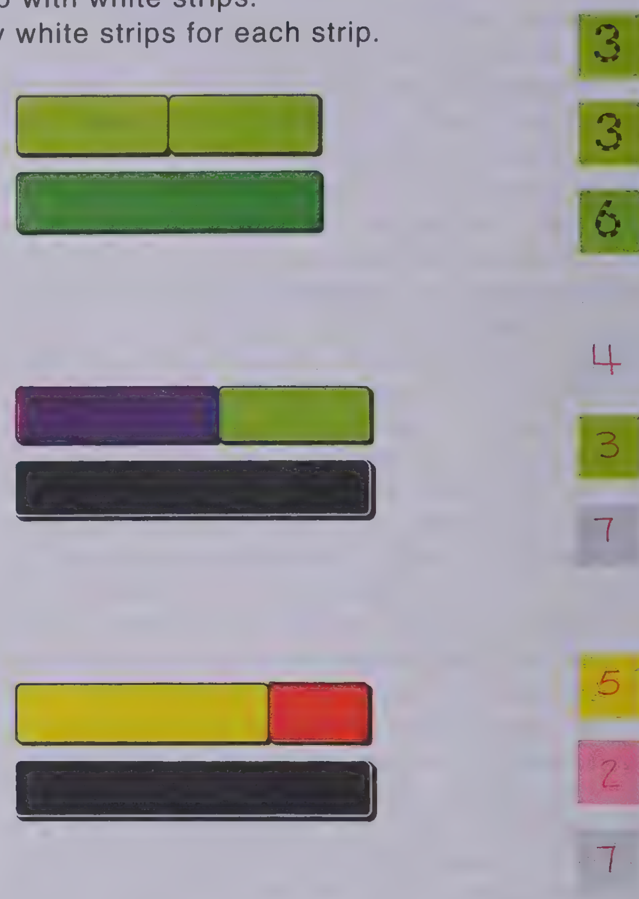
Direct the children to cover each of the light green strips with as many white strips as they need. Then direct their attention to the dashed threes and explain that here is where they can record how many white strips cover each light green strip.

Continue similarly with the remaining frames. Each time have the children record their answers in the appropriate boxes. As they work, point out that in each frame the train of two strips matches the the single strip. Also ask questions such as: "Name two strips that match the black strip." "Can you find two other strips not shown here that match the black strip?"

Encourage children to explore other combinations which match the dark green and black strips. Also, you may want to start referring to the strips by their number names as well as color names. Allow the children to begin using the number names at their own rate.



Cover each strip with white strips.
Write how many white strips for each strip.



Readiness for greater sums

Purposes

To provide background for introduction of combinations of 6 and 7

To develop an intuitive grasp of number combinations as shown with the strips

Preparation

Materials

counters for page b-26 (optional)
set of strips for each child

Various activities with the strips would best prepare the children for this investigation. If children have not already done so, have them build a staircase with the strips for discussion. Ask the children to compare the strips according to length. Ask such questions as: "Which is longest?" "Which is shortest?" Then use a set of demonstration strips to develop the idea of a "train." (Demonstration size strips similar to the children's centimetre strips may be made from felt or construction paper to use on the flannelboard.) Show a few examples of trains, such as a train with red and purple strips or with white and yellow strips, and then ask the children to make some trains. Also show examples of how a train may be matched with a single strip. (See below.) Simply refer to the strips by their color names.

Sample trains

red	yellow		purple	l. gr.		wh.	purple

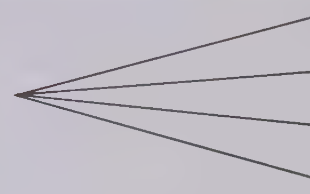
Sample trains matched with single strips:

l. gr.	purple		yellow	wh.
black			dark green	

Let's talk

Discussion
Page b-26

This discussion picture deals with the same number combinations as the investigation. Ask the children to count how many students they see in the library. When someone answers "six," write the numeral 6 on the chalkboard. Talk about what these students might be doing in the library and suggest that some of them might want to sit down. Point out the two groups of six chairs and have the children describe various ways in which the students might all be seated. Record the pairs of numerals on the chalkboard.



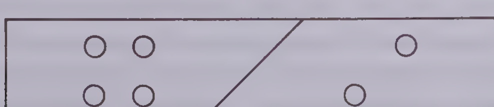
Note that combinations such as $4 + 2$ and $2 + 4$ are considered combinations for the same grouping.

As children suggest these combinations, have them place their counters on their text page. You might also place two groups of six chairs in the centre of the classroom and have children act out these combinations as if they were the students in the library.

When the combinations for 6 have been sufficiently discussed, talk about the combinations that are possible if the librarian also wanted to sit at a table.

Follow-up

Some children would benefit from further work with counters. Prepare a worksheet which shows two empty sections and incomplete equations underneath. Give each child six or seven counters. Have them represent each equation by grouping the counters into the proper sections on the page. Children should do this as many times as needed to complete all of the equations.



$3 + 3 = \square$	$1 + 6 = \square$
$2 + 5 = \square$	$7 + 0 = \square$
$4 + 2 = \square$	$0 + 6 = \square$

The following activity may be used in relation to a particular season. Guide the children in folding a sheet of newsprint to make four similar sections. Write or give oral directions similar to the following:

Tie the directions in with the seasons. If the Christmas holidays are near, sample directions might be:

1) Make 4 red  and 3 green . Write an equation to show how many in all.
2) Draw 7 balls. Color 4 yellow and 3 red. Write an equation to show how many in all.
3) Draw 6 Christmas trees. Color 5 green and 1 white. Write an equation to show how many in all.

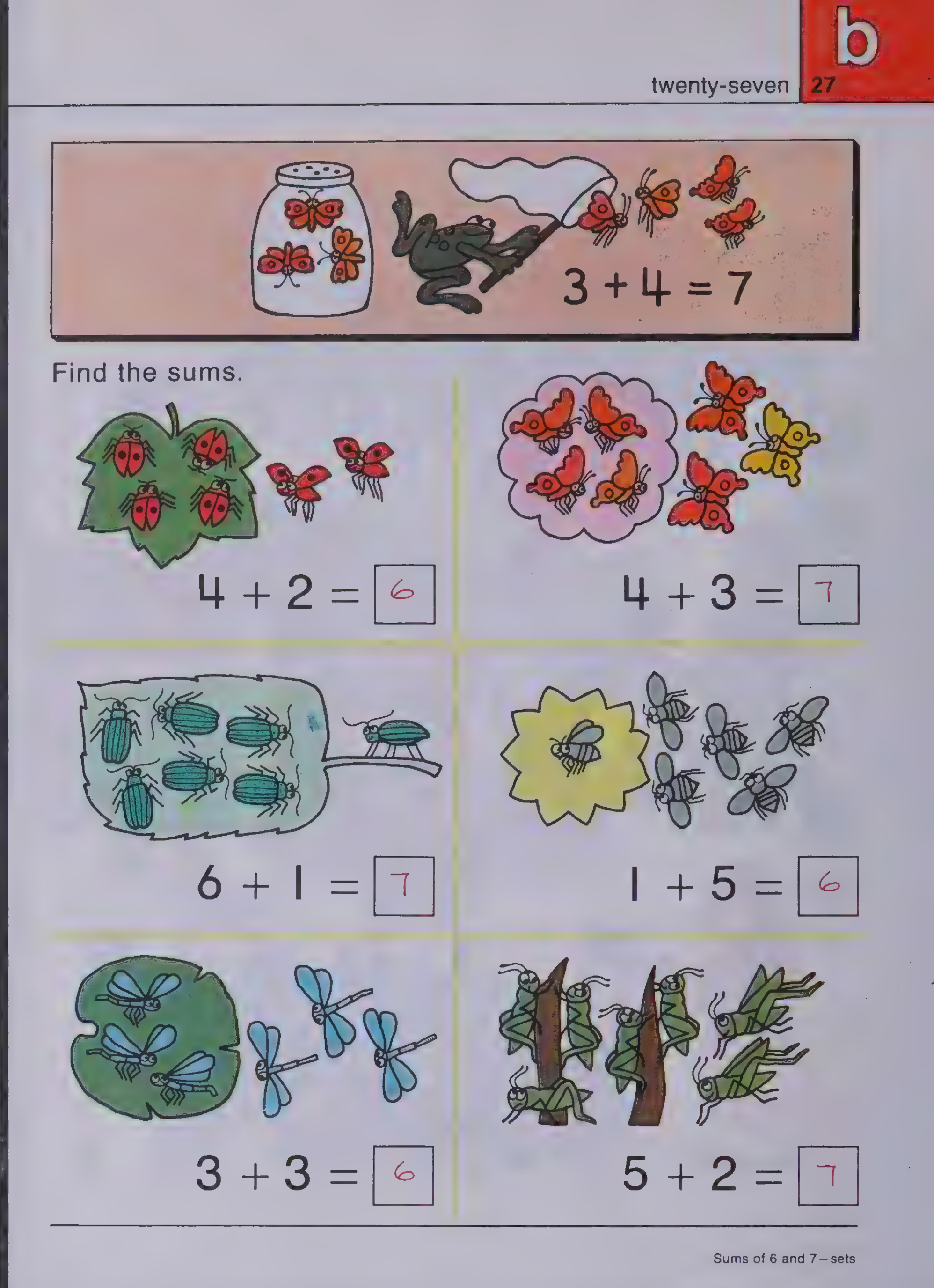
Resources for Active Learning

A CLOUDBURST, Vol. 1, Nos. 1111–2, Midwest Publications

Teaching
Page b-27

Use the demonstration art to explain the directions to the children. Then call attention to the first frame. Ask them to look at both sets, the set of four and the set of two. Then help them relate these sets with the symbols 4 + 2. Finally, ask "How many bugs are there in all?" When someone responds "six," instruct the children to write the numeral 6 in the empty box. Ask the children to complete the page on their own.

As they work, you might ask individual children to explain a particular combination and how it relates to the pictures.


b
twenty-seven 27

$3 + 4 = 7$

Find the sums.

$4 + 2 = \boxed{6}$

$4 + 3 = \boxed{7}$

$6 + 1 = \boxed{7}$

$1 + 5 = \boxed{6}$

$3 + 3 = \boxed{6}$

$5 + 2 = \boxed{7}$

Sums of 6 and 7 – sets

Objective

Given an addition equation with two addends whose sum is six or seven, the child will be able to find the sum by studying a related set illustration.

Although some children will need to continue using counters and other manipulative devices, many will be able to use the illustrated sets to help them solve the equations.

Pre-Book Activity

Materials

counters, at least 7 per child

Oral exercises combined with manipulation of counters will help the children better understand the combinations of 6 and 7, and provide further review of the combinations under five. Place a set of six small objects on the overhead projector, or use larger objects on a demonstration table, to help the children visualize "sixness." Distribute the counters and encourage children to build sets with the same number as your demonstration sets. Encourage them to use the counters to help them find combinations for six and seven. Begin the activity by asking children to place six counters on their desk and to move these so they look like two sets. Ask someone to tell how many are in each set he has found for six. Use this combination in an equation on the chalkboard. Continue with other combinations for six. Finally work through combinations of seven similarly.

Find the sums.

$2 + 4 = \boxed{6}$

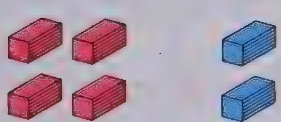
$3 + 4 = \boxed{7}$

$3 + 3 = \boxed{6}$

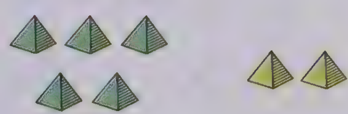
$2 + 5 = \boxed{7}$

$1 + 6 = \boxed{7}$

$6 + 0 = \boxed{6}$



$4 + 2 = \boxed{6}$



$5 + 2 = \boxed{7}$

Teaching
Page b-28

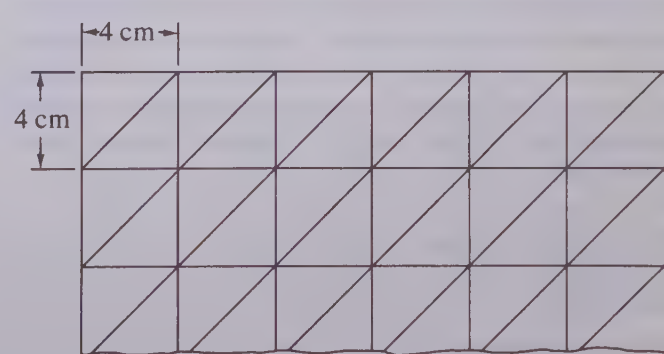
Although this page is essentially a continuation of page b-27, you might want to work through one or two equations with some children. As you speak of the illustrations in the sets, you may use whatever names you prefer and which you think the children can best understand. For example, you might call the items in the first frame pyramids, or, simply refer to the items by color.

Allow any children who still have difficulty with the use of symbols to manipulate counters as they try to solve each equation. Do not try to hasten children who are not yet ready to work solely with symbols.

Follow-up

As a class activity, let the children collect, or make, some "quiet counters" (circles of cork, milkbottle tops, or other scrap items that do not rattle when handled). These are especially useful when one section of the class is studying independently. If "ready-made" quiet counters are not available, substitute counters may be made from heavy construction paper.

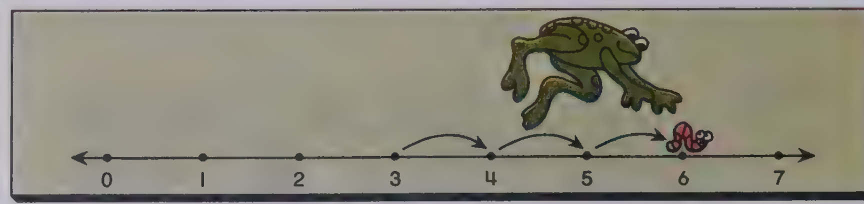
Prepare a duplicating master by marking off 24 four-centimetre squares. Draw diagonals from the top right-hand corner to the bottom left-hand corner of each square to make 48 triangles. Run this pattern on white construction paper, and ask the children to cut the triangles apart or use a paper cutter to do this beforehand. (Because it will be useful for them to have 144 counters apiece later, in developing place value, run three sheets for each child. But, because it takes time to cut even one sheet, postpone cutting the others until they are needed.) Suggest that each child initial his triangles before he cuts them apart, and show the children how to store their counters in envelopes.



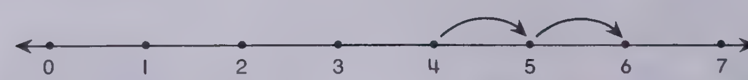
Teaching
Page b-29

Use the illustration at the top of page b-29 to talk about jumps along a number line and the equation $3 + 3 = 6$. As suggested earlier, refer to the frog by name and encourage children to make up stories about him and the worm he's trying to catch. Ask for a story which could fit the first equation $4 + 2 = \square$. When you discuss this equation, instruct the children to move their pencils along the number line as you think aloud with them. You may say something such as: "Start on four. We're supposed to add 2; so count, one, two jumps (show the jumps on your demonstration number line.) We've landed on 6, so $4 + 2 = 6$."

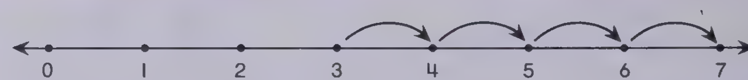
Work through the next equation, $3 + 4 = \square$ similarly, or ask a child to explain it. Instruct the children to solve the remaining equations independently. Encourage them to use one of the number lines at the top of the page or the blank number line on page b-30. As children work, give help to any who are having difficulty.



Solve the equations.



$4 + 2 = \boxed{6}$



$3 + 4 = \boxed{7}$

$3 + 3 = \boxed{6}$	$5 + 1 = \boxed{6}$
$2 + 3 = \boxed{5}$	$2 + 5 = \boxed{7}$
$1 + 6 = \boxed{7}$	$7 + 0 = \boxed{7}$
$0 + 6 = \boxed{6}$	$4 + 3 = \boxed{7}$
$2 + 4 = \boxed{6}$	$1 + 5 = \boxed{6}$

Objective

Given addition equations with sums of 7 or less and a number line labelled from 0 to 7, the child will be able to solve the equations by referring to the number line.

Although some children may successfully find sums by thinking of sets, it would be helpful for them to understand how the number line may be used as an aid in finding sums. Note that here the purpose of the number line is not so much to supply a pictorial representation of an equation, as it is to serve as a counting device to aid in finding sums.

Pre-Book Activity

Materials

individual numeral cards

To review how the number line may be used as an aid in addition, use your demonstration number line to play "What's my equation?" Use a number line which is labelled at least as far as seven. Tell the children that you are going to mark and jump along the number line according to an equation that you are thinking of. They should carefully watch what you do and see if they can guess your equation. It would be helpful for them to have paper and pencil so they can record what you do, or you might want them to use the plastic numerals or individual numeral cards to show the equations. Then slowly work through an equation, preferrably a simple one at first. For example, if you are thinking of $3 + 2 = 5$, draw a small arrow underneath the 3. Then jump twice. Finally, circle the 5.

Find the sums.

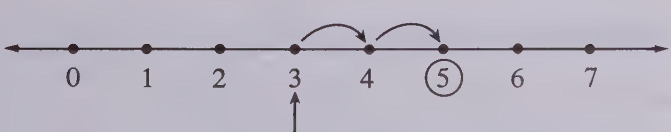
$1 + 5 = \boxed{6}$	$0 + 6 = \boxed{6}$
$4 + 2 = \boxed{6}$	$2 + 5 = \boxed{7}$
$3 + 4 = \boxed{7}$	$5 + 1 = \boxed{6}$
$6 + 1 = \boxed{7}$	$3 + 2 = \boxed{5}$
$3 + 3 = \boxed{6}$	$1 + 6 = \boxed{7}$
$5 + 2 = \boxed{7}$	$5 + 0 = \boxed{5}$

$\begin{array}{r}6\\+0\\\hline 6\end{array}$	$\begin{array}{r}3\\+4\\\hline 7\end{array}$	$\begin{array}{r}4\\+2\\\hline 6\end{array}$	$\begin{array}{r}0\\+7\\\hline 7\end{array}$	$\begin{array}{r}5\\+1\\\hline 6\end{array}$	$\begin{array}{r}2\\+2\\\hline 4\end{array}$
$\begin{array}{r}4\\+3\\\hline 7\end{array}$	$\begin{array}{r}2\\+5\\\hline 7\end{array}$	$\begin{array}{r}2\\+4\\\hline 6\end{array}$	$\begin{array}{r}3\\+2\\\hline 5\end{array}$	$\begin{array}{r}5\\+2\\\hline 7\end{array}$	$\begin{array}{r}7\\+0\\\hline 7\end{array}$

Sums of 6 and 7 – number line

Teaching
Page b-30

Point out the blank number line at the top of the page. Explain to the children that it is purposely blank so that they can use it for the equations below. Also call attention to the exercises at the bottom that are written in vertical notation or "with one numeral above the other." Encourage children to do this page independently and to use the number line rather than counters as an aid. However, as always, if a child wants to use the counters, he should be allowed to do so.



Ask a volunteer to write your equation on the chalkboard.

$$3 + 2 = 5$$

Follow-up

Purchase, or urge the children to make, a set of flash cards with the combinations through seven on one side and the answers on the other. If they make their own cards, you should check their answers. Or you might partition a sheet into 5-by-7-cm sections (approximately) and write one combination vertically in each space. Duplicate the sheets on light-colored A-4 (210 mm x 297 mm) construction paper, and ask the children to cut them on the dotted lines. Remind them to put the answer on the back of each "card." Be sure to check their answers yourself, or assign one of the better students to be "checker for the day."

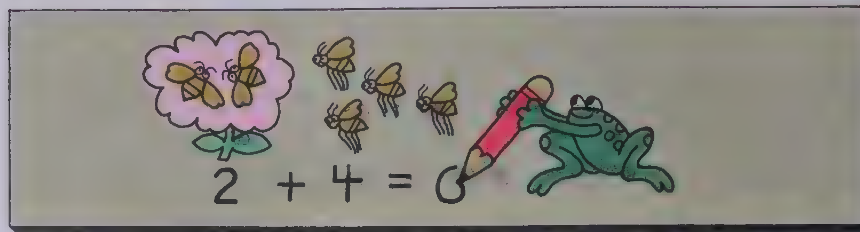
5 cm

7 cm

$\begin{array}{r}1\\+2\end{array}$	$\begin{array}{r}2\\+2\end{array}$	$\begin{array}{r}2\\+3\end{array}$	$\begin{array}{r}2\\+4\end{array}$
$\begin{array}{r}1\\+3\end{array}$	$\begin{array}{r}1\\+4\end{array}$	$\begin{array}{r}1\\+5\end{array}$	$\begin{array}{r}3\\+3\end{array}$

Teaching
Page b-31

After discussing the art, call attention to the first equation, $6 + 1 = \square$, and ask a child to explain what he is expected to do. Then point out the exercises in standard vertical notation at the bottom of the page. You might want to work through one or two of these together. Allow the children to use counters or a number line if they want to.



Find the sums.

$6 + 1 = \boxed{7}$	$5 + 1 = \boxed{6}$
$3 + 4 = \boxed{7}$	$6 + 0 = \boxed{6}$
$4 + 1 = \boxed{5}$	$3 + 3 = \boxed{6}$
$2 + 4 = \boxed{6}$	$4 + 3 = \boxed{7}$
$1 + 5 = \boxed{6}$	$2 + 5 = \boxed{7}$

5	1	7	3	3	2
+1	+6	+0	+3	+4	+3
6	7	7	6	7	5

2	4	4	1	2	2
+4	+3	+1	+5	+2	+5
6	7	5	6	4	7

Practice – sums less than 8

Objective

Given addition equations with sums of 7 or less, or such exercises written in standard vertical notation, the child will be able to find the sums.

Since no new concepts are treated in this lesson, you might use much of the lesson time to give remedial help to those who lack sufficient understanding of addition and subtraction concepts. Children who have a fairly well developed understanding of these concepts would benefit from a variety of supplementary games and activities. However, you may prefer to work several of the exercises on these two pages with the children as a review.

Pre-Book Activity

Materials

sets of colored strips

Give each child his set of strips and suggest that he build a rocket or a ship with them. Then you might direct the children through some of the following activities. (The color names will be used here although with many groups of children you will want to start using the number names.)

1) "Find the dark green strip. Make a train of two strips which is as long as the dark green strip. (If children are still not familiar with the number names you might ask them to use their white strips to figure out what combinations of six their strips match.)

b 32 thirty-two

Complete each table.

Add 4	
2	6
3	7
0	4

Add 3	
3	6
2	5
4	7

Add 5	
1	6
0	5
2	7

Complete the matching.

4 + 2	5	6	7
3 + 4	5	6	7
3 + 2	5	6	7
4 + 3	5	6	7
1 + 5	5	6	7
6 + 1	5	6	7
1 + 4	5	6	7
5 + 2	5	6	7
0 + 6	5	6	7
3 + 3	5	6	7

Practice—sums less than 8

TEACHING
Page b-32

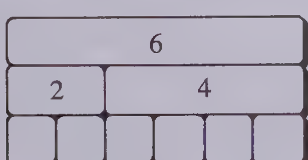
Before introducing this page, you will probably want to work through some addition tables. For example, write the following on the chalkboard or overhead projector.

Add 1	
2	
5	
4	
1	
3	

Add 2	
3	
0	
1	
4	
5	

Then fill in a few answers and ask if anyone can fill in the remaining part of the table. When someone does this successfully, have him explain what he did. Then ask children to look at page b-32. Find out how many can complete the next table without help. Have a child explain why a dashed six is written in the first box. Ask the children to complete the six. If children understand how to work with the tables, proceed with directions for the bottom section of the page. Explain that each addition combination matches one of the numbers 5, 6, or 7. Suggest that they ring or mark with an X the correct numeral for each sum. Guide them in completing the dashed circle around the 6 for the combination 4 + 2. Encourage them to complete the page independently.

Write down that combination of six as an equation."



$2 + 4 = 6$

2) Repeat number one and ask children to make a train of two strips they have not used.
3) Repeat number one and ask children to make a train with two strips of the same color.

If some children have difficulty with an activity such as this, do not over-emphasize its importance. It would be better for such children to continue free play with the strips.

FOLLOW-UP

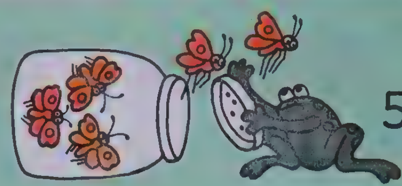
Make a bulletin board displaying a large Santa with four or five sacks of mail. (Adapt the mailing theme to Halloween or whatever holiday is closest.) The sacks can be grocery sacks or envelopes made of construction paper. Label one sack for each of the numerals "3," "4," "5," "6," and "7." On old Christmas-card envelopes, paste a variety of names for 3, 4, 5, 6, and 7, using simple addition and subtraction combinations such as: 2+4 and 5 − 3. Let the children take turns being "Santa's helper" and sorting the mail by putting the envelopes into the right sacks.

Directions to the children could be: "Help Santa sort the mail so it gets to the right house for Christmas. Put the envelopes into the correct bag."

Teaching
Page b-33

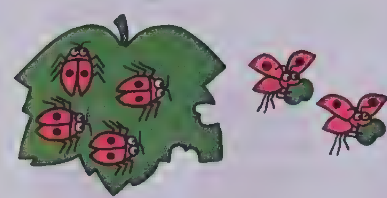
Have the children study the first frame and think of a take away or subtraction action for it. Have the children first count how many bugs in all. Then point out that two of the bugs are flying away and count how many are left. Since four are left, they should write 4 in the appropriate frame.

With most children you will want to work through each set in a similar manner. Note that the sets on this page are similar to those on page b-27 where sums of 6 and 7 are studied. This similarity emphasizes that both addition and subtraction concepts may be generated from the same subsets of 6 and 7.



$5-2=3$

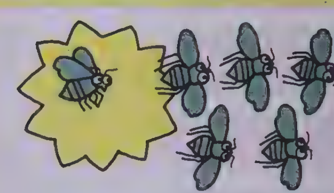
Find the differences.



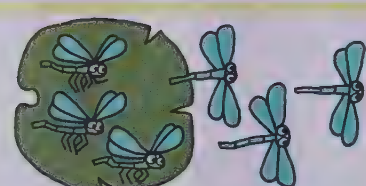
$6-2=\boxed{4}$

$7-3=\boxed{4}$

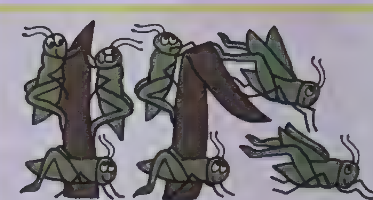
$7-1=\boxed{6}$



$6-5=\boxed{1}$



$6-3=\boxed{3}$



$7-2=\boxed{5}$

Differences related to sums of 6 and 7

Objective

Given subtraction equations related to sums of 6 and 7, the child will be able to find the differences by using appropriately illustrated sets.

Although by now children should have worked extensively with sets of 6 and 7, you will want to provide them with activities which suggest subtraction. It is important that children study the subtraction combinations of 6 and 7 through the use of counters or other manipulative materials.

Pre-Book Activity

Materials

counters, 7 per child

Give each child at least seven counters. Ask the children to build sets of 7 on their desks or tables. Then ask that they remove or simply hide with their hands 3 of the counters. Ask: "How many are left?" and see if children can describe the subtraction equation for this situation. Write $7-3=4$ on the chalkboard. Similarly ask children to build a set of 6 on their desks. Then ask that they cover four of the counters and tell how many are left. See if the children can describe this take away situation by using an equation and write $6-4=2$ on the chalkboard. Work through several similar examples. Always help children relate their action with the counters to the written equation.

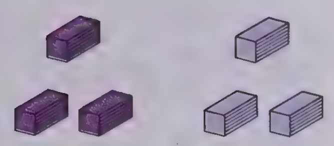
Follow-up

Draw a long double column at each end of the chalk-

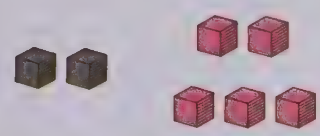
Solve the equations.

$6 - 4 = \boxed{2}$

$7 - 4 = \boxed{3}$



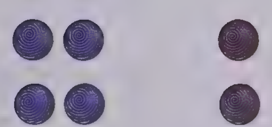
$6 - 3 = \boxed{3}$



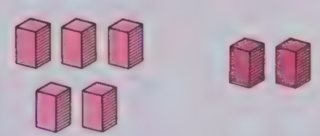
$7 - 5 = \boxed{2}$

$7 - 6 = \boxed{1}$

$6 - 0 = \boxed{6}$



$6 - 2 = \boxed{4}$



$7 - 2 = \boxed{5}$

Differences related to sums of 6 and 7

Teaching
Page b-34

Since the exercises on this page are similar to those on page b-33, you might want the children to work on them independently. As on page b-28, choose your vocabulary according to what children are capable of handling. That is, use tent, box, ball, block, can, etc. or refer to the geometric terms pyramid, sphere, cube, cylinder, etc. Continue to allow children to use counters if they want, although you should encourage them to think about the illustrated sets.

board. At the top of each double column, write "Add 1" and go down the first column writing the digits 0 through 6 in order. Then (still moving down the column) repeat the digits at random. Leave the second columns blank.

Next, organize a "Blue" team and a "Green" team. Give the leader of each team the appropriate color chalk. At a signal, the first member of each team should write the first answer in the right column, give the chalk to the next child in line, and go to the end of his line. The game continues until the players reach the end of the columns of figures. Give three points to the team that finishes first and one point for each correct answer. The rest of the class may sit as a jury to determine the correct answers.

To vary the game, erase the "Add 1" direction and write "Subtract 2" or some other direction at the top. Adjust the first column as necessary. Then erase the answers and proceed as before. If the group is very mature and knows the number facts well, complete the answer column and ask everyone to try to fill in the missing parts on the first column.

Resources for Active Learning

Using a math balance:

A CLOUDBURST, Vol. 1, No. 1511, Midwest Publications

MATHEX: Numeration No. 2, pp. 22–25, Encyclopaedia Britannica Publications Ltd.

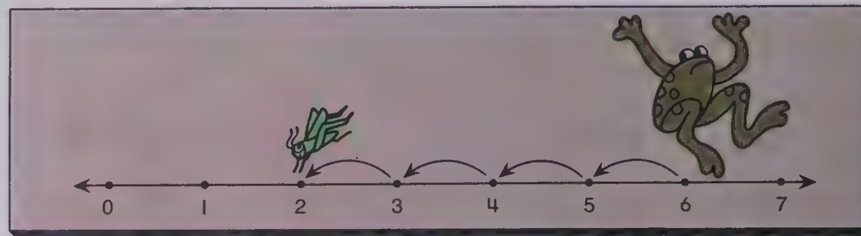
Nuffield Project: COMPUTATION AND STRUCTURE ②, pp. 65, Wiley

USING . . . MATHEMATICAL BALANCE, Math Media

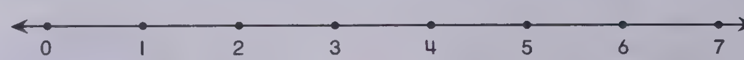
Teaching
Page b-35

Call attention to the illustrated number line and ask children to describe what their number line friends are doing. Encourage the children to make up stories about them. Emphasize the fact that subtraction is shown by jumps to the left and relate the illustration to the equation $6 - 4 = 2$.

Point out the blank number line to the children and explain that it is provided for them to use to find the differences. Also have them notice that the exercises at the bottom are written in the standard notation, but that they can think about them in the same way they do the equations.



Find the differences.



$6 - 2 = \boxed{4}$	$6 - 3 = \boxed{3}$
$7 - 2 = \boxed{5}$	$7 - 3 = \boxed{4}$
$5 - 2 = \boxed{3}$	$7 - 5 = \boxed{2}$
$6 - 4 = \boxed{2}$	$6 - 5 = \boxed{1}$

$$\begin{array}{r}7\\-2\\\hline 5\end{array}\quad\begin{array}{r}6\\-1\\\hline 5\end{array}\quad\begin{array}{r}4\\-2\\\hline 2\end{array}\quad\begin{array}{r}7\\-6\\\hline 1\end{array}\quad\begin{array}{r}7\\-4\\\hline 3\end{array}\quad\begin{array}{r}6\\-0\\\hline 6\end{array}$$

$$\begin{array}{r}6\\-4\\\hline 2\end{array}\quad\begin{array}{r}7\\-5\\\hline 2\end{array}\quad\begin{array}{r}6\\-2\\\hline 4\end{array}\quad\begin{array}{r}7\\-3\\\hline 4\end{array}\quad\begin{array}{r}5\\-3\\\hline 2\end{array}\quad\begin{array}{r}6\\-3\\\hline 3\end{array}$$

Subtraction – number line

Objectives

Page b-35: Given a number line labelled to 7 and subtraction equations related to sums of 6 and 7, the child will be able to find the differences by using the number line.

Page b-36: Given addition and subtraction equations on the same page, the child will be able to find the sums or the differences.

Pages b-35 and b-36 contain two different objectives. With some children, you may want to combine the lessons, but with others you will want to treat the pages as separate lessons.

Pre-Book Activity

Materials

demonstration number line

Show a demonstration number line on the chalkboard or the overhead projector. Write an equation such as $4 - 3 = \square$ and help children recall how the number line may be used to solve it. Emphasize that for subtraction, we jump "backward" (to the left) on the number line. Then work through some subtraction equations for 6 and 7 to prepare children for page b-35.

To prepare children for page b-36, present them with a mixture of addition and subtraction equations. You might spend a few minutes on oral drill with addition and subtraction equations. Then introduce some story problems such as:

Solve the equations.

$4 + 2 = \boxed{6}$	$7 - 3 = \boxed{4}$
$2 + 4 = \boxed{6}$	$6 - 2 = \boxed{4}$
$4 + 3 = \boxed{7}$	$7 - 4 = \boxed{3}$
$3 + 3 = \boxed{6}$	$6 - 4 = \boxed{2}$
$5 + 2 = \boxed{7}$	$6 - 3 = \boxed{3}$
$1 + 6 = \boxed{7}$	$7 - 2 = \boxed{5}$
$4 + 3 = \boxed{7}$	$4 + 2 = \boxed{6}$
$7 - 3 = \boxed{4}$	$6 - 2 = \boxed{4}$
$2 + 4 = \boxed{6}$	$3 + 4 = \boxed{7}$
$6 - 5 = \boxed{1}$	$7 - 5 = \boxed{2}$

Teaching
Page b-36

The exercises on this page for the first time include both addition and subtraction equations. Have the children describe the sets pictured in the first frame and help them relate these sets to the equation $4 + 2 = \square$. Then have them describe how the picture in the next frame shows subtraction.

Explain to the children that some of the equations on this page are addition equations and some are subtraction equations. Remind them to read the symbols carefully so they know what to do.

1) Jimmy had five pennies. His father gave him two more. How many pennies does Jimmy have now?
2) Susan had six kittens. She gave four of them away. How many kittens does she have now?
3) Anne brought six cookies to school. She gave some of them to Jane and found that she had three left. How many did she give to Jane?
4) Billy gave Tom three baseball cards to start a collection. Joe gave Tom four cards. How many cards does Tom have in all?

Write sample equations on the chalkboard.

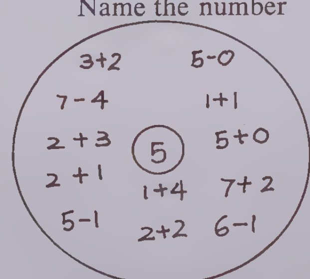
$3 + 2 = \square \qquad 7 - 4 = \square \qquad 6 - 1 = \square \qquad 2 + 5 = \square$

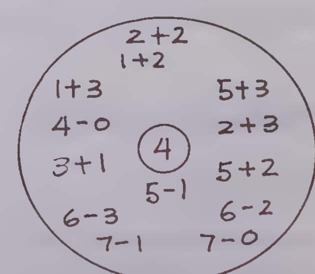
and ask children to ring the symbol in each equation which tells them whether to add or subtract. Emphasize that the plus sign, +, means to find the sum, that is, to add; and that the minus sign, −, means to find the difference, that is, to subtract.

Follow-up

Duplicator masters or drawings on the chalkboard may be used to provide children with further practice in finding sums and differences. Suggest to the children that they put a ring around every sum or difference which matches the number in the centre.

Name the number





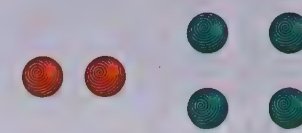
Teaching
Page b-37

Have the children look at the two equations in the top frames and then ask them to explain what they should do on this page. Emphasize that these two frames deal with addition and that they should find the sums. Then point out the minus signs in the equations of the next two frames. Be sure they understand that for these equations they should find the differences. If children have learned to read addition and subtraction equations correctly they should not be confused by the exercises on this page. However, stress that they should study the equations carefully.

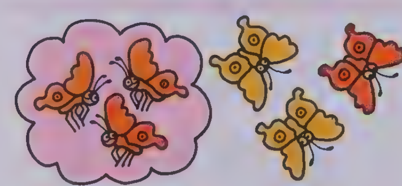
Show you know

Solve the equations.

$5 + 2 = \boxed{7}$



$2 + 4 = \boxed{6}$



$6 - 3 = \boxed{3}$

$7 - 3 = \boxed{4}$

Find the sums.

$3 + 3 = \boxed{6}$

$6 + 1 = \boxed{7}$

$\begin{array}{r} 5 \\ +2 \\ \hline 7 \end{array}$ $\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$ $\begin{array}{r} 4 \\ +2 \\ \hline 6 \end{array}$

Find the differences.

$7 - 2 = \boxed{5}$

$6 - 2 = \boxed{4}$

$\begin{array}{r} 6 \\ -4 \\ \hline 2 \end{array}$ $\begin{array}{r} 6 \\ -3 \\ \hline 3 \end{array}$ $\begin{array}{r} 7 \\ -1 \\ \hline 6 \end{array}$

Objective

The child will demonstrate his ability to work with the concepts presented in this module by finding sums and differences of 6 and 7.

Note that the change of pace activity which follows the review exercises again utilizes the strips. Although page b-38 should be handled with a light touch, it can indirectly be considered as a preparation for the following module.

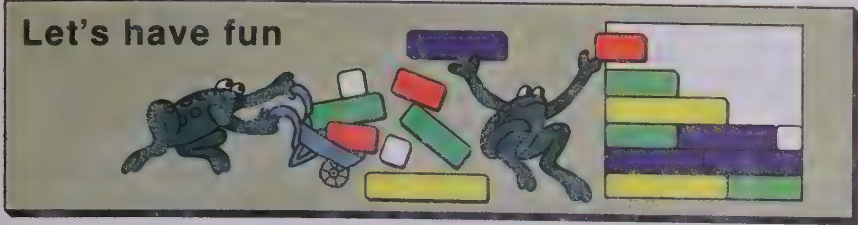
Pre-Book Activity

Materials

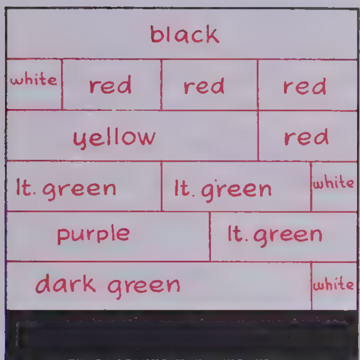
sets of centimetre strips
tagboard of stiff paper, at least 1 sheet per child

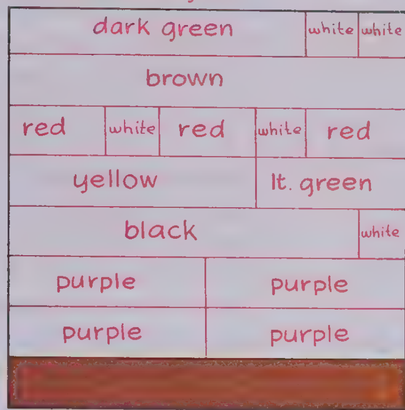
Write equations for sums and differences of 7 or less on pieces of 5-by-15-cm tagboard and pass these out to the children at random. On the chalkboard, write the numerals 1, 2, 3, 4, 5, 6, and 7. Leave some distance between the numerals. Ask a child to call out one of these numbers. Everyone should then examine his card to see if the answer to his equation is the number just called out. All who have cards for this number should place them on the chalk tray below the numeral. If they are placed on the chalk tray one by one, the class can check to see that the cards are placed under the numeral they match.

Let's have fun



Fill each square with your strips. Answers will vary. Examples are given.





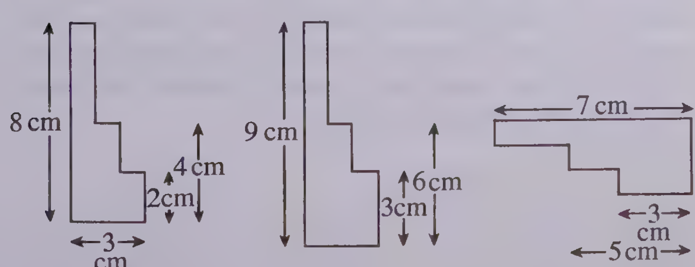
Readiness for sums of 8 and 9

Teaching
Page b-38

Distribute a set of strips to each child. Encourage some free play with the strips before explaining how to use this page. Then ask a child if he can guess what to do with the squares. Help the children understand that they should use as many different colored strips as they can to cover each square. As on page b-24, no mention need be made of specific number combinations even though this activity prepares the way for a study of the sums and differences of 8 and 9.

Follow-up

If children enjoy covering areas with the strips, you might prepare a duplicating master with areas of the following dimensions:



Resources for Active Learning

A CLOUDBURST, Vol. 1, No. 2911, Midwest Publications

LIGHT GREEN MODULE, UNIT B

Sums of 8 and 9

Pages b-39 to b-52

General Objectives

To increase understanding of addition and subtraction

To increase understanding of equations and skill in using them

To introduce the addition and subtraction combinations for 8 and 9

To begin development of skill in adding and subtracting combinations for sums equal to or less than 9

To develop a feeling for number combinations in story problem situations

The structure of this module closely parallels that of the previous module. The opening investigation introduces the child to combinations for 8 and 9, by use of the strips and by discussion of a carefully planned illustration. Then the addition combinations are presented in terms of sets and by use of the number line. The subtraction combinations of 8 and 9 are similarly developed. Finally on pages b-49 and b-50 children are presented with both addition and subtraction equations. The module concludes with a review page and a change of pace.

Mathematics

As in the previous module no new mathematical concepts are introduced. The addition and subtraction concepts are further developed to include new combinations. It is worth noting that the combinations for ten are postponed until children study place value and learn the meaning of the first two-digit numeral, 10.

Teaching Light Green Module, Unit B

Approximate Time: 7 to 10 days

Materials

counters, at least 9 for each child

flannelboard and set of felt objects, letters, numerals, and other symbols

magnetic strip board for each child if possible (see description on page 120 of this teacher's edition)

number line for demonstration

overhead projector (if available)

plastic numerals (or individual number cards) for each child

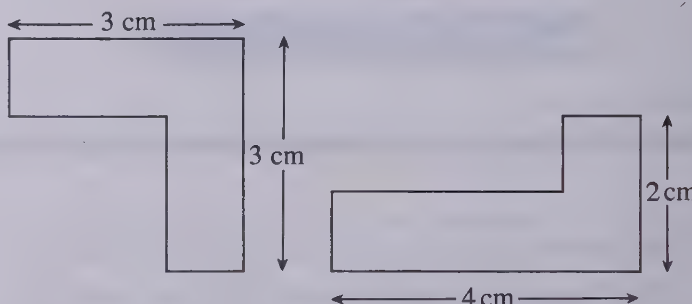
set objects for demonstrations

set of strips for each child

Although there is no specific vocabulary for this module, be sure children correctly understand and use terms such as "minus," "plus," and "equals."

Oral story problems are often an effective way to help children relate their study of addition and subtraction to real life situations. Make repeated use of stories similar to those suggested in the teaching notes for page b-49. Do not restrict yourself to the samples provided. Whenever you have extra time for arithmetic, initiate activities that provide a background for story problems. Draw the children into these stories by having them participate in activities that illustrate number combinations.

Since each child has a set of strips, you would be wise to have the children use them for many activities. You can make task cards or duplicated worksheets as shown below on which the strips are outlined to provide a study of certain number combinations or to show relationships among the strips. Activity cards or booklets sold for use with the Cuisenaire Rods would be excellent for these activities. However, your work with the strips should help you generate ideas of your own if these commercial cards are not available to you. For example, a worksheet with the following kinds of outlines may be used with two sets of directions: 1) Use only the white strips. How many white strips did you use to fill in each outline? 2) Try to cover each outline with only two strips. What two strips did you use?



Other worksheets might show outlines of trains which children can build for each strip. The children may then write an equation for each train. Color names may be used at first for such an activity. Have the children agree upon abbreviations for each color. If children are ready, number names might be used.

$p + p = n$

$g + y = n$

$r + d = n$

$w + k = n$

Color code: w = white, r = red, g = light green, p = purple, y = yellow, d = dark green, k = black, n = brown, e = blue, o = orange

Evaluation of Progress

You can evaluate whether or not the children have attained skills with addition and subtraction combinations from the way they handle pages b-44, b-48, b-49, b-50 and b-51.

The children should also be able to make trains with the strips and to use the number line for addition and subtraction.

Resources for Active Learning

General Activities:

Combining groups:

WORKJOBS, pp. 148–155, 160–161, 164–165, 168–169, 172–173, 194–197, 202–209, Addison-Wesley

For materials to study facts refer to Book B. Module 3 Introduction under "Resources for Active Learning."

Nuffield Project: COMPUTATION AND STRUCTURE ②, "The Operation of Addition," pp. 58–69, Wiley

Manipulative Devices:

Cubical Counting Blocks, (Milton Bradley; school supplier)

Hainstock Blocks (Creative Publications; Lakeshore)

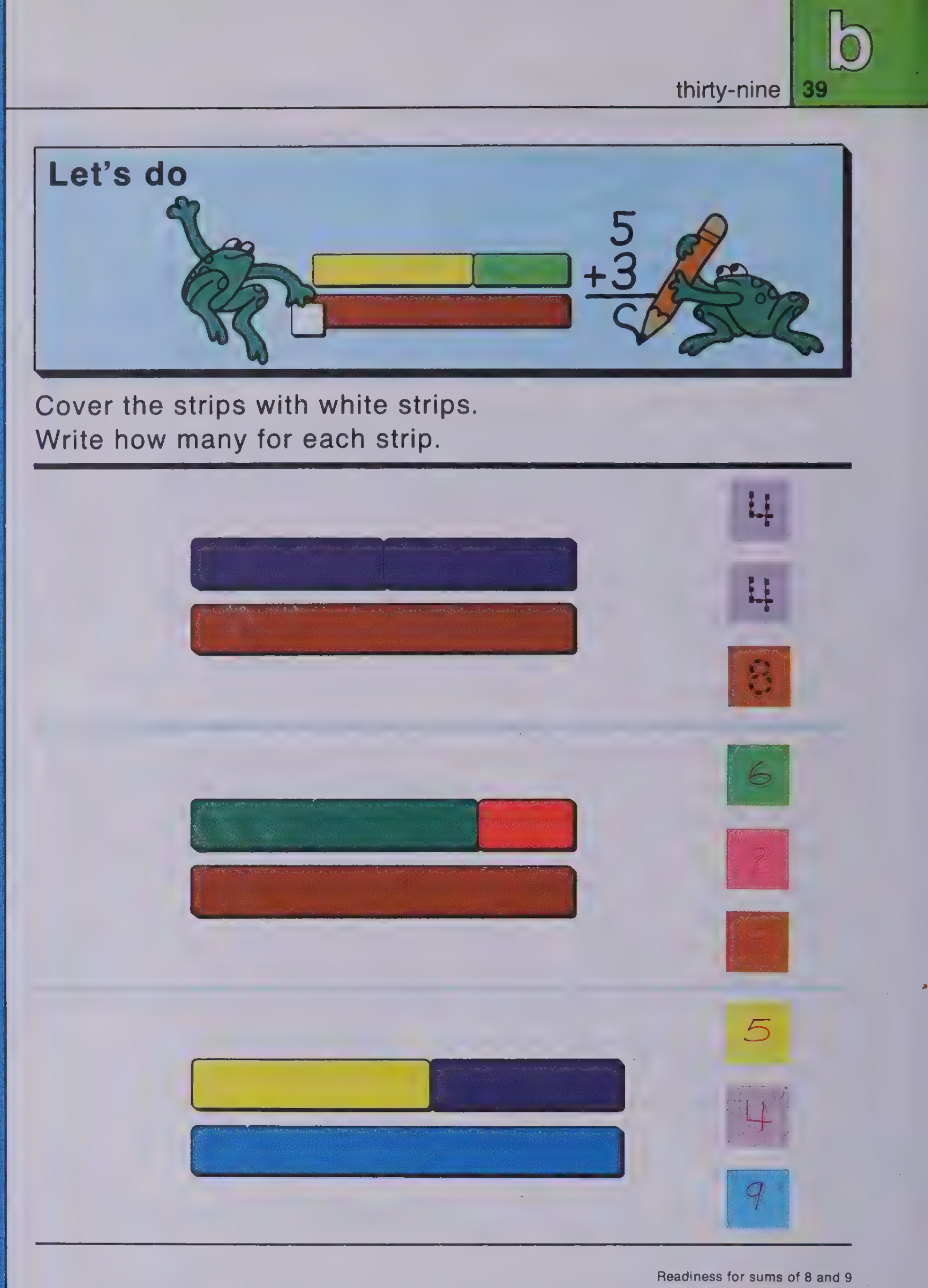
Commerical Games:

Exploring Number and Space games (Science Research Associates)

Investigation
Page b-39

This investigation follows the same format as the investigation on page b-25. Direct the children to cover the three illustrated strips in the first frame with white strips. Then ask them to count how many white strips are needed to cover one purple strip. Point out the dashed numeral 4 in the yellow box and ask the children to trace over it. Continue similarly by asking how many white strips are needed to cover the other purple strip; and finally the brown strip. You might ask more mature children if they can think of a way of writing 4, 4, and 8 in an equation. Help such children see how $4+4=8$ can be related to the strips, but do not emphasize this. It is not intended that this relationship be stated in equation form at this time.

If children are capable, encourage them to continue covering each set of illustrated strips and recording the numbers appropriately. Give only as much guidance as necessary. Some children, however, will benefit from your direction, so work through the remaining frames together with these children.


b
thirty-nine 39

Let's do

Cover the strips with white strips.
Write how many for each strip.

Readiness for sums of 8 and 9

Purposes

To provide background for introduction of combinations of 8 and 9

To further develop an understanding of number combinations by use of the strips

Preparation

Materials

set of strips for each child

By now children should be fairly familiar with the strips. If the children have begun to refer to some of the strips with the number names, it would be helpful to review these. For example, say: "If we call the white strip *one*, what should we call the red strip?" (Two.) Tell the children that you want them to build a staircase with the strips as you call out the numbers. Then slowly call out the numbers from one to nine. (If children want to include the orange strip and call it ten, allow them to do so, but do not stress ten since they have not yet studied the numeral for ten.) When they have completed the staircase, ask them to pick up the four strip, or the five strip. Ask questions such as: "What color is the nine strip?" "The six strip?" "What is the number name for the purple strip?" "The light green strip?" and so on. Finally, ask them to build a few simple trains. Example: Use two strips to make a train as long as the dark green or six strip.

If you think the children are not ready yet to use the number names so extensively, give them similar activities but use the color names. There is no harm in delaying the use of the number names at this level. Greater stress should be on the relationships among the strips.

Let's talk

Readiness for sums of 8 and 9

Discussion
Page b-40

Ask the children to describe the picture. Be sure they include how many bees (how many bees in flight, how many not flying, and how many altogether). Call attention to the two flowers and have children suggest different ways the eight bees might light on the two flowers. As you discuss different combinations, repeat the question "How many bees on both flowers?" to help children keep in mind that they are discussing combinations of 8. Next suggest that the bee on the leaf might join the other bees on the flowers. Have children describe some possible combinations of how the nine bees might distribute themselves between the two flowers.

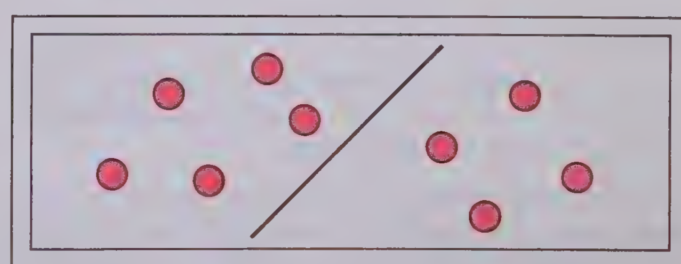
Throughout this discussion of combinations for 8 and 9, many children would benefit from actually using small counters (such as the white strips) for the bees. Direct them to place the counters on the flowers to show the ways the bees might light. Continue to stress the sum of each of the combinations to be sure children relate them correctly to either 8 or 9. Write the combinations they mention on the chalkboard.

8 → 7,1
8 → 6,2
8 → 5,3
8 → 4,4
8 → 8,0

9 → 9,0
9 → 7,2
9 → 8,1
9 → 6,3
9 → 5,4

Follow-up

Provide children with further opportunities to work with the counters. The suggested follow-up on page b-26 would be suitable for use here with combinations of 8 and 9.

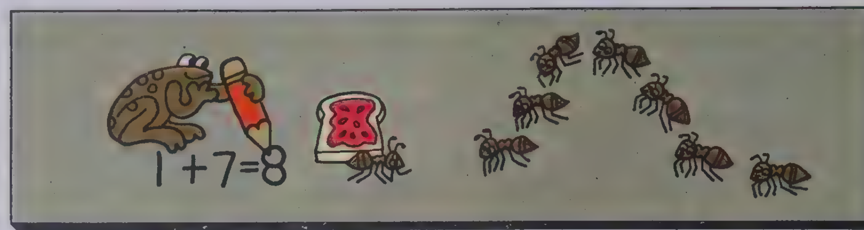


7 + 2 = ___	2 + 6 = ___
8 + 0 = ___	6 + 3 = ___
4 + 4 = ___	5 + 4 = 9
7 + 1 = ___	8 + 1 = ___

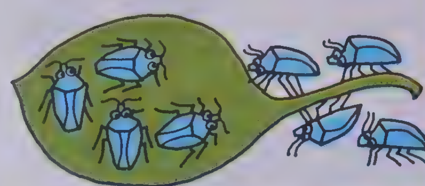
Teaching
Page b-41

Again use the demonstration art as an introduction. Then call attention to the first frame. Ask the children to count how many bugs are on the leaf and how many on the stem. Help them relate the equation symbols $4 + 4$ to the two sets. Then have them explain how many insects there are altogether in this frame and what number they should write in the empty box. When they have completed the equation $4 + 4 = 8$, explain that they should study the sets in the remaining frames and complete the equations on their own. As they are working, ask individual children to explain a particular equation to you. For example, you might ask a child if he can show you why $7 + 2 = 9$. Help him explain that the set of seven when put with the set of two may now be thought of as a set of nine.

If some children wish to use counters to solve the equations, encourage them to do so.

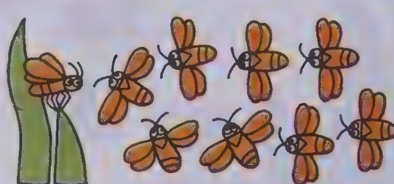


Find the sums.

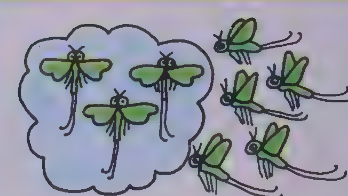


$4 + 4 = \boxed{8}$

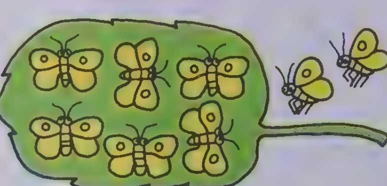
$4 + 5 = \boxed{9}$



$1 + 8 = \boxed{9}$



$3 + 5 = \boxed{8}$



$6 + 2 = \boxed{8}$

$7 + 2 = \boxed{9}$

Objective

Given an addition equation with two addends whose sum is 8 or 9, the child will be able to find the sum by studying a related set illustration.

The set illustrations are provided for use by the child as he solves the equations. However, if some children still want to work with counters, allow them to do so.

Pre-Book Activity

Materials

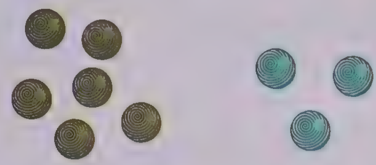
flannelboard
felt objects
individual numeral cards

Use a flannelboard or chalkboard and build equations for number stories. For example: "Five children were at the book corner." "Three more soon joined them." "How many were there in all?" After children respond, "Eight," ask if anyone can build an equation to record the number action. You might also have all the children use numeral cards at their desks. Other suitable stories follow:

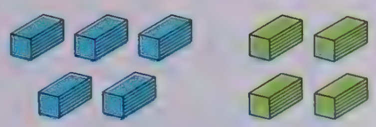
1) The first grade class had four yellow balls for playground use. Then the sixth grade athletic club gave them four more balls. How many balls do the first graders now have?
2) Dan painted five pictures for the art bulletion board. Helen painted four. How many pictures did they have?
3) Tom walked seven blocks and then rested. Then he walked two more blocks. How many blocks did he walk in all?

Find the sums.

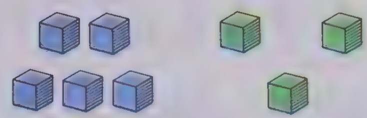
$2 + 6 = \boxed{8}$



$6 + 3 = \boxed{9}$



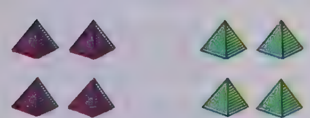
$5 + 4 = \boxed{9}$



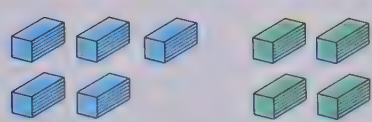
$5 + 3 = \boxed{8}$

$2 + 7 = \boxed{9}$

$1 + 7 = \boxed{8}$



$4 + 4 = \boxed{8}$



$5 + 4 = \boxed{9}$

Teaching
Page b-42

This page is essentially a continuation of page b-41; that is, the concepts and manner of solving equations are the same. Work through at least one or two of the frames with all of the children. With some children you might want to work through all of the frames; others will benefit from completing the page independently. Use whichever terms you think your children can best relate to when you are referring to the illustrations.

Follow-up

List the addition facts for 8 and 9 on the chalkboard. Distribute 5-by-7-cm cards and instruct the children to add to their set of flash cards the combinations for 8 and 9. If you choose, you might duplicate these combinations on light colored construction paper and ask the children to cut them on the dotted lines. Remind them to put the answer on the back of each card.

5 cm

7 cm

$\begin{array}{r}1\\ \underline{+7}\end{array}$	$\begin{array}{r}2\\ \underline{+7}\end{array}$	$\begin{array}{r}2\\ \underline{+6}\end{array}$	$\begin{array}{r}3\\ \underline{+6}\end{array}$
$\begin{array}{r}1\\ \underline{+8}\end{array}$	$\begin{array}{r}3\\ \underline{+5}\end{array}$	$\begin{array}{r}4\\ \underline{+5}\end{array}$	$\begin{array}{r}4\\ \underline{+4}\end{array}$

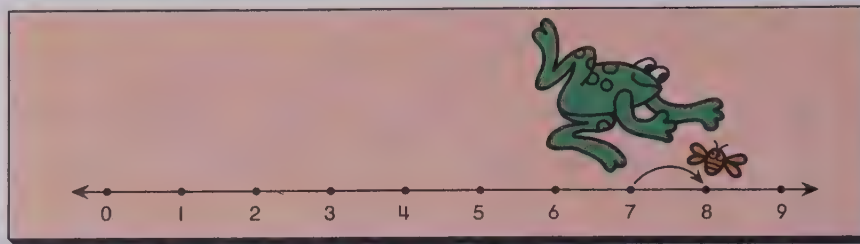
Resources for Active Learning

MATHS MINI-LAB, Card 2, Selective Educational Equipment

TEACHING
Page b-43

Call the children's attention to the number line in the illustration and use it as basis for some number line stories. Be sure to relate the illustration to the equation $7 + 1 = 8$. Then point out the blank number line and the fact that it is labelled from 0 to 9. Ask the children to look at the first equation $2 + 6 = \square$. Have a volunteer explain how to use the number line as an aid in solving the equation. Be sure the children understand that the first number is their starting point and that they use the second number as the number of jumps. Finally they read off the answer from their landing point. Remind the children to count their jumps carefully, particularly now that the numbers they are using are not quite so small.

Also, call their attention to the vertical notation used for the problems at the bottom of the page. Children may use the number line to find these sums.



Find the sums.

0 1 2 3 4 5 6 7 8 9

$2 + 6 = \boxed{8}$	$6 + 3 = \boxed{9}$
$5 + 3 = \boxed{8}$	$3 + 4 = \boxed{7}$
$8 + 0 = \boxed{8}$	$3 + 6 = \boxed{9}$
$5 + 4 = \boxed{9}$	$6 + 2 = \boxed{8}$
$4 + 2 = \boxed{6}$	$1 + 8 = \boxed{9}$

$\begin{array}{r}3\\+6\\\hline 9\end{array}$ $\begin{array}{r}7\\+1\\\hline 8\end{array}$ $\begin{array}{r}5\\+3\\\hline 8\end{array}$ $\begin{array}{r}2\\+7\\\hline 9\end{array}$ $\begin{array}{r}6\\+3\\\hline 9\end{array}$ $\begin{array}{r}5\\+2\\\hline 7\end{array}$

$\begin{array}{r}4\\+5\\\hline 9\end{array}$ $\begin{array}{r}3\\+3\\\hline 6\end{array}$ $\begin{array}{r}4\\+4\\\hline 8\end{array}$ $\begin{array}{r}6\\+2\\\hline 8\end{array}$ $\begin{array}{r}0\\+9\\\hline 9\end{array}$ $\begin{array}{r}2\\+6\\\hline 8\end{array}$

OBJECTIVE

Given addition equations with sums of 9 or less and a number line labelled from 0 to 9, the child will be able to solve the equations by referring to the number line.

PRE-BOOK ACTIVITY

If children enjoyed the activity "What's my equation?" suggested on page b-29, use it again. Make up stories about the number line frog and his insect friends hopping along the line. Point out to the children that the number line has been labelled to 9. Relate stories such as the following and ask children to record the action in an equation. "The frog spotted the number line bee at the point labelled 7. Since he didn't want to be caught he hopped one space and landed at 8. Can anyone name my equation?" If, while you tell the story, you emphasize 7 as the starting point, the 1 jump, and the landing point, 8, and remind the children to record what the bee is doing, many first graders will be able to give the correct equation $7 + 1 = 8$.

Complete each table.

Add 4	
3	7
5	9
4	8
2	6

Add 3	
6	9
0	3
4	7
3	6

Add 5	
1	6
4	9
3	8
2	7

Complete the matching.

4 + 4	7	8	9
5 + 3	7	8	9
4 + 3	7	8	9
2 + 6	7	8	9
5 + 4	7	8	9
2 + 7	7	8	9
7 + 0	7	8	9
6 + 3	7	8	9
2 + 5	7	8	9

Practice—sums of less than 10

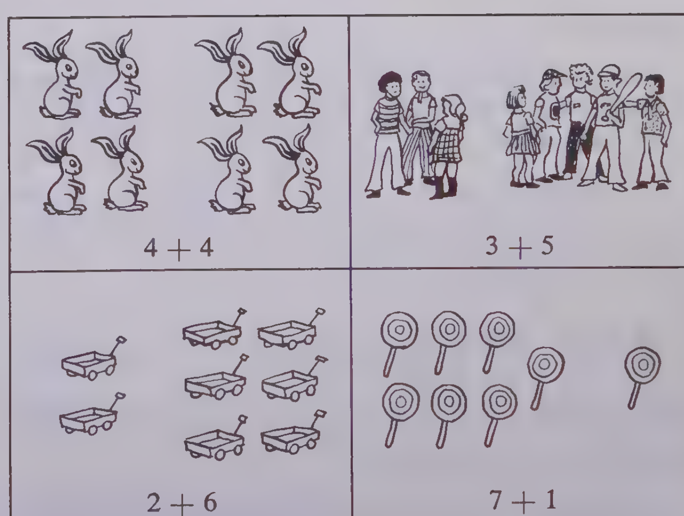
Teaching
Page b-44

Since the exercises on this page have a slightly different format, you will want to give the children careful guidance. Read the directions for the top frame. Ask the children to tell you the meaning of the directions at the top of each table. Do the first problem on the "Add 4" table and have children trace over the dashed numeral. If they understand what to do, instruct them to follow the directions carefully and complete each table.

Children should not have difficulty understanding what to do on the bottom half of the page. Direct them to ring the numeral on the right which matches the sum on the left.

Follow-up

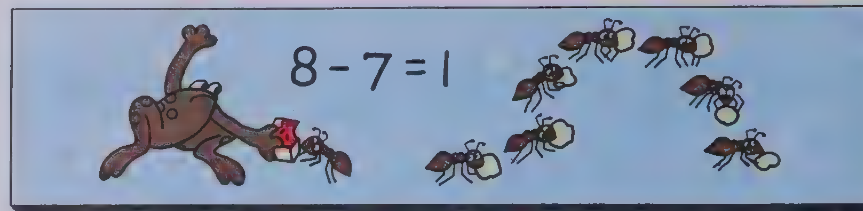
Supply each child with crayons and a large sheet of newsprint. Guide the children in folding the paper into four sections. Suggest that they write either a large numeral 8 or a large numeral 9 at the top of their paper. Then, in each section, draw a different set which shows a combination for their number.



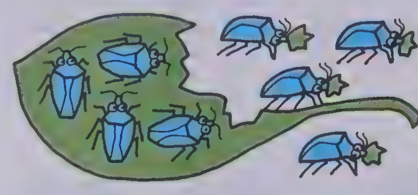
Teaching
Page b-45

Call attention to the first frame. Have children describe how the pictured sets help them think of take away or subtraction. Suggest that they cover with their counters, or with their fingers, four of the insects. They should then count how many are left. Relate this number to the difference $8-4$. Since four are left, the children should write 4 in the answer box.

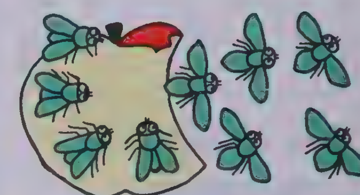
Work through the remaining frames in a similar manner. If the children are capable, you might want them to work individually. However, most will benefit from teacher guidance for these subtraction equations.



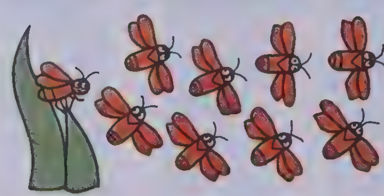
Find the differences.



$8 - 4 = \boxed{4}$

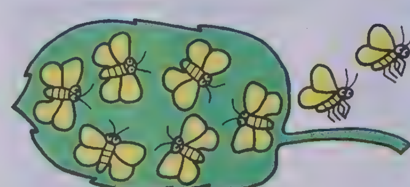


$9 - 5 = \boxed{4}$

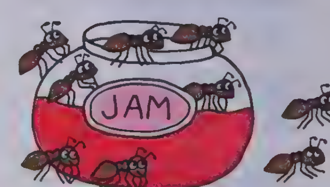


$9 - 8 = \boxed{1}$

$8 - 5 = \boxed{3}$



$8 - 2 = \boxed{6}$



$9 - 2 = \boxed{7}$

Objective

Given subtraction equations related to sums of 8 and 9, the child will be able to find the differences by referring to illustrated sets.

It is important to provide the children with opportunities to use counters and other manipulatives in these subtractive situations.

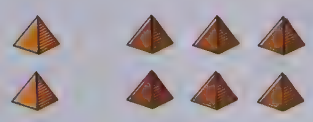
Pre-Book Activity

Materials

9 counters for each child

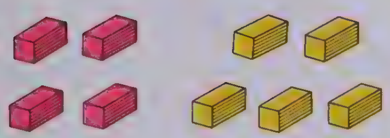
Give each child nine counters. Ask the children to build a set of nine on their desk or table. Then ask that they remove, or simply hide with their hand, five of the counters. Ask: "How many are left?" and see if children can describe the subtraction equation for this situation. Write $9-5=4$ on the chalkboard. Similarly ask children to build a set of eight counters on their desk. Then ask that they cover six of the counters and tell how many are left. See if the children can then describe this take away situation by using an equation. Write $8-6=2$ on the chalkboard. Work through several similar examples. Always help children relate their action with the counters to the written equation.

Find the differences.

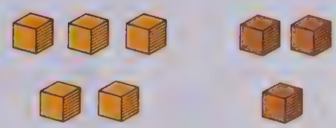


$8 - 6 = \boxed{2}$

$9 - 3 = \boxed{6}$

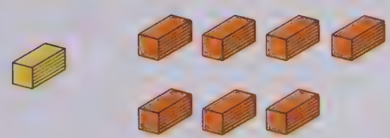


$9 - 5 = \boxed{4}$

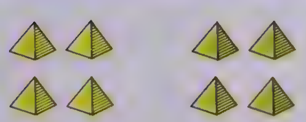


$8 - 3 = \boxed{5}$

$9 - 7 = \boxed{2}$



$8 - 7 = \boxed{1}$



$8 - 4 = \boxed{4}$

$9 - 4 = \boxed{5}$

Differences related to sums of 8 and 9

Teaching
Page b-46

Page b-46 is basically the same as page b-45. Thus, you might want children to try to solve the equations independently. However it is very important to help children relate the numerical symbols of the equations to the appropriate set illustration. Guide the children in the subtractive interpretation of the illustrated sets by helping them realize for example, that 8 may be thought of as a set of 6 and a set of 2. Those who want to use sets of counters should be allowed to do so.

Follow-up

Write the subtraction combinations for 8 and 9 on the chalkboard. Have the children add these to the sets of fact cards they have made. If you choose, duplicate these combinations on light colored construction paper and ask the children to cut them on the dotted lines. Let the children check their cards against a master list to be sure they did not forget any facts or use wrong answers. Encourage them to work with the cards in their free time, either by themselves or with a partner.

5 cm

7 cm

$\begin{array}{r}8\\-4\\\hline\end{array}$	$\begin{array}{r}9\\-5\\\hline\end{array}$	$\begin{array}{r}8\\-3\\\hline\end{array}$	$\begin{array}{r}9\\-1\\\hline\end{array}$
$\begin{array}{r}9\\-6\\\hline\end{array}$	$\begin{array}{r}8\\-2\\\hline\end{array}$	$\begin{array}{r}9\\-7\\\hline\end{array}$	$\begin{array}{r}8\\-7\\\hline\end{array}$

Resources for Active Learning

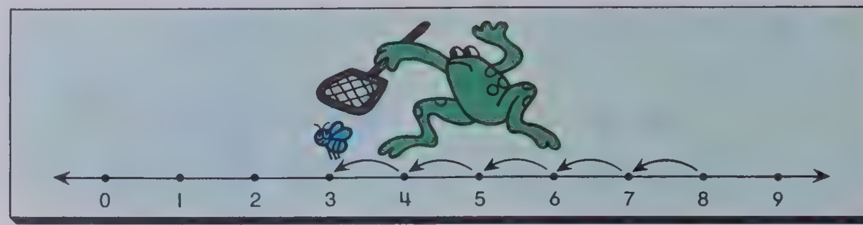
EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Addition," Nos. 31–33; "Subtraction," Nos. 54–55, Responsive Environments Corp.

For using the math balance refer to Unit B, Red Module, page g-34 under "Resources for Active Learning"

Teaching
Page b-47

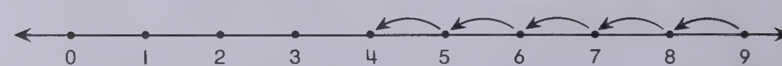
Use the illustration at the top of the page as a basis for discussion. Ask the children to make up stories about the frog and bee jumping along the number line and discuss the equation $8 - 5 = 3$. Then direct the children's attention to the equation $8 - 3 = \square$ and its accompanying number line. Point out that the first number of the equation should be the starting point on the number line. Then, since this is a subtraction equation they should make their three jumps to the left. Finally, they should read their answer from the landing point, 5, and write the numeral 5 in the answer space provided.

Work through the next equation similarly. You might call attention to the fact that all of the equations on this page deal with the facts for 8 and 9. Encourage children to solve the equations by using the number line although if some prefer other methods such as using counters, allow them their preference.



Solve the equations.

$8 - 3 = \boxed{5}$



$9 - 5 = \boxed{4}$

$8 - 7 = \boxed{1}$	$9 - 7 = \boxed{2}$
$9 - 4 = \boxed{5}$	$9 - 2 = \boxed{7}$
$8 - 6 = \boxed{2}$	$8 - 2 = \boxed{6}$
$9 - 6 = \boxed{3}$	$8 - 5 = \boxed{3}$
$8 - 4 = \boxed{4}$	$9 - 8 = \boxed{1}$

Differences—number line

Objective

Given a number line labelled to 9 and subtraction equations related to sums of 9 or less, the child will be able to find the differences by using the number line.

Pre-Book Activity

Materials

roll of paper for floor number line (masking tape may be used)

Roll out along the classroom floor a piece of paper about 500 to 700 centimetres wide and 3 metres long on which you have marked a number line from 0 to 9. Write an equation on the chalkboard and suggest that children try to "jump" the equation on the floor number line. For example, if you write $8 - 5 = \square$, a child should start at 8 and then jump back 5 times. He should give the answer by reading the numeral he is standing on; here, for example, he should be standing on the space marked 3. Encourage children to suggest addition and subtraction equations, and then have a child "jump out" each equation on the floor number line. Others may check to see if the jumps and the resulting answer are correct.

Find the differences.

8 − 3 = 5	7 − 4 = 3
9 − 2 = 7	9 − 4 = 5
6 − 4 = 2	8 − 1 = 7
9 − 5 = 4	6 − 2 = 4
8 − 5 = 3	9 − 8 = 1
7 − 2 = 5	8 − 4 = 4

9	7	8	6	9	8
−7	−5	−8	−5	−1	−7
2	2	0	1	8	1

9	6	8	8	7	9
−3	−3	−2	−6	−3	−6
6	3	6	2	4	3

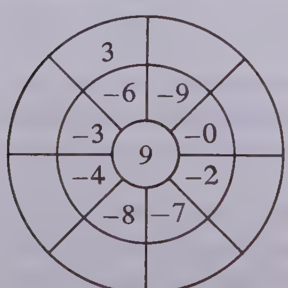
Teaching
Page b-48

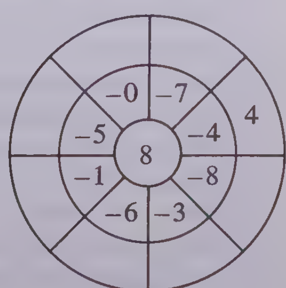
Point out the number line at the top. Explain that it is provided so the children can use it as an aid in solving the equations. Encourage children to complete the page independently. You might point out the vertical notation of the exercises at the bottom. Also, children should understand that these exercises will review subtraction facts of 6 and 7 as well as the facts of 8 and 9.

Follow-up

Duplicated number wheels often provide children with an enjoyable form of practice.

Instruct the children to subtract each of the numbers between the spokes of the wheel from the number in the centre. Direct them to write the answers in the outer spaces.



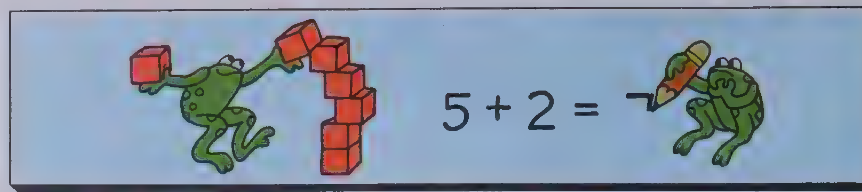


Bulletin board charts can be used for games which are useful in motivating children to learn the combinations. For example, choose four teams matched as evenly as possible. Then let each team choose a distinct outline of a prehistoric animal. These animals should be traced and cut out of different colored construction paper. The teams compete in solving equations at the chalkboard (one child at a time from each team). Make a chart with a path for each dinosaur. The chart should show dinosaurs advancing along the path toward a distant water hole at the rate of one space for each point. Show volcanoes, jungle ferns, vines, tar pits, and other dinosaurs along the path. For this lesson, concentrate on subtraction combinations related to eight and nine. This game can be continued for several days if the children's interest is sustained.

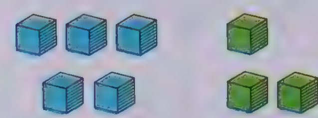
Teaching
Page b-49

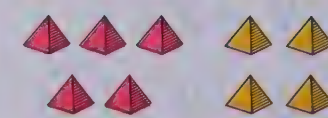
Call attention to the illustrated sets in the top frames. Ask a child to read the addition equation. Help the children relate $5 + 3 = \square$ to the set of 5 and the set of 3. Have them count how many items in all and write the answer, 8, in the box. Work through the subtraction equation, $9 - 4 = \square$ similarly. Help children realize how the pictured sets may be related to subtraction. Have them complete the equation with the correct numeral.

You will want to work several of the exercises together with some children. Make sure that they are correctly and carefully reading the equations. With other children, you might want to encourage them to do all of these exercises on their own. If you choose to assign these exercises for independent work, move around the classroom to be sure the children add for the addition equations and subtract for the subtraction equations. If you notice an error, ask the child to read the equation out loud to himself to see if he can discover his own mistake.



Solve the equations.





$5 + 3 = \boxed{8}$	$9 - 4 = \boxed{5}$
$7 + 1 = \boxed{8}$	$9 - 6 = \boxed{3}$
$4 + 4 = \boxed{8}$	$8 - 5 = \boxed{3}$
$3 + 6 = \boxed{9}$	$9 - 2 = \boxed{7}$
$8 - 6 = \boxed{2}$	$9 - 5 = \boxed{4}$
$6 + 2 = \boxed{8}$	$4 + 4 = \boxed{8}$
$9 - 3 = \boxed{6}$	$4 + 5 = \boxed{9}$
$8 - 7 = \boxed{1}$	$8 - 4 = \boxed{4}$

Objective

Given addition or subtraction equations related to sums of 9 or less, the child will be able to find the sums or differences.

With few exceptions children's work up to this point has been either with addition or with subtraction. On these two pages addition and subtraction exercises are presented side by side. It will be important to emphasize to the children how careful they must be in examining each equation so that they perform the correct operation.

Pre-Book Activity

Varying the process needed to solve simple oral stories will help children adapt their thinking from an addition situation to a subtraction situation. It would be helpful to write both addition and subtraction equations on the chalkboard and ask children to choose the equation which fits each story. The following are examples.

$9 - 4 = \square$	$5 - 3 = \square$
$3 + 2 = \square$	$4 + 2 = \square$
$6 - 2 = \square$	$8 - 3 = \square$

1) Jim, Jack and Joe rode their bikes to the playground. Pete and Tom were driven to the playground by their parents. How many boys in all met at the playground?
2) By 2:00 o'clock, eight boys were at the playground. One parent picked up three of the boys around 3:00 o'clock. How many boys were left playing in the playground?
3) Nine girls were at a birthday party. Four of the girls won prizes. How many did not win a prize?
4) Suzy had four balloons. Her brother then gave her

Find the sums and differences.

0 1 2 3 4 5 6 7 8 9

$4 + 3 = \boxed{7}$

$3 + 5 = \boxed{8}$

$4 + 4 = \boxed{8}$

$6 + 2 = \boxed{8}$

$6 + 3 = \boxed{9}$

$8 - 2 = \boxed{6}$

$9 - 3 = \boxed{6}$

$9 - 7 = \boxed{2}$

$8 - 6 = \boxed{2}$

$9 - 2 = \boxed{7}$

5	5	6	1	2	8
+4	+3	+2	+8	+5	+1
9	8	8	9	7	9

9	7	8	9	8	9
−9	−0	−3	−6	−7	−8
0	7	5	3	1	1

7	8	7	4	9	2
+2	−3	−5	+5	−6	+6
9	5	2	9	3	8

Practice—addition and subtraction

Teaching
Page b-50

Call the children's attention to the number line at the top of the page. Have someone explain why it is there. Help the children realize that it is provided as an aid for those who wish to use it. However, if children prefer to work with counters, be sure they feel free to do so. Again tell children to watch the signs carefully, particularly at the bottom of the page, since addition and subtraction exercises are presented side by side.

two more. How many did she have then?

5) Ron had six balloons. His dog popped two of them. How many balloons did Ron have left?
6) Judy's older brother wants to be a drummer in a band so he likes to practice five hours a day. Today he has already practiced three hours. How many more hours should he practice?

Follow-up

If children require more exercises of mixed practice, prepare a duplicating master such as the one shown in the next column.

$3 + 5 = \square$

$8 - 3 = \square$

$9 - 6 = \square$

$4 + 5 = \square$

3	7
+6	−2
8	4
−4	+3

Add 4	
5	
1	
3	

Subtract 5	
9	
7	
8	

Resources for Active Learning

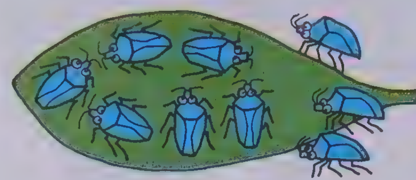
DEVELOPMENTAL MATH CARDS, "Addition Satellites," B-10, Addison-Wesley

Teaching
Page b-51

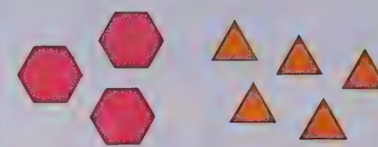
Call the children's attention to the two top frames. Point out that both of these equations are addition equations and that the illustrated sets should be studied as they try to solve the equations. Then have the children notice that the next two frames deal with subtraction; again illustrated sets are provided to help them solve the equations. In the last two sections of the page, encourage children to follow the sign of each exercise even though all the addition equations are grouped together and likewise all the subtraction equations are grouped together. Move around the room as the children work to be sure that they are following the directions properly. Children's achievement on this page as well as on several previous pages should help you evaluate their understanding of addition, subtraction, equations and number combinations of 9 or less. You might want to use the light green module of Unit C for building skills of addition and subtraction at this time for those who would benefit from more practice.

Show you know

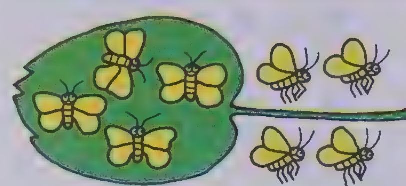
Solve the equations.



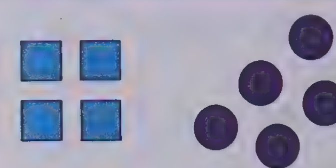
$6 + 3 = \boxed{9}$



$3 + 5 = \boxed{8}$



$8 - 4 = \boxed{4}$



$9 - 5 = \boxed{4}$

Find the sums.

$6 + 2 = \boxed{8}$

$4 + 5 = \boxed{9}$

$$\begin{array}{r} 2 \\ +7 \\ \hline 9 \end{array} \qquad \begin{array}{r} 4 \\ +4 \\ \hline 8 \end{array} \qquad \begin{array}{r} 8 \\ +1 \\ \hline 9 \end{array}$$

Find the differences.

$8 - 2 = \boxed{6}$

$9 - 6 = \boxed{3}$

$$\begin{array}{r} 9 \\ -4 \\ \hline 5 \end{array} \qquad \begin{array}{r} 8 \\ -3 \\ \hline 5 \end{array} \qquad \begin{array}{r} 9 \\ -2 \\ \hline 7 \end{array}$$

Objective

The child will demonstrate his ability to work with the concepts presented in this module by finding sums or differences for 9 or less.

Pre-Book Activity

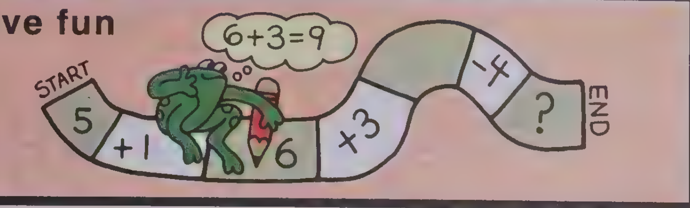
Write on the chalkboard a list of equations, some true and some untrue. Make it very clear to the children that some of the equations on the board are not correct. Then ask children to come up and ring each incorrect equation. Finally, ask for volunteers who can rewrite each untrue equation so that it becomes true.

3 + 1 = 5	3 + 2 = 5
2 + 6 = 8	
9 − 5 = 4	
7 − 4 = 2	7 − 5 = 2

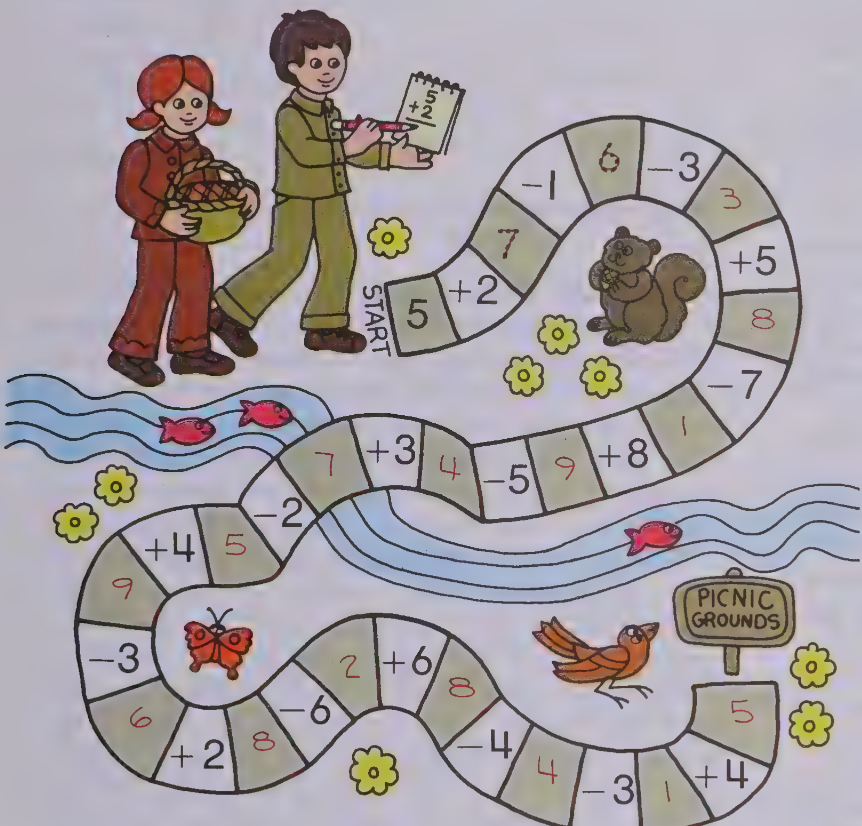
If children enjoy working in teams, you might have four lists, each with the same number of untrue equations. Then a point may be scored for every equation correctly identified as untrue and for each of these that is correctly rewritten. Use games such as these to show children how tally marks can be used for scoring points. A record of a game might look something like the following.

Bears	~~||||~~ |	Lions	~~||||~~ |||
Tigers	~~||||~~	Wildcats	~~||||~~

Let's have fun



Follow the trail. You should end with 5.



Maintenance activity

Teaching
Page b-52

Emphasize that this page is for fun. Ask the children to see if they can follow the trail. Direct them to find the "start" and then follow the clues in the white spaces. Tell them that the trail starts with 5, and the first clue says "Add two." Ask, "How much is five plus two?" When they answer "seven," tell them to trace over the dashed numeral 7. Continue with, "The next clue says 'minus one.' How much is seven minus one?" When they answer "six," agree, and tell them to trace the numeral 6 in the third yellow space.

Direct the children to continue following the trail and the clues. Point out that they should end with 5 if they have done the work carefully. Some children may need to manipulate blocks or counters to be able to engage in this activity.

Follow-up

Suggest a "treasure hunt" and provide the children with scratch paper and pencil. Draw a winding path on the board. Label ten spots on the path from 0 through 9 and add landmarks such as trees, mountains, swamps, and so forth to each spot. For directions, say to the children: "Let's see if you can follow the clues and find the buried treasure. Listen carefully and do what the clues tell you to do." A sample set of clues follows.

"Start at 3, . . . add 4, . . . subtract 2, . . . Where is the treasure?" Since the answer to this chain game is 5, the treasure has been buried at the spot near 5. Thus after you ask "Where is the treasure?" ask a child who thinks he knows where it is, to come and place an "X" beside the correct numeral. For the example above, his "X" would be near 5. Then ask this child to whisper to you a number where he wants to bury the treasure he has just found. Then call out clues so that the answer is the new burial spot the child gave you. Another volunteer can then mark this spot and so on. Be sure that when you give the clues you go very slowly so children will have time to figure them out.

Measurement

Pages b-53 to b-62

General Objectives

To introduce the concept of length

To develop understanding of the terms "longer than," "shorter than," and "as long as"

To familiarize the child with the centimetre as a unit of linear measure

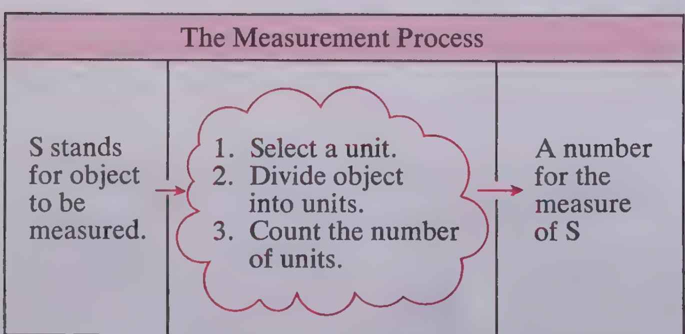
This module should provide experience with the basic process of physical measurement; that is, the process of placing the unit of measure along side the object to be measured until the extent of the sequence of units and the object to be measured are the same, and then counting the number of units laid down.

The child is introduced to the centimetre unit. He first uses separate unit pieces to measure length. Then he associates a "train" of these units with a ruler. Commercial centimetre rulers may be introduced.

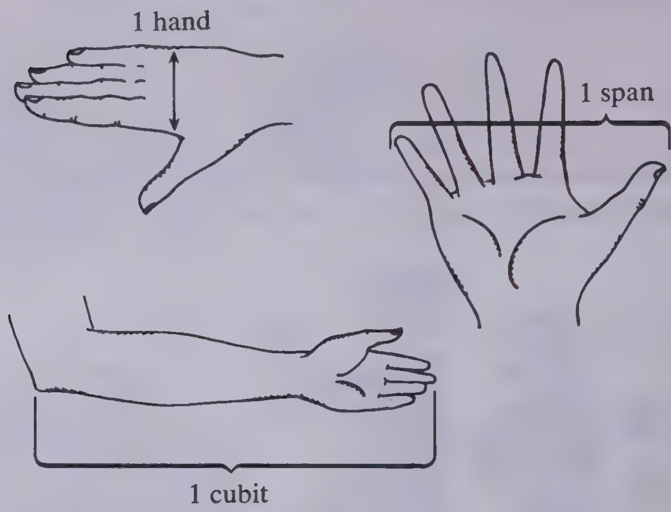
Pages b-61 and b-62 provide a review and change of pace for the dark green module. Pages b-63 and b-64 provide a review for the entire Unit B.

Mathematics

Measurement is a process of associating numbers with certain objects. Before an object in the real world can be measured, a unit must be chosen. The number 1 is assigned to this unit. The measuring process then consists of comparing the object to be measured with this unit, usually by counting the number of units it takes to "cover" or "fill up" the object. Thus the choice of the unit determines which numbers are assigned to other objects. This process is illustrated below.



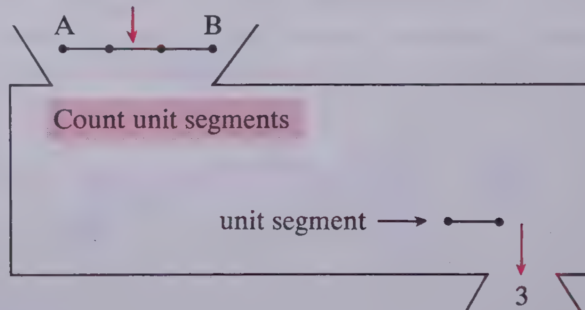
We should observe that *the choice of units is arbitrary,* as shown in the following figure.



Units for measuring length are frequently related to parts of the body. For example, the width of the palm, roughly 10 centimetres, makes a *hand.* Horses are measured by hands. If you spread your thumb and little finger as far apart as possible, you have a unit of measure called a *span,* about 23 centimetres. A biblical unit of measure, the *cubit,* is the distance from the elbow to the tip of the fingers, about 45 centimetres. Goliath is said to have had a height of "6 cubits and a span." Sailors still use a measure called a *fathom,* about 2 metres. This was originally the distance between outstretched fingertips when the arms are stretched out to each side at shoulder height.

Words like *height* and *distance* are often used to describe the idea of *length.* For example, we speak of the length of a piece of string, the distance between Calgary and Edmonton, and the height of a flagpole; but the same idea is present in all of these instances. We mean that we choose a unit of measure and then give a number that tells approximately how many of these units it would take, lying end to end along a straight path, to reach from one point to another.

The length of segment AB is a number that compares the size of the segment to be measured with the size of a unit segment. Perhaps the following illustration will help clarify the concept of measurement of length.



The input is line segment AB. The machine performs the operation of counting unit segments, and the output, 3 in this example, is the number of unit segments in the input.

Teaching Dark Green Module, Unit B

Approximate Time: 6 to 8 days

Materials

centimetre rulers
centimetre units
colored strips
paper clips, small size

Vocabulary

centimetre	high	measure	tall
far	long	ruler	unit

Throughout this module it is important that each child actively participate in the counting and manipulative activity involved in the measurement process. Although the material in the text and the ideas suggested in this manual do not assume that the child can count beyond nine, you may want the children who are capable to do activities which demand an ability to count further.

Evaluation of Progress

Since this module is basically an introduction to the process of measurement, very little emphasis should be placed on evaluation. Although the review page, b-61, may indicate to you which children have mastered the concepts and skills presented in the module, many children will have grown in their level of intuitive understanding of measurement even though their skill in using the measurement process may need much more development.

Resources for Active Learning

General Activities:
Balance and weight activities:

ACTIVITIES IN GEOMETRY FOR PRIMARY PUPILS, p. 60, Addison-Wesley
The BALANCE BOOK, Webster, McGraw-Hill
A CLOUDBURST, Vol. 1, Nos. 5421, 5441, Midwest Publications
DEVELOPMENTAL MATH CARDS, $A^{2}6$, $B^{2}2$, Addison-Wesley
EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, pp. 82–87, Herder and Herder
MATHEX: Measurement and Estimation No. 5, pp. 37–38, Encyclopaedia Britannica Publications Ltd.
Nuffield Project: BEGINNINGS ∇, pp. 58–64; COMPUTATION AND STRUCTURE ②, pp. 23–33, Wiley
WORKJOBS, "Weight Boxes," pp. 88–89, Addison-Wesley
MATHEX: Measurement and Estimation No. 5, "Developing Measurement Concepts," pp. 1–6, Encyclopaedia Britannica Publications Ltd.

Manipulative Devices:

Discovery Blocks (Educational Teaching Aids)
Equal pan balance/balance scale (Creative Publications; Edmund Scientific; Selective Educational Equipment)
Geo Blocks (Selective Educational Equipment; Webster, McGraw-Hill)
Geo-boards (Addison-Wesley)
Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)
Poleidoblocs Set G (Responsive Environments Corp.; Selective Educational Equipment)
Trundle wheels (Educational Teaching Aids; Math Media)

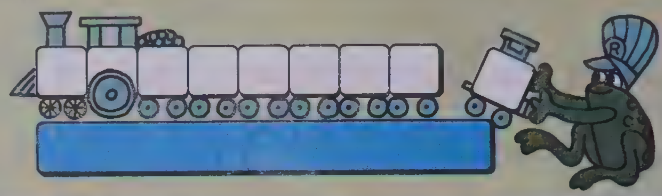
Investigation
Page b-53

Ask the children to look at the picture at the top of the page. Have a volunteer describe what he thinks is happening. Suggest that the children line up their units on the illustration to see if they can make a train as long as the one in the book. Check that the children do not overlap unit pieces. Ask them to count the units to find out how many are needed to match the blue strip. Then read the question at the top of page b-53. Explain that the dotted outlines on the top of each pictured strip are included as guidelines for their trains. Children should try to fit the units inside these dashed lines, but if those who are less co-ordinated have difficulty, do not overemphasize this point. After children have figured out how many units match a given strip, they should write that number in the space provided. As the children work, you might occasionally refer to the units as *centimetre units*.

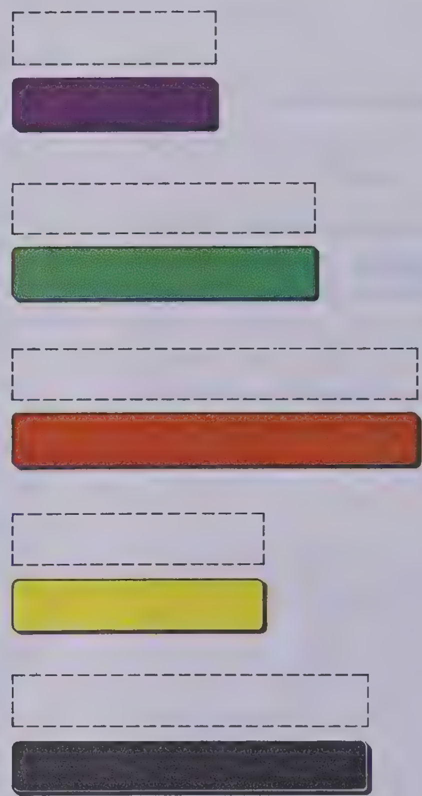
You might want some children to try to make trains as long as the other strips that are not shown on this page. However, if some children have difficulty doing this without the dotted lines, postpone this suggestion until later.

Throughout the investigation, continue to stress the correct use of the phrase "as long as." For example, "the purple strip is as long as a train of four units."

Let's do



Can you make a "unit train" as long as each strip?



4 units

6 units

8 units

5 units

7 units

Purposes

To introduce the concept of a unit for finding length

To help the child develop correct use of the phrases "is shorter than," "is longer than," "is the longest," "is the same as"

To help the children develop the skill to use the phrase "as long as" and describe a given object in terms of a number of the unit objects

Preparation

Materials

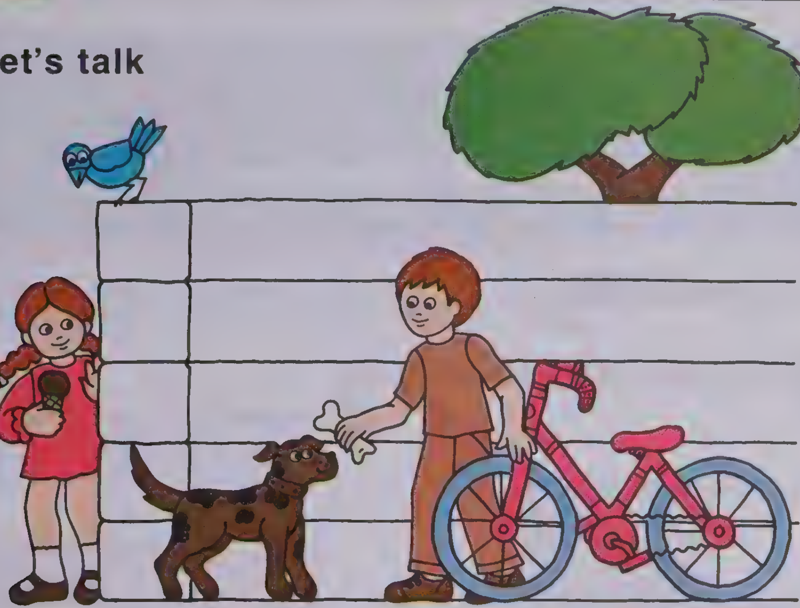
centimetre units, about 15 for each child
set of strips for each child (optional)

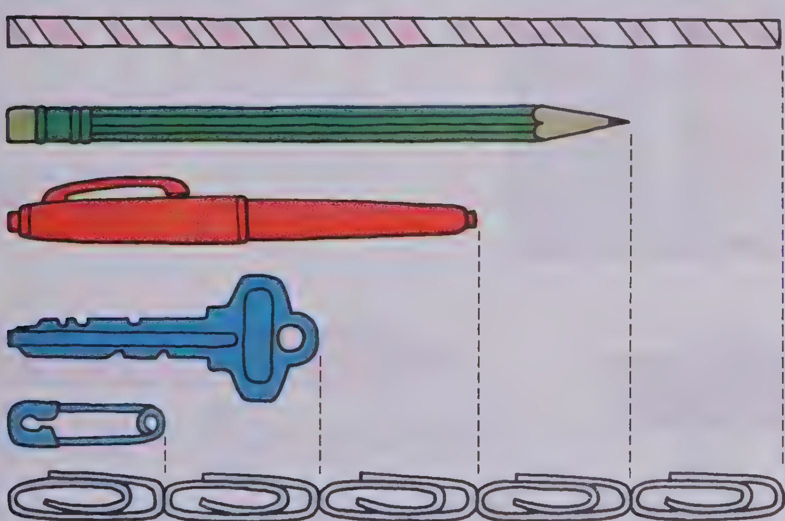
Distribute the centimetre units so that each child has about 15. Note their similarity with the white or *one* strip. Explain to the children that since they are going to use these strips in a new way you will be talking about them as "unit" strips, although this means the same as calling them *one* strips. Depending on the reading ability of the children, you might want to teach the word *unit* as a sight word.

Follow-up

An activity in which children compare their heights may be recorded on a long chart placed along a wall. For example, have the children decide who is the shortest in the class. This child should then stand beside the chart and place his name to mark his height on the chart. Then

Let's talk





Readiness for measurement concepts

Discussion
Page b-54

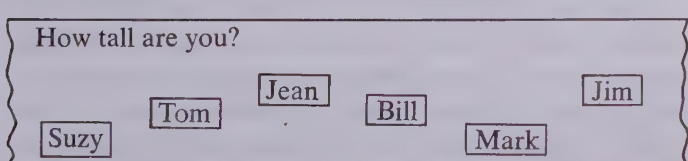
The key notion of this lesson is the concept of a unit. An on-going objective of this module is that children will understand that various choices for a unit are possible. However, some children at first grade age will lack the maturity necessary to understand this.

Ask children to look at the pictures at the top of page b-54. Specific questions that might help the children are: "Which object at the top is the tallest?" "Which object is the shortest?" "Name something that is taller than the girl?" "Name an object that is shorter than the girl." "Name something that is higher than the girl but not taller." "The bicycle is as tall as how many blocks in the fence."

Use similar questions to help the children compare the other objects on the page. Point out the chain of paper clips at the bottom. Ask: "How many paperclips does it take to make a train as long as the pencil?" "The straw is as long as how many paperclips?" Finally, relate the discussion to the investigation by asking children to try to make a train of units about as long as the safety pin. Point out that we can describe the safety-pin as being one paperclip long or as being about three strip units long.

Demonstrations with other units, such as a red strip of paper 5 cm long, may be used to measure objects on the flannel-board.

have others compare themselves to the mark. You can give them strips of paper about 3 cm wide and 5 cm long to use to mark their height. If done carefully, the chart can show a gradual slope from shortest to tallest, but this is not necessary. (The same type of activity may be "charted" on a life-size model of the tallest child or in some other manner invented by the children.)



Use such a chart for comparisons. For example, ask, "Name someone taller than Bill." "Name someone shorter than Jean." Use sentences such as: "Tom is taller than Suzy, but Jim is taller than Tom."

Resources for Active Learning

Experiences in linear and area measurement:

A CLOUDBURST, Vol. 1, No. 5111, Midwest Publications

DEVELOPMENTAL MATH CARDS, $A^2$4, 16, $A^3$19; $B^2$13, 15, 17, Addison-Wesley

ELEMENTARY SCHOOL SCIENCE Primer (TE), "Short-long; Short-tall," T25–28; Book 1 (TE), "Measuring Distance, Units of Measurement, and Standard Measures," pp. 21–39, Addison-Wesley

EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, pp. 59–69, 87–91, Herder and Herder

MATCH AND MEASURE, pp. 9–38, Webster, McGraw-Hill

Nuffield Project: BEGINNINGS ▽1, pp. 68–81, Wiley

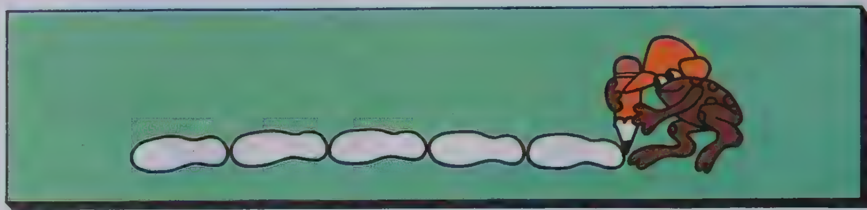
Discussion
Page b-55

For this lesson arrange the class into four groups. Every child should measure each of the four objects pictured on this page, using a cutout of his shoe as the unit of measure. You may want to call this unit the "shoe" or "shoe unit." Each child needs nine cutouts of his shoe. Space should be made available to lay the baseball bat and the metre piece of tape on the floor so the children can make "shoe" trains to match them.

For the other two measurements the child is at the door of the classroom or at his desk.

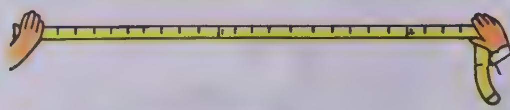
Have the children record the measurements they get. A person with a long shoe should get different results than a person with a short shoe. Discuss the results with the class. You may even want a child with a long shoe and a child with a short shoe to measure the same object in front of the class.

Note: Since these measurements will not be exact, you will want to round off intuitively to the nearest shoe length.



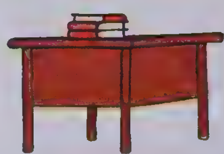
Answers will vary.

About ____ units long



About ____ units long

About ____ units tall



About ____ units long

Objective

Given an object, the child will be able to find its approximate length using arbitrary units of measure.

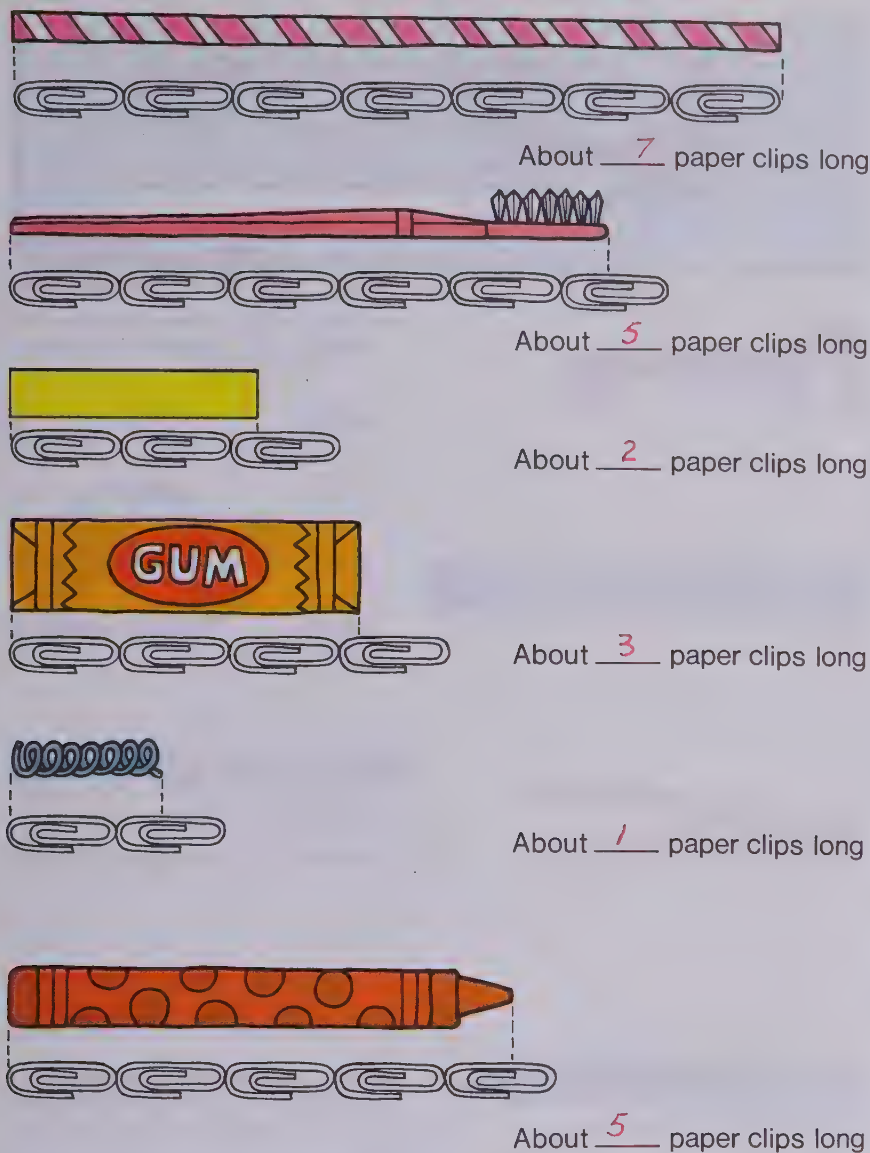
Pre-Book Activity

Materials

9 cutouts of each child's shoe
a baseball bat
a piece of tape about 1 metre long

Have each child construct 9 cutouts of his or her shoe. All the cutouts for one child should be about the same length. Then have the children compare the lengths of the different "shoes." Help the children decide whose "shoe" is shortest and whose is longest. You might also have the children prepare a "shoe" chart as they prepared a height chart on page b-54.

b 56 fifty-six



Readiness for measurement concepts

Discussion
Page b-56

The primary concern of this lesson is to give the children an opportunity to use arbitrary units in the measurement process.

Emphasize the steps in the measurement process. Question the children about what unit is being used (the paper clip). Discuss the steps in setting up the matching train: start even with the object; have the units touch but not overlap.

Be sure to question the children about picking the number of units that come closest to the length of the object.

Work with the children to find the measures of the first two objects. Then have the children complete the page. Remind them to record how long each pictured object is in the space provided.

Follow-up

Having ordered the cutouts by length take the longest and shortest and measure some objects. Have the children guess how many of each "shoe" will be needed in the respective matching trains. Measure objects such as the width of a window, the width of the teacher's desk, the height of some of the children, the length of some of the children's arms, etc.

Resources for Active Learning

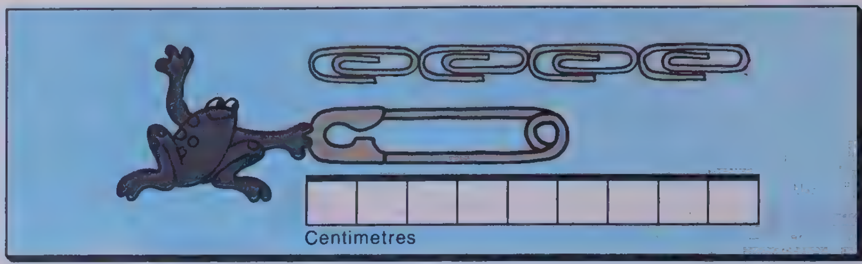
DEVELOPMENTAL MATH CARDS, A-4, A-8, A-16, Addison-Wesley

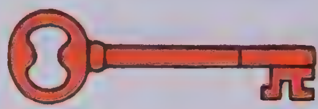
MATHEMATICS IN MODULES: M1 • Measurement • Length (A), pp. 4-5, Addison-Wesley

Teaching
Page b-57

Direct the children's attention to the illustration at the top of the page. Ask if anyone can tell you about how many paper clips long the safety pin is. Point out that we want to measure the pin in centimetre units as well. Ask if anyone can tell you about how many centimetres long the safety pin is.

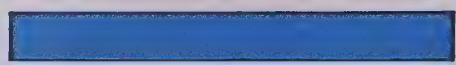
Explain to the children that they are to try to find out how long each pictured object is using *both* the paper clips and the centimetre units. Work with the children to measure the key. Have the children line their paper clips up beside the key. When they have recorded how many paper clips were needed to match the key, have them match centimetre units to the key and record this. Then have the children complete the page on their own.





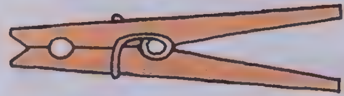
About 3 paper clips long

About 6 centimetres long



About 4 paper clips long

About 9 centimetres long



About 3 paper clips long

About 7 centimetres long

About 4 paper clips long

About 8 centimetres long

Objective

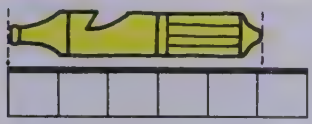
Given an object, the child can find its length using either an arbitrary unit or a centimetre unit.

Pre-Book Activity

Materials

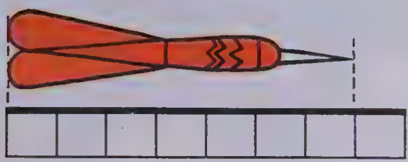
small paper clips (about 2 cm), 9 per child
centimetre units, 9 per child

Distribute paper clips and centimetre units, 9 of each, to each child. Have the children match paper clips to items they have around them: pencils, crayons, their thumbs or fingers, etc. Go over the measuring idea with them, stressing the fact that you match one *end* of the object you are measuring with the *end* of a paper clip.



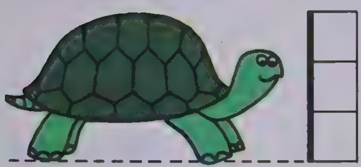
The whistle is about

5 centimetres long



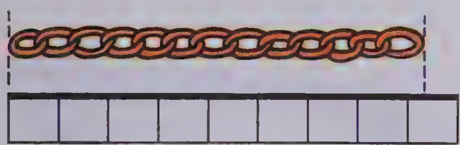
The dart is about

7 centimetres long



The turtle is about

3 centimetres high



The chain is about

8 centimetres long

The toothpaste is about

9 centimetres long

Teaching
Page b-58

Point out the centimetre units matched to the whistle at the top of the page. Review the measuring process with the children. Then have them work the page independently.

Be careful about the turtle. The children should know by now that the process is the same regardless of whether one is measuring length, width, or height.

Follow-up

If further work is desirable with some students, provide them with a page of objects to be measured and paper clips of 2 different sizes. Have them measure the same object with both sizes of paper clips.

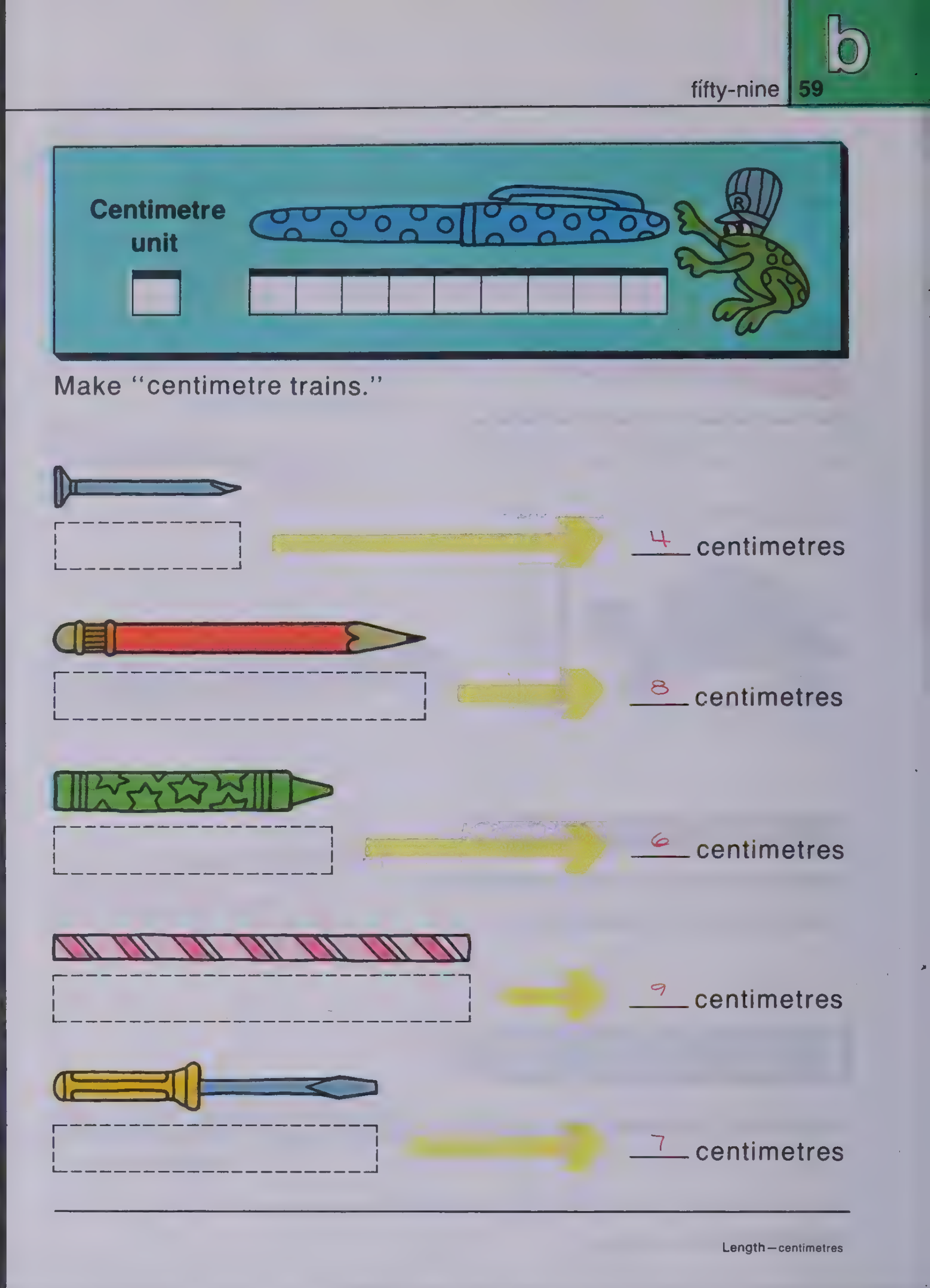
Resources for Active Learning

MATHEMATICS IN MODULES: M1 • Measurement • Length (A), pp. 6-7, Addison-Wesley

Teaching
Page b-59

Direct the children's attention to the illustration of the pen at the top of the page. Point out that this is a picture of one way that someone might measure a pen. Ask if anyone can tell you how long the pen is. Suggest that they line up their centimetre units along those pictured. Help the children realize why we say "The pen is nine centimetres long." (We are using centimetre units and the pen matches nine of them.)

Explain that they are to try to find out how long each pictured object is by measuring it with the centimetre units. If the children are ready for it, the centimetre ruler could be used here instead of the centimetre units. If necessary, work through some of these exercises with the children, but if possible have the children do them on their own. Remind them to record how long each pictured object is in the space provided.


fifty-nine 59 b

Centimetre unit

Make "centimetre trains."

4 centimetres

8 centimetres

6 centimetres

9 centimetres

7 centimetres

Length—centimetres

Objective

Given an object whose length is a whole number of centimetres, the child will be able to use the centimetre units to measure and report the length of the object.

Although the centimetre unit is a standard unit of measure with which the child should eventually become familiar, one of the main purposes of this lesson is to help the child understand that to measure we count units.

Pre-Book Activity

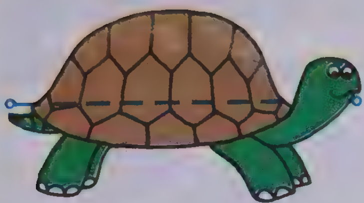
Materials

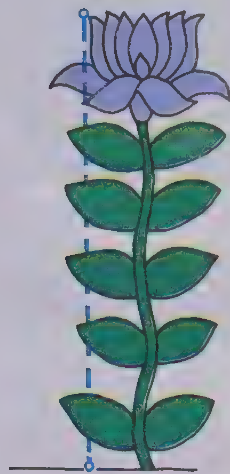
centimetre rulers
centimetre units
set of colored strips

Distribute a set of strips and the centimetre units to each child. Ask the children to put a yellow strip on their desk or table. Then ask them to use their centimetre units to make a train as long as the yellow strip. When children decide that the yellow strip matches a train of five of the units, explain that since these are centimetre units, we say that the yellow strip is five centimetres long. Ask them to make a train to match the blue strip and tell how many centimetres long it is. Continue with other strips. Help the children realize that we say they are *measuring* these strips with centimetre units. Children might record some measurements on the chalkboard.

Use your rulers.

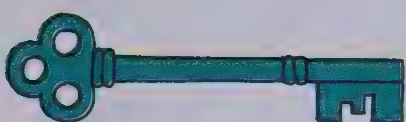
How **long**? How **high**? How **tall**?





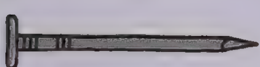
7 centimetres 7 centimetres 9 centimetres

Use a ruler to measure each object.



8 centimetres

9 centimetres



5 centimetres

Centimetres

Teaching
Page b-60

Call the children's attention to the turtle. Ask them to measure the length of the turtle by placing their centimetre ruler along the blue line. Point out that every blue dashed line should be measured with the centimetre ruler. When children agree on the length of the turtle, have them record the number of centimetres on the blue line. Encourage them to continue the page on their own. Again move around the room to be particularly sure that the children place the edge of the ruler exactly at one end of the line they are measuring.

Distribute a centimetre ruler to each child. Have the children compare the centimetre units with the ruler as they have been comparing the strips. Those children who feel comfortable with the rulers may use them for measuring on these pages.

Follow-up

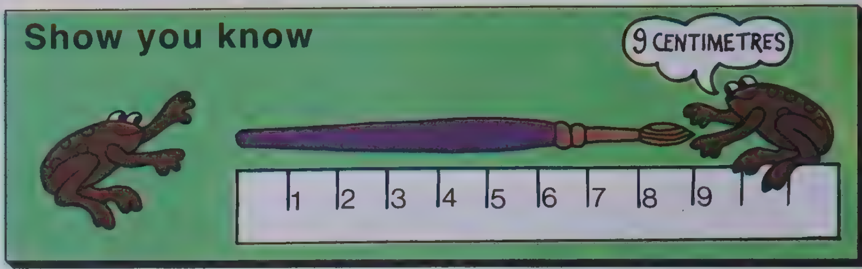
Distribute to each child (or pair of children, if you want them to work in groups) about 12 or 15 pipe cleaners. Suggest that they make stick figures or pipe cleaner dolls of varying heights. They can then use these to make comparisons. For example, they can name and label the dolls and then make a chart naming each doll in order from shortest to tallest. Or, they may simply draw one straight line on the bottom of a large sheet of paper and mark the heights in order or glue the dolls in place to show the order of the heights. You might challenge more capable children with the problem of measuring these heights with centimetres. (Some may try to make a train from the bottom line to the height mark; others may want to try to use a ruler; a piece of string may give an idea to another child.)

As children work on such an activity, you might move around the classroom and try to evaluate children's ability to conserve length, that is to understand that an object keeps its same length even if its position is changed. You might do this by asking them to find two pipe cleaners (or crayons, pencils, etc.) that are the same length. Then move the position of one and ask "Which one is longer?" The child who can conserve length will realize that one is still as long as the other. (See *How Children Learn Mathematics*, Copeland, pages 195–198, for reference.)

Teaching
Page b-61

Direct the attention of the children to the paint brush in the demonstration art at the top of page b-61. Point out the ruler beneath the brush and discuss how the frogs are measuring it. Instruct the children to take their own centimetre ruler and place it below the bolt. Ask them to measure the bolt and write their answer on the space provided. Ask them to measure the crayon similarly, again using their centimetre ruler. Then they should measure the chain, the comb, and the brown strip and write how many centimetres long each is in the space provided. Give as much guidance as necessary with the directions for this page, but encourage children to do the measuring independently.

Show you know

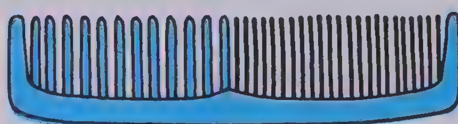


Use a ruler to measure each object.

7 centimetres

8 centimetres

5 centimetres



9 centimetres

8 centimetres

Objective

The child will demonstrate his ability to work with the concepts presented in this module by finding the length of pictured objects in centimetres.

As usual this module contains a concluding lesson which consists of a review or evaluation page and a change of pace page. However, since the major intent of this module is to serve as an *introduction* to measurement, you should not place too much emphasis on evaluating children's skill in finding length.

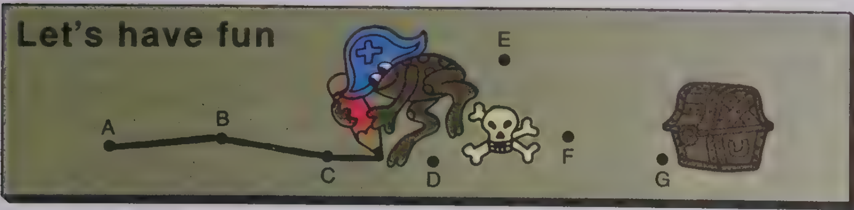
Pre-Book Activity

Materials

centimetre rulers

To review the concept of linear measure, introduce the children to units such as a cubit, a span, and a hand. In the centre of the room, place an object appropriate for linear measure, such as a strip of cardboard about 10 or 12 centimetres wide and about 1 metre long. Hold this up and ask the children to suggest how they could tell someone how long it is. After they have suggested the use of the centimetre ruler, pose the problem of measuring this strip without the use of any instrument. That is, could they use parts of their body to describe the length of the strip. After children have shared their ideas, describe the units pictured in the next column and write the words on the chalkboard. Then have children measure the strip with these units. Be sure they understand why everyone's measurement of the strip will not be the same.

Let's have fun



Draw the paths.

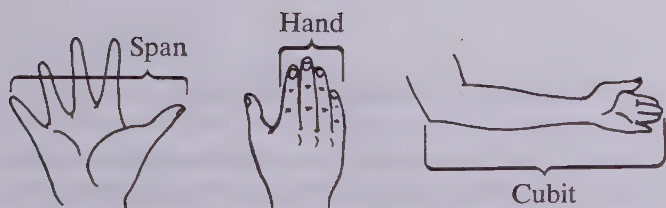
How long is each path? 9 centimetres ten centimetres

Who gets the treasure? turtle

Length of paths

Teaching
Page b-62

Page b-62 should be treated with a light touch. Encourage the children to describe the picture and make up stories about it. Help them identify the pirate turtle and pirate frog. Elicit from them that both the turtle and the frog want to reach the treasure. Explain that each path has been marked in centimetres and that by using their centimetre ruler they should be able to find out which path is longer. Then encourage them to try to discover who gets the treasure, (assuming that both travel at the same speed).



Resources for Active Learning

MATHEMATICS IN MODULES: M1 • MEASUREMENT • Length (A), p. 8, Addison-Wesley

Teaching
Page b-63

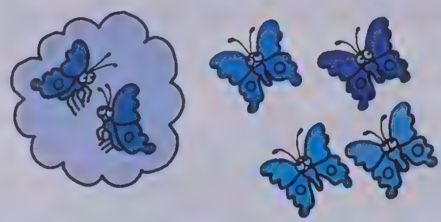
Use the number line at the top of the page to review addition. Help the children relate the equation to the illustrated sets. Point out how this addition problem may be written in either of the two given forms, but for either form they think: "two plus four is . . ." Also be sure children understand that the number line is provided for their use. Any combinations they are not sure of may be figured out on the number line.

Finally, direct the children to find the sums for both the middle and bottom sections.

Looking back

Find the sums.

0 1 2 3 4 5 6 7 8 9



$2 + 4 = \boxed{6}$

$$\begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array}$$

$3 + 2 = \boxed{5}$	$6 + 2 = \boxed{8}$
$6 + 1 = \boxed{7}$	$7 + 2 = \boxed{9}$
$4 + 4 = \boxed{8}$	$3 + 4 = \boxed{7}$
$2 + 5 = \boxed{7}$	$5 + 4 = \boxed{9}$

$$\begin{array}{r} 3 \\ +6 \\ \hline 9 \end{array} \quad \begin{array}{r} 5 \\ +0 \\ \hline 5 \end{array} \quad \begin{array}{r} 1 \\ +4 \\ \hline 5 \end{array} \quad \begin{array}{r} 4 \\ +3 \\ \hline 7 \end{array} \quad \begin{array}{r} 2 \\ +5 \\ \hline 7 \end{array} \quad \begin{array}{r} 7 \\ +2 \\ \hline 9 \end{array}$$

$$\begin{array}{r} 4 \\ +2 \\ \hline 6 \end{array} \quad \begin{array}{r} 8 \\ +1 \\ \hline 9 \end{array} \quad \begin{array}{r} 5 \\ +3 \\ \hline 8 \end{array} \quad \begin{array}{r} 0 \\ +6 \\ \hline 6 \end{array} \quad \begin{array}{r} 6 \\ +2 \\ \hline 8 \end{array} \quad \begin{array}{r} 3 \\ +3 \\ \hline 6 \end{array}$$

Cumulative review

Objective

The child will demonstrate his ability to work with the concepts developed throughout Unit B.

Pages b-63 and b-64 present a cumulative review of the main topics treated in Unit B.

Pre-Book Activity

A variety of activities are possible for use to review the sums and differences of 9 or less. If the children have been using their flash cards and they are handy, you might declare this "Flash-Card Day." Assign partners and give the children enough time to go through their flash cards with their partners quickly. They might put all of the combinations they find troublesome in a pile separate from those they have mastered. Set aside this pile for special study.

It would also be helpful to review combinations on a number line. Draw a number line on the chalkboard or overhead projector and label it from 0 through 9. Remind the children of the number-line frog. After a few stories about the frog and his insect friends, try to get the children to make up their own stories and encourage the other children to act them out on the floor number line.

Use both addition and subtraction equations. If children have trouble using specific numbers, you might suggest numbers by saying something like, "Can anyone make up a story about the frog using the numbers 8 and 2 and 6?"

Find the differences.

0 1 2 3 4 5 6 7 8 9

$6 - 2 = \boxed{4}$

$$\begin{array}{r} 6 \\ -2 \\ \hline 4 \end{array}$$

Solve the equations.

$5 - 2 = \boxed{3}$

$7 - 3 = \boxed{4}$

$4 - 3 = \boxed{1}$

$8 - 4 = \boxed{4}$

$9 - 5 = \boxed{4}$

$6 - 4 = \boxed{2}$

$9 - 6 = \boxed{3}$

$6 - 3 = \boxed{3}$

Find the differences.

$$\begin{array}{r} 8 \\ -3 \\ \hline 5 \end{array} \quad \begin{array}{r} 9 \\ -2 \\ \hline 7 \end{array} \quad \begin{array}{r} 5 \\ -4 \\ \hline 1 \end{array} \quad \begin{array}{r} 6 \\ -1 \\ \hline 5 \end{array} \quad \begin{array}{r} 7 \\ -4 \\ \hline 3 \end{array} \quad \begin{array}{r} 9 \\ -4 \\ \hline 5 \end{array}$$

$$\begin{array}{r} 3 \\ -0 \\ \hline 3 \end{array} \quad \begin{array}{r} 8 \\ -6 \\ \hline 2 \end{array} \quad \begin{array}{r} 9 \\ -8 \\ \hline 1 \end{array} \quad \begin{array}{r} 7 \\ -5 \\ \hline 2 \end{array} \quad \begin{array}{r} 9 \\ -7 \\ \hline 2 \end{array} \quad \begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array}$$

Cumulative review

Teaching
Page b-64

Treat this page as you did page b-63. Help children interpret how the butterflies flying away relate to the subtraction equation $6 - 2 = \square$. Discuss the answer to this sample problem and have children write the correct response, 4, in the two appropriate places. Again be sure children realize that the number line is provided for their convenience. If any children want to work with sets or strips to find the differences, allow them to do so. In fact, if a child seems to be having difficulty with the page, supply him with counters for finding differences. Most children, however, should be encouraged to use the number line.

Follow-up

If some children need further work with basic combinations, provide them with practice or opportunities to work with counters or the strips. Strip activities for combinations such as those suggested in the introduction to the light green module would be helpful. Practice wheels such as those suggested in the Follow-Up section on page b-47 might again be useful.

YELLOW MODULE, UNIT C

Two-digit Place Value

Pages c-1 to c-14

General Objectives

To introduce the concept of place value for two-digit numerals

To develop the understanding that the symbol 10 not only represents the number ten but means one ten and zero.

A clear understanding of place value ranks as one of the most important objectives to be accomplished in the teaching of first-grade mathematics. Therefore, the *Investigating School Mathematics* series treats place value thoroughly and continues development of the concept throughout the program.

Of all number symbols, the symbol 10 for the number ten is perhaps the most difficult to understand. For this reason we delay the introduction of this symbol until we have explored the over-all concept of two-digit place value.

Place value is introduced by using the word *ten* (rather than 10). After the child has learned the word *ten* for the number ten, the idea of several sets of ten is introduced. That is, we ask the children to determine how many sets of ten there are in a given set collection. Following this, we give them sets of ten objects accompanied by nine or fewer objects and ask: "How many sets of ten and how many more?" This leads to the general concept of "so many tens and so many ones."

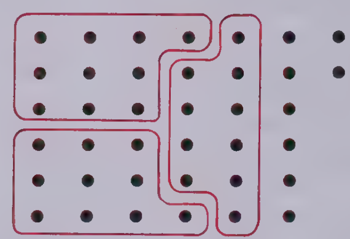
The meaning of the symbol 10 is introduced after the concept of so many tens and so many ones has been developed. From this concept the children are led to the idea of so many tens and zero ones, and then to the idea of one ten and zero ones. In this way, the children are led to understand why the symbol 10 is written for the number ten; they are led to see that the symbol 10 is merely an instance of the two-digit place-value scheme.

Mathematics

When we write the symbol 4357, the numerals 4, 3, 5, and 7 stand for 4000, 300, 50, and 7 respectively. This illustrates an important property of our numeration system; the use of place value. Place value simply means that the number a digit represents depends upon where it occurs in the symbol.

Another important fact, which simplifies our calculations and makes it easy to learn arithmetic, is that by using only ten symbols, 0 through 9, we can represent any whole number. Each of these digits used by itself represents a single number; it is only when we write a number greater than nine that a given digit may name different numbers according to its position in the numeral. Thus, in 636, one 6 represents six hundred and the other represents six.

The place-value system that we use is based on ten, which means that we group by tens. That is, given a collection of objects, we might ask how many disjoint sets of ten can be formed. For example, consider the set of dots shown in the illustration; we see that there are three sets of ten and eight left over.



The importance of place value is evident when we attempt to write the numeral for this number of dots. Instead of writing "three sets of ten and eight more," we simply write 38 and agree that the numeral in the "second place" (3 in this case) represents sets of ten. Of course, when working with larger numbers, we must group the sets of ten. Following the same pattern of grouping by tens, we have sets of 10 tens, each of which we call one hundred. For example, we might have a set of objects grouped as follows: 5 sets of *one hundred,* 3 sets of ten, and 7 more; we write 537.

Teaching Yellow Module, Unit C

Approximate Time: 7 to 10 days

Materials

brass paper fasteners
counters, about 30 per child
flash cards in multiples of ten
individual numeral cards (or plastic numerals)
objects that can readily be grouped by tens (pencils, straws, sticks, pipe cleaners, checkers, etc.), ideally about 50 objects per child
rubber bands, string, or pipe cleaners for bundling strips, 1 set per child

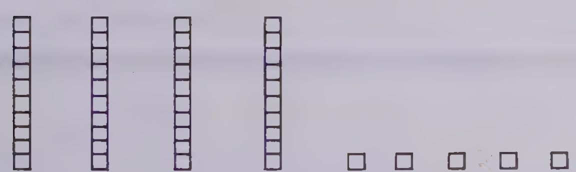
Vocabulary

digit	forty	place value	ten
eighty	multiple	seventy	thirty
fifty	ninety	sixty	twenty

We introduce the word *digit* in order to distinguish between the numerals for numbers less than ten and those for ten or greater. *Digit* refers to the numerals zero through nine, whereas the *numeral* refers to any of the symbols for the entire set of whole numbers. For example, in the numeral 23, we may refer to the digit 2 in the tens' place or to the digit 3 in the ones' place.

You will notice that we abuse mathematical language somewhat when we refer to two-digit numbers. However, the correct phrase "the number represented by a two-digit numeral" is awkward and excessive; the children will understand that this is what we mean when we say "two-digit" number.

You will notice that many pre-book activities suggest that children group sets of objects into tens. It is most important that children be given the opportunity to actually work with concrete objects, to make bundles of ten, and to sort sets of objects into tens with some left over. If it is not possible for each child to have a set of 30 or 40 objects, have children work in groups of two or three. Some follow-up activities suggest use of the strips to show place value: These activities can be extended as you see fit. Since the uncolored side of each strip is marked, you might want children to focus attention on the ten spaces on the back of the orange strip. Other 10-strips may easily be made by gluing centimetre graph paper onto white cardboard and cutting centimetre strips with a paper cutter so that each strip is 1 centimetre wide and 10 centimetres long.



45

Evaluation of Progress

When evaluating achievement in this unit, you should consider the following two points carefully. First, the children should be able to respond correctly to questions on the meaning of tens and ones in all two-digit numerals. Page c-13 can be used to measure this objective.

Second, the children's insight into the general idea of grouping by tens must be evaluated. This point may seem to be the same as the first; however, some children will be able to respond readily concerning the meaning of a two-digit numeral and yet not clearly understand the idea of grouping or the quantitative aspect involved in their response. This kind of understanding must be appraised in day-by-day oral and written evaluations.

Resources for Active Learning

General Activities:

Multi-base arithmetic blocks:

THE DIENES M.A.B. Tasks and Manual, Herder and Herder

NUMBER-BLOX, Creative Publications

Place value:

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Place Value," No. 65, Responsive Environments Corp.

MATHEX: Operations No. 3, pp. 9–16, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: COMPUTATION AND STRUCTURE 2, pp. 70–74, Wiley

Manipulative Devices:

Cogno-Board (Teaching Resources)

Colored beads and pattern cards (Teaching Resources; Lakeshore)

Cuisenaire Rods (Cuisenaire Co.)

Multi-base arithmetic blocks (Educational Teaching Aids; Herder and Herder)

Pattern Blocks (Selective Educational Equipment; Webster, McGraw-Hill)

Pegboards (school supplier)

Commercial Games:

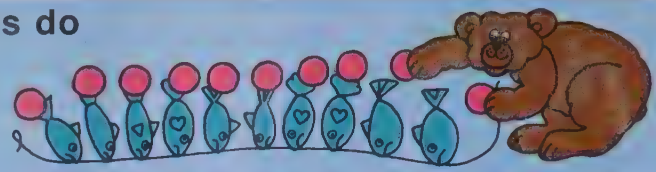
Configurations (Hammett; Holt, Rinehart and Winston; Wff 'N Proof)

INVESTIGATION
Page c-1

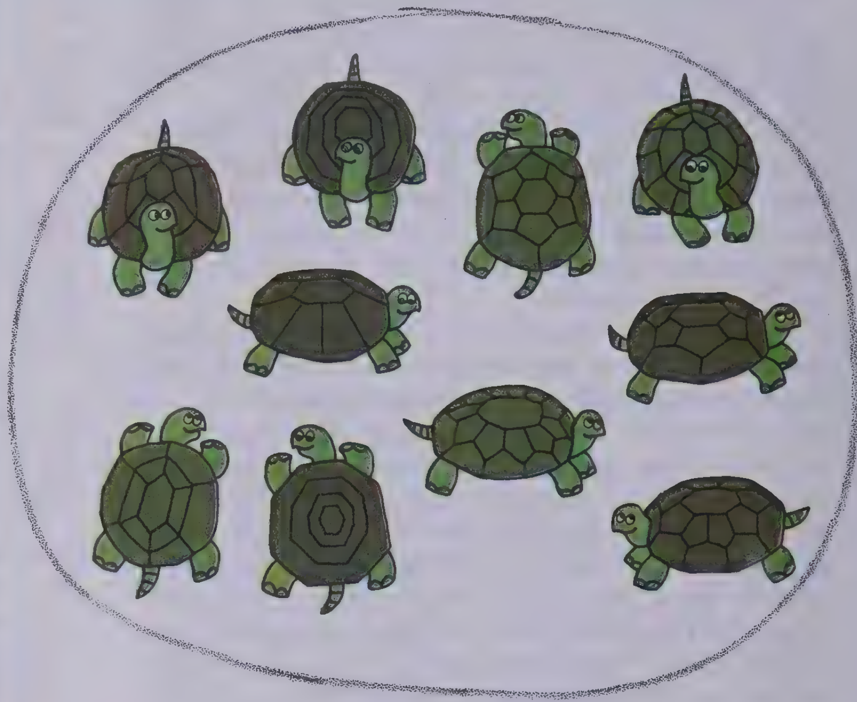
Distribute to the children various size sets ranging from 15 to 45 counters each. Explain that it is not necessary for them to know how many counters they have. Then direct their attention to the investigation page. Read the directions to them explaining that they should match a counter from their set with each turtle in the pictured set and then put this group of counters in a pile. Be sure they understand that they are to find out how many times they can match the set of turtles before they have no more counters. You might suggest that they think of their counters as rocks and figure out how many each turtle could hide under. Although the children are actually grouping by ten, it is not intended that you teach the number ten. If the word comes up, use it, but do not stress it. Also, encourage children to share their results with others since they are using sets of various sizes.

When the children have built their sets, call attention to the questions. Since children worked with different sized sets, their answers to the questions at the bottom of the page will differ. Ask a few volunteers to share their answers and to explain how they built their sets. Stress both the use of one-to-one matching with the given set to build each set, and the concept of grouping their counters into sets of a certain size with some left over.

Let's do



How many times can you match this set.?



Answers will vary.

How many sets did you make?

How many were left over?

Readiness for grouping by 10 and place value

PURPOSES

To introduce the concept of grouping objects by tens
To introduce counting to ten

PREPARATION

Materials

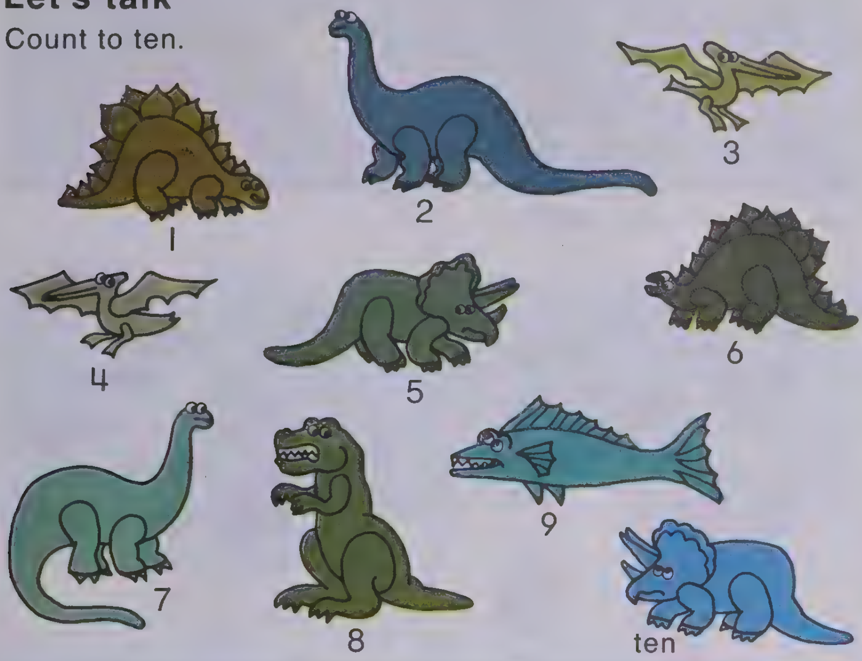
sets of counters, 15 to 45 for each child

No specific preparation is essential for this investigation. Therefore, unless you wish to review sums and differences of 9 or less, begin immediately with the investigation.

C 2 two

Let's talk

Count to ten.



How many animals? Write the word.

ten ten ten

Can you make some ten patterns?

Sets of 10

Discussion
Page c-2

Before continuing with page c-2, it would be helpful to have the children count several sets containing nine or fewer objects. Then display a set of nine objects and have the children count together as you point to each object in turn. Put one more object with the set and count together again. When you reach ten, have the children observe that there are ten objects in this set. Then have them count the set of turtles on page c-1. Write the word *ten* on the board and tell the children that this is the word for the number ten. Finally, call attention to the pictured animals on page c-2. Count this set together with the children and teach them to print the word *ten*.

The bottom section of the page might best be handled as a group activity. Three or four children might work together using counters to build patterns of ten. Distribute newsprint or other paper and suggest that they show with a picture the patterns they and others in their group make.

Follow-up

After children have worked with the counters to make patterns of ten, suggest that they draw objects in patterns of ten.

Give the children large sheets of newsprint and crayons. Tell them to use their favorite colors and to make as many different arrangements of ten objects as they can. Suggest that they use balloons or blocks or anything easy to draw, so that the mechanical phase of the project does not detract from the recognition of patterns.

Mathematics

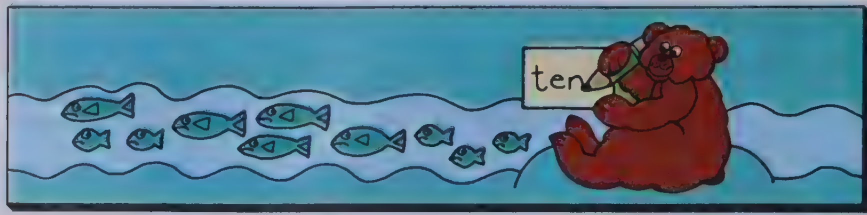
Although most children can and have counted to ten, and have seen and perhaps even written the symbol "10," it is recommended that 10 be avoided until the concept of two-digit numerals is developed. Initially, children should be encouraged to write the word *ten*. Simply tell the children that we want them to use the word ten for a while and that soon they will study numerals which use two of the symbols, 0, 1, 2, . . . 9 that they've already studied.

Resources for Active Learning

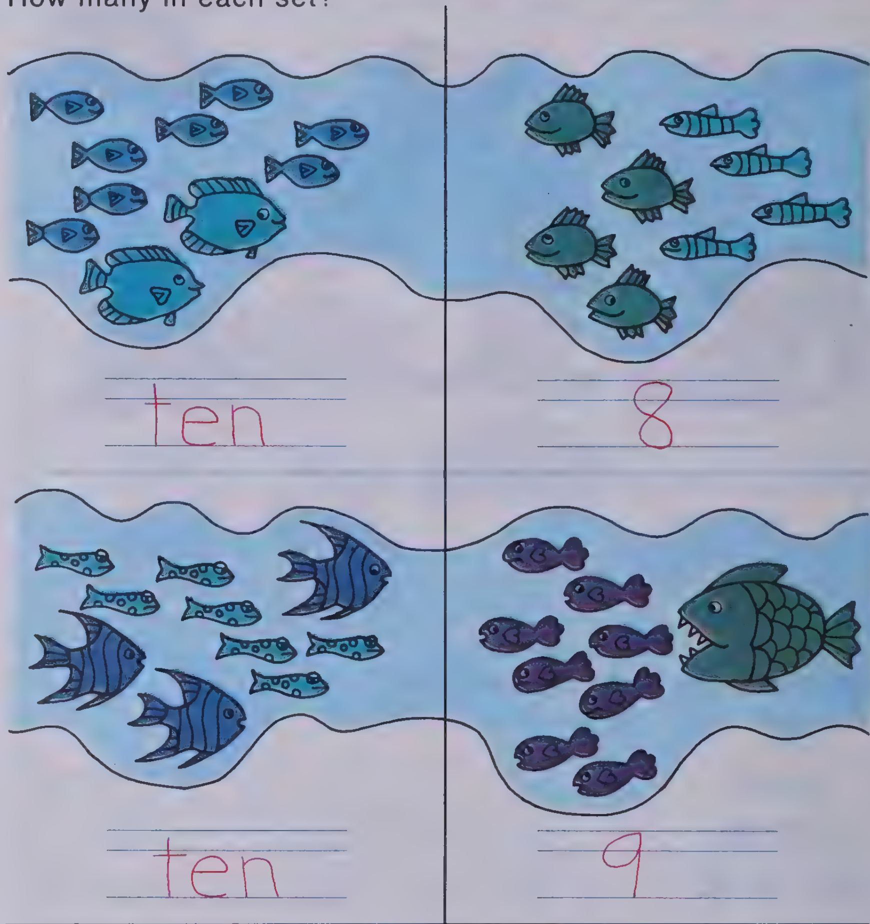
Place Value:
WORKJOBS, "Number Dots," pp. 146–147, Addison-Wesley

Teaching
Page c-3

Call attention to the ten fish at the top of the page and point out the model the bear is writing for the word ten. Then instruct the children to examine each set and very carefully count how many are in each set. Direct them to write the numerals for 8 and 9 but the word for ten. However, if a child *wants* to write "10" allow him to do so, but do not over-emphasize this with the other children. When the children have identified the number in each set, ask everyone to count the number of frames which show sets of ten. Be sure to stress that two of the pictures on this page show sets of ten.



How many in each set?



Objective

Given pictured sets of ten, the child will be able to identify the number in each set and the number of sets of ten.

Although many children will have developed an ability to interpret pictorial representation of sets, it is very important that they be given opportunity to work with objects they can group in tens.

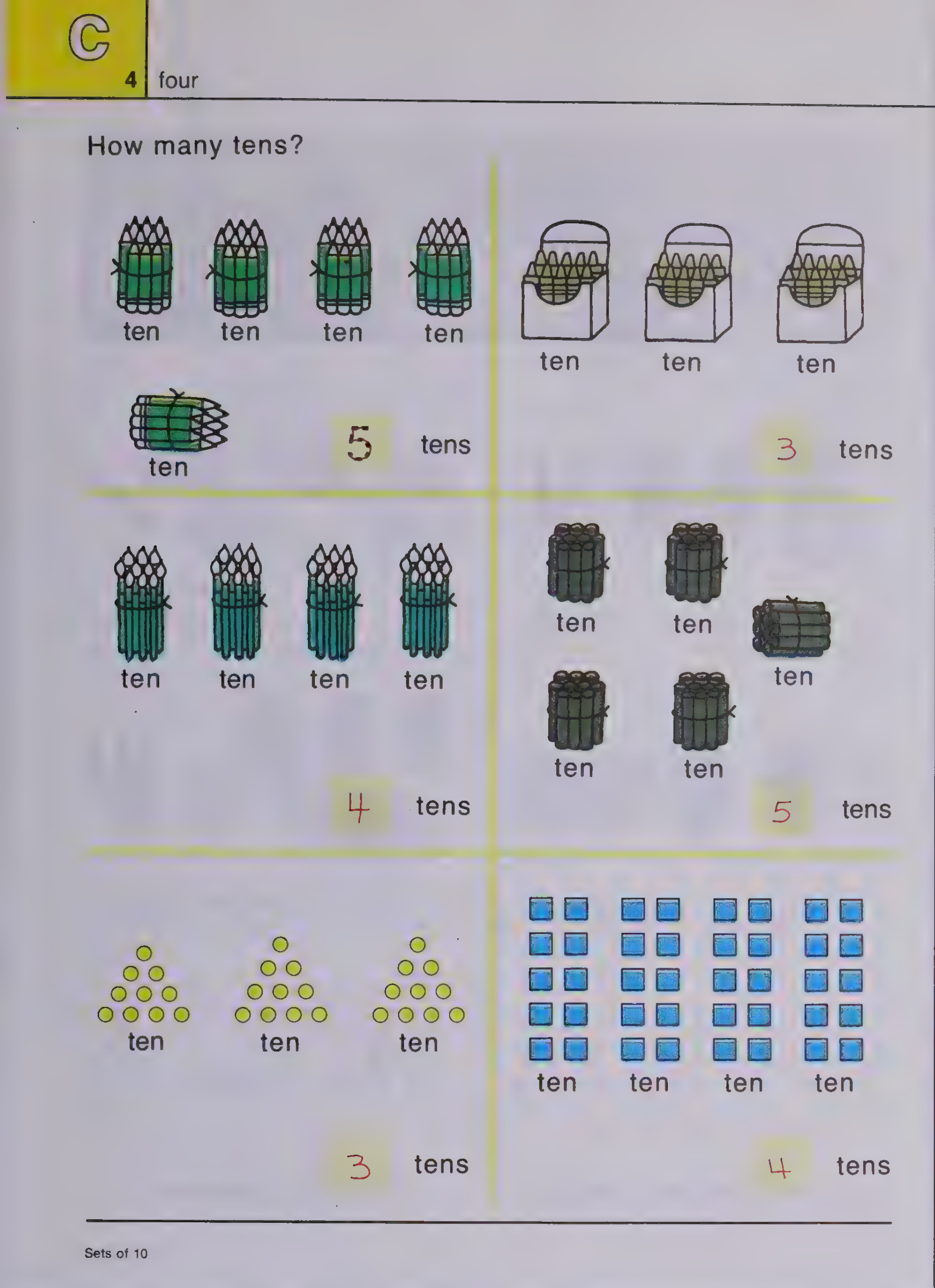
Pre-Book Activity

Materials

sets containing 30, 40, or 50 objects (pencils, sticks, checkers, coins, or some other objects)

Give the sets of objects to the children. Divide the class into groups of two or three, if you wish. Ask the children to count out sets of ten. When they have a set of ten, instruct them to put this group aside and continue counting until all the objects are put in groups of ten. If appropriate, show them how to put a rubber band or a string around each set of ten sticks or pencils. (Large pipe cleaners used as ties make bundling easier for children who are not co-ordinated well enough to use rubber bands successfully.) At this time, the children should be limited to grouping in multiples of ten with no objects left over.

You might also display sets of ten objects grouped in the familiar pattern of two rows of 5; and in the 4, 3, 2, 1 pyramid; so that the children will become accustomed to these two arrangements which will be used frequently. They should learn to recognize these patterns of ten so that they can concentrate on the number of sets of ten



Teaching
Page c-4

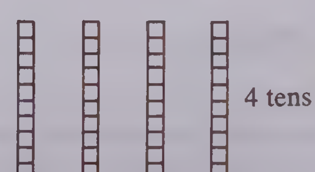
Call attention to the first frame on the page. Ask the children how many pencils they think are in each bundle. Note that they need not count the objects in each small set on their page. They should realize that these sets as pictured always contain ten. Relate the frame to the pre-book activity by displaying one of the sets of ten which the children counted. Just as they grouped their objects into sets of ten, so the illustrations show pictures of objects which have the label *ten* under each group. Explain that since there are 5 bundles of ten, we say 5 tens. Have children trace over the dashed numeral. Continue working through the next frame with the children. Then suggest that they complete the page on their own. Even though the emphasis on this page is more on how many sets of ten (rather than counting to ten), continue stressing and practicing counting to ten.

rather than count each object in a given set. However, the most important point in this pre-book activity is for children to actually experience the grouping of objects into sets of ten.

Follow-up

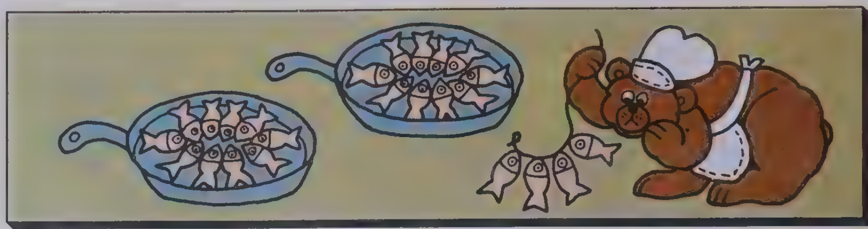
Give each child his set of strips. Ask him to build a staircase using one of each color of his strips (smallest on top). Begin with the top white strip and count down until you come to the orange strip. Ask the children what number they think will be assigned to the orange strip, if the white strip is the unit. Suggest that they check their guess by lining up as many white strips as they need to measure the orange strip. They might also turn their orange strips over and count the sections on the uncolored side to see that the orange strip matches ten units.

When all have verified for themselves that the orange strip should be assigned the number ten, begin exercises to strengthen their recognition of the strips by number names. For example, ask the children to show you the five-strip, or the seven-strip, or the ten-strip. Also include examples such as: "Can you show me 3 ten-strips?" "Can you make a train of 4 ten-strips?" Since such a train will be lengthy, show how they can place the strips parallel to one another to portray 4 tens.

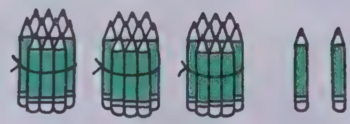


Teaching
Page c-5

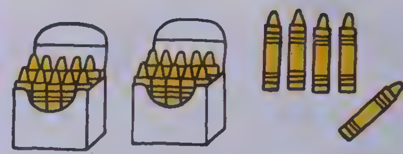
Use the illustration at the top of the page to summarize how a set can be grouped into tens and some left over. Then call attention to the first frame and ask the children how many bundles of ten there are and how many are left over. Ask the children if they can explain what the 3 and the 2 represent. Help them realize that the 3 tells how many tens and the 2 tells how many are "left over." Have them complete the dashed line numerals. Then work through the remaining frames together. Each time ask the children how many tens there are and how many left over, and wait until they write the correct numerals. Notice that in the bottom left frame, there are zero left over. An understanding of this frame and its correspondence to the phrase *3 tens and 0* will aid in the child's understanding of the symbol 10 for 1 ten and 0, which will soon be introduced.



How many?



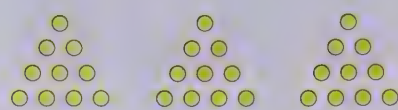
3 tens and 2



2 tens and 5

2 tens and 4

4 tens and 3



3 tens and 0

1 tens and 4

Objective

Given objects grouped in tens and some left over, the child will be able to express how many by giving the number of tens and the number of ones.

In the previous lesson children learned to identify the number of a collection as a certain number of tens. This lesson simply extends this understanding to include collections which, when separated into groups of ten, have some left over. Note also that actual writing of the two-digit numeral is not brought in at this time.

Pre-Book Activity

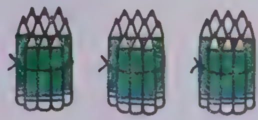
Materials

sets of pencils, sticks, checkers, and so on which cannot be grouped evenly into tens (32, 45, 27, 41, etc.)

Place the sets on the children's tables and ask them to group the sets by tens. When they have made as many tens as they can, have them observe how many sets of ten and how many single objects, or ones, they have. For example, suppose one of the sets has 34 objects. Ask a group of two or three children to count and group the objects by ten. (If the objects are pencils, put a rubber band or a pipe cleaner around them; if they are checkers or coins, stack them.) When the children have found all the sets of ten, observe with them that there are 3 sets of ten and 4 more. Have them repeat this activity until the children are quite familiar with the idea of so many tens and so many left over. Each time, of course, the number left over must be less than ten.

How many?

2 tens and 1



3 tens and 0

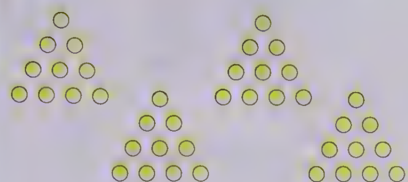
1 tens and 3

2 tens and 3

4 tens and 0

3 tens and 4

2 tens and 5



4 tens and 0

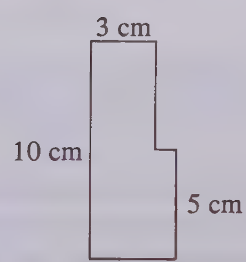
Teaching
Page c-6

Since this page is a continuation of the exercises on page c-5, you might want the children to complete it on their own. Call attention to the first frame. Ask the children how many tens there are and how many left over. Be sure they understand how they should write the numerals showing so many tens and so many left over. Notice that some frames have only groups of tens with zero left over. Observe carefully that the children write the number of tens and zero. This type of frame is preparation for understanding and writing numerals such as 30, 20, and finally 10.

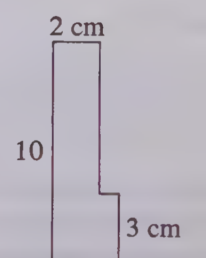
Follow-up

Ask the children to show with the strips 3 tens and 5, or 2 tens and 8, or 4 tens and 3. Even though they would be able to correctly show these amounts with the ten strips and one other strip, suggest that they use only orange strips and the white strips. Thus, to show 3 tens and 5, they should use 3 orange strips and 5 white strips.

You might provide a duplicating master on which children can fill in areas with their orange and white strips. Sample areas are shown in the next column.



3 tens and 5



2 tens and 3

(The dimensions in centimetres are provided.)

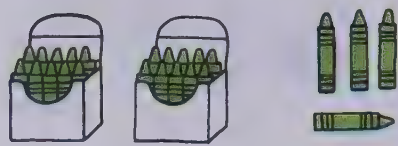
Resources for Active Learning

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Natural Numbers," No. 23, Responsive Environments Corp.

Teaching
Page c-7

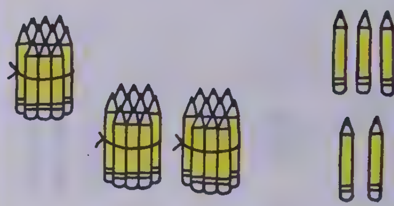
Use the illustration at the top to help the children relate what they did in the pre-book activity with the numeral cards to the writing of 32. Ask them to explain what the 3 and the 2 mean and that 32 means 3 tens and 2. Then call attention to the first frame and develop it similarly. The children should realize that there are 2 tens and 4. Instruct them to complete the dashed numerals. Then explain that 24 is the short way to write 2 tens and 4. Have them complete these dashed numerals. Continue to work through the three other frames. Each time, be sure children find how many tens and how many left over and record this in the "____ tens and ____" form. Finally, make sure they write the two-digit numeral.

How many?



2 tens and 4

We write 24.



3 tens and 5

We write 35.

4 tens and 5

We write 45.

5 tens and 4

We write 54.

Introduction to place-value notation

Objective

Given objects grouped in tens and some left over, the child will be able to write the base-ten numeral to express how many.

In this lesson, children actually write two-digit base-ten numerals. Notice however that examples with zero are not yet introduced. Zero and the symbol for ten, 10, will be treated in the next lesson.

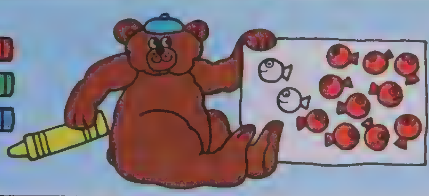
Pre-Book Activity

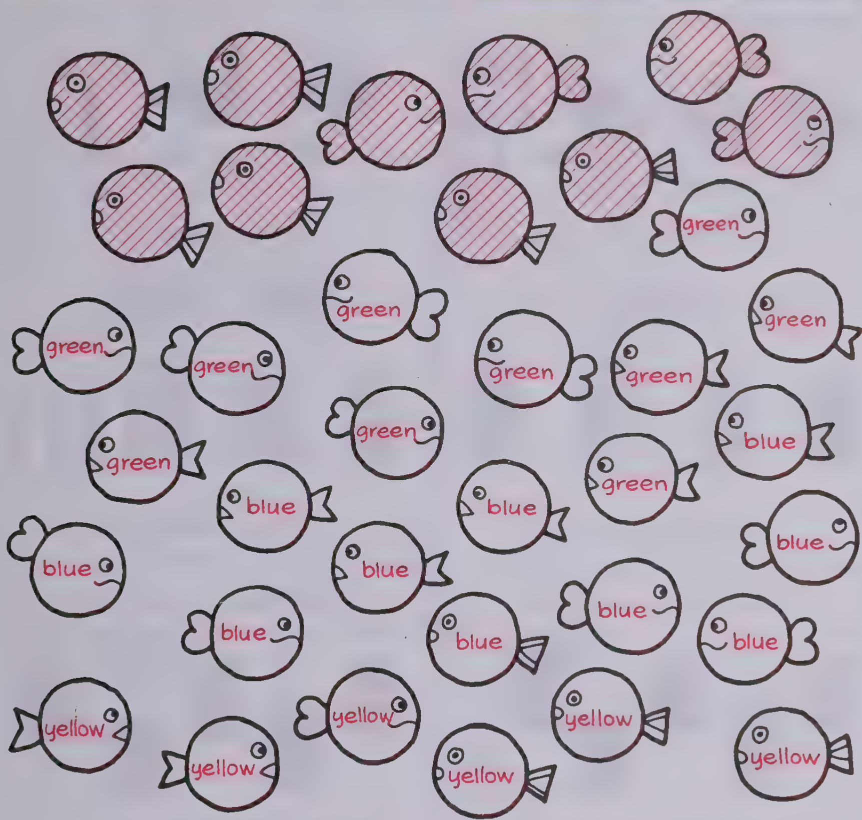
Materials

individual numeral cards for the digits 1 through 9
sets of pencils, sticks, checkers, and so on which cannot be grouped evenly into tens

Distribute the materials so that every 3 children have a set of objects and each child has a set of numeral cards. Ask the children to count and group their set into sets of ten and some left over. Then ask them if they can figure out some way of recording how many they have by using nothing other than their numeral cards. As they work, remind them that they have been using two numerals, or digits, in the phrases such as *3 tens and 4*. Now you want to see if they can figure out a way to use just the numerals. Explain that if they want to first record how many they have by using the phrase ____ tens and ____, it might be helpful. Encourage them to talk with each other about how they might use the numeral cards, and encourage creativity. For example, the tens' digit might be placed at the top of the desk and the ones' at the bottom. Or the tens' digit might be put on top of the ones', or above the ones':

Color ten (red)
Color ten (green)
Color ten (blue)
Color the others (yellow)





How many?

__3__ tens and __6__ We write __36__.

Introduction to place-value notation

Teaching
Page c-8

Although this page develops the same concepts as page c-7, it also supplies a slight change of pace for the children. Help children interpret the directions at the top. Explain that they should color ten of the fish red, ten of them green, and ten of them blue. Any left over fish should be colored yellow. Emphasize that they must count carefully so that they color exactly ten for each color, red, green, and blue. Finally, point out the question at the bottom and ask the children to answer the question "How many?" in the two ways indicated.

3 tens and 5 $\boxed{3}$
$\boxed{5}$

When children have chosen a way of using the cards, suggest that they explain what they did to the whole class. Ask a child whose group placed the numerals side by side to explain their understanding of the numerals. Emphasize this placement as that used in writing numerals. For example, if a group worked with 35 objects they would place their cards side by side.

$\boxed{3}$ $\boxed{5}$

Work through examples until you are sure the children understand that these numerals may be read as so many tens and so many left over. Thus $\boxed{3}$ $\boxed{5}$ may be read 3 tens and 5. (The terms twenty, thirty, forty, and so on will be introduced in a later lesson.)

Follow-up

Prepare a worksheet similar to the Follow-Up suggested on page c-6, but include space for children to write the base-ten numeral by using the phrase "**We** write ____" below each area to be covered.

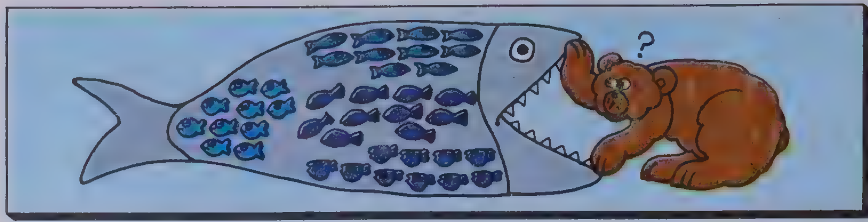
You might also write on the chalkboard a list of numerals (less than 50) and ask children to "build" each one with their strips. Children might do this activity in pairs. Simply write a list of two-place numerals each followed by the phrase "____ tens and ____". Together they should build each number and then complete the phrase. For example, if 52 appeared on the list, the children would show 5 ten-strips and 2 units, and then complete the phrase:

__5__ tens and __2__.

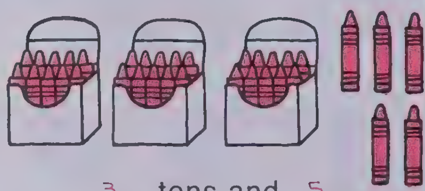
Teaching
Page c-9

Since this page is partially review and contains a dashed numeral for the new idea in the second frame on the left, attempt to have the children do the page with a minimum of guidance. Note that the numerals for forty and fifty are introduced. However, it is not necessary that the words *forty* and *fifty* be taught here. You might use them as they come up naturally, but do not emphasize them. The important point is that children realize that 40 means 4 tens and zero, and 50 means 5 tens and zero.

As the children work, move around the room giving individual help as needed. When the children finish both this page and page c-10, use their answers to discuss the more important points.

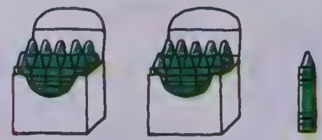


How many?



3 tens and 5

We write 35.



2 tens and 1

We write 21.

4 tens and 0

We write 40.

3 tens and 3

We write 33.

5 tens and 1

We write 51.

5 tens and 0

We write 50.

Place-value notation

Objectives

Given objects grouped in tens with any number left over, the child will be able to write the base-ten numeral.

Given the symbol 10, the child will understand that it means 1 ten and 0 as well as simply ten.

Note that one of the main objectives of this lesson, the understanding of the numeral for ten, is introduced on page c-10. Be sure that all the children have opportunity to complete the exercises on page c-10.

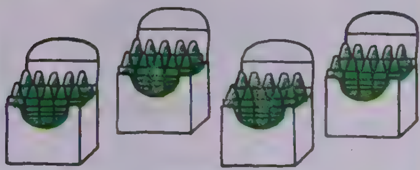
Pre-Book Activity

Materials

sets of pencils, sticks, or checkers containing less than 50 objects, some containing exactly 20, 30, or 40 individual numeral cards from 0 to 9

Distribute the sets so that every fourth or fifth child is given a set of 20, 30, or 40. (If materials are insufficient for each child to have his own set, have the children work in groups.) Each child should also have a set of numeral cards. Explain to the children that you would like them to make as many sets of ten as they can and to see if they can show with two numeral cards how many they have in the set they have grouped. As children work, ask them questions such as: "How many tens do you have?" "How many are left over?" Since the chief new idea in this lesson deals with writing 10 and multiples of 10, stress those numerals as children find them. Keep in mind however that this lesson is part of an ongoing objective to be able to group any set into sets of ten and a set of fewer than ten, and then to write the numeral for the set.

How many?

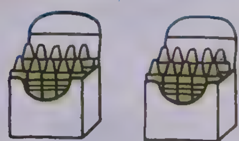


4 tens and 0

We write 40.

3 tens and 0

We write 30.



2 tens and 0

We write 20.

1 tens and 0

We write 10.

1 tens and 1

We write 11.

1 tens and 2

We write 12.

Place-value notation

Teaching
Page c-10

Direct the children's attention to the first frame. Remind the children that they should think of each crayon box as representing ten. Ask them to study the pictured sets and explain the use of the phrase *4 tens and 0*. You might write 40 on the chalkboard and ask a child to explain the meaning of the 4 and the 0 written in this way. Then have the children work through the remaining frames on their own.

When everyone has finished, ask the children to read the answers to the exercises together. Emphasize particularly the answers to the last three exercises. Many children are familiar with the numeral for ten; however, this is the first opportunity they have to see why it is written as 10. Emphasize that in writing the symbol for ten, we think *1 ten and 0*. You might have the children cross out the *s* in the word "tens" to emphasize that here they are dealing with only *one* group of ten.

Also point out that the numeral 11, which we call eleven, means 1 ten and 1 and that the numeral 12, which we call twelve, means 1 ten and 2. Both of these numerals will be familiar to many.

Follow-up

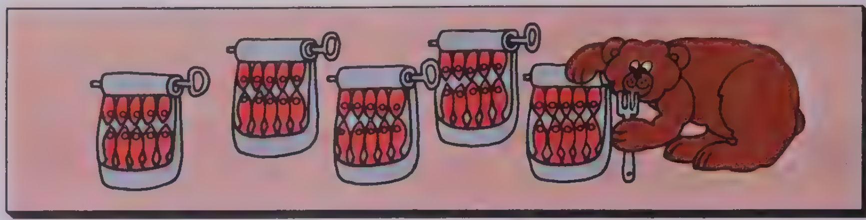
Write a matching game on the chalkboard or duplicate a worksheet similar to the following one:

58	3 tens and 4	73	8 tens and 2
27	5 tens and 8	16	9 tens and 8
34	4 tens and 9	82	7 tens and 3
49	2 tens and 7	98	1 ten and 6
61	6 tens and 1	24	2 tens and 4

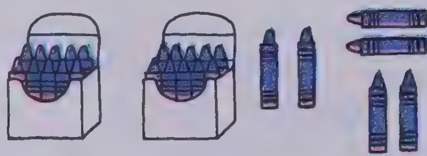
Instruct the children to match the numeral at the left with the phrase at the right that shows its meaning.

Teaching
Page c-11

Since this page is similar to many previous pages, you will probably want the children to do it independently. After using the illustration at the top as an introduction, call attention to the top frame. Ask children to describe how many objects are pictured. As usual, stress that each box represents ten crayons. Instruct them to write how many tens and how many left over and then show the way we write this numeral on the chalkboard so all can check what they have written. Assign the remaining exercises as independent work.



How many?



2 tens and 6

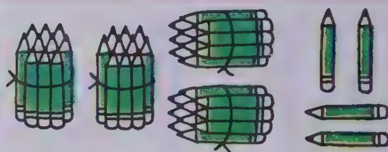
We write 26.

3 tens and 1

We write 31.

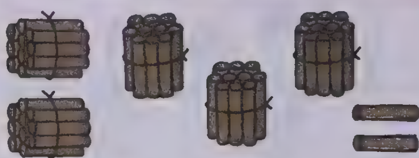
2 tens and 0

We write 20.



4 tens and 4

We write 44.



5 tens and 2

We write 52.

1 tens and 0

We write 10.

Place-value notation

Objective

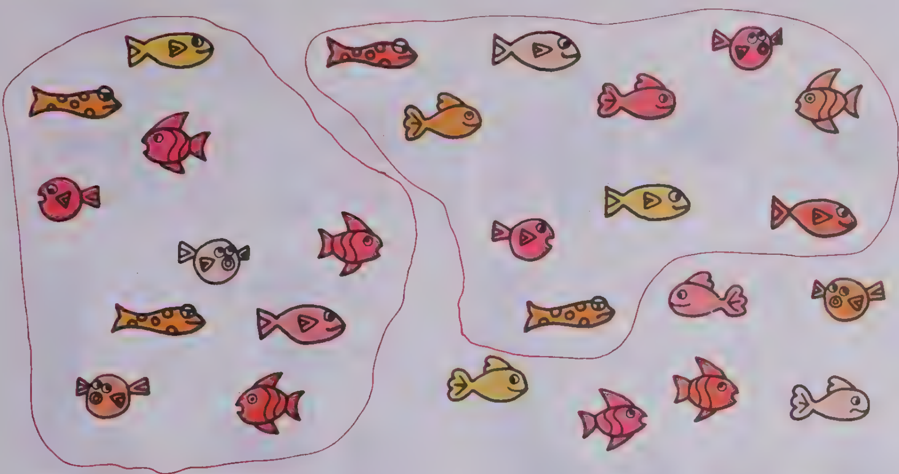
Given pictures of objects grouped by tens and some left over, the child will be able to identify how many tens and ones, and write the corresponding base-ten numerals.

Pre-Book Activity

Since these pages are primarily a review of the ideas covered so far in the study of place value, this is an excellent time to introduce the names for the numbers. For example, begin your activity by writing the numerals for the decades 10, 20, 30, and on up to 90, on the chalkboard. Lead the children in reciting the names, pointing to each numeral as its name is given. The most difficult names are those in which the number is not clearly mentioned (twenty, thirty, and fifty). The names sixty through ninety are much easier, since these begin with the word that names the number of tens in them.

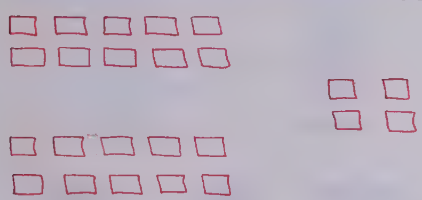
Prepare flash cards with these numerals on them. Ask the class to read the number names aloud as you show each card. Some children will need considerable practice before they master the names. When most of the children have mastered the names of the decades, you can begin giving them examples of numbers such as 62, 35, 84, and so on. You might do this by saying: "I'm thinking of six tens and four. Can you read this number?" and write 64 on the chalkboard. Gradually, lead the children to practice reading a list of two-digit numbers you have written on the board. Note that the names for the numbers from 13 to 19 have not yet been formally introduced.

Ring sets of ten with different colors.



2 tens and 6 How many? 26

Make some sets of ten and some "extras." Answers will vary. An example is given.



2 tens and 4 How many? 24

Grouping by tens

TEACHING
Page c-12

Give careful directions for the different format of this page. Explain to the children that they should count out ten fish as many times as they can and circle each group of ten with a different colored crayon. Then they should count the groups of ten, and the fish left over, and use the blanks provided to give the number of tens and how many left over and the corresponding two-digit numeral.

Call attention to the bottom section of the page. Read the directions and explain that they are free to draw whatever objects they want. Since they are to form groups of ten, suggest that they use something small, such as letters of the alphabet, in their drawings. The important point is that they find groups of ten and can give the number of the set they have drawn.

FOLLOW-UP

Oral games, such as "I'm Thinking of a Number," similar to that suggested in the pre-book activity, can provide practice in learning place value. Begin by saying: "I'm thinking of a number that has five tens and six. Who can write its name on the board?" Call on a child to answer. If he writes the correct numeral, he continues the game by saying, "I'm thinking of a number . . . ."

Vary the game for a very alert group by saying: "I'm thinking of a number one more than twenty-nine" or "I'm thinking of a number one less than sixty," and so on.

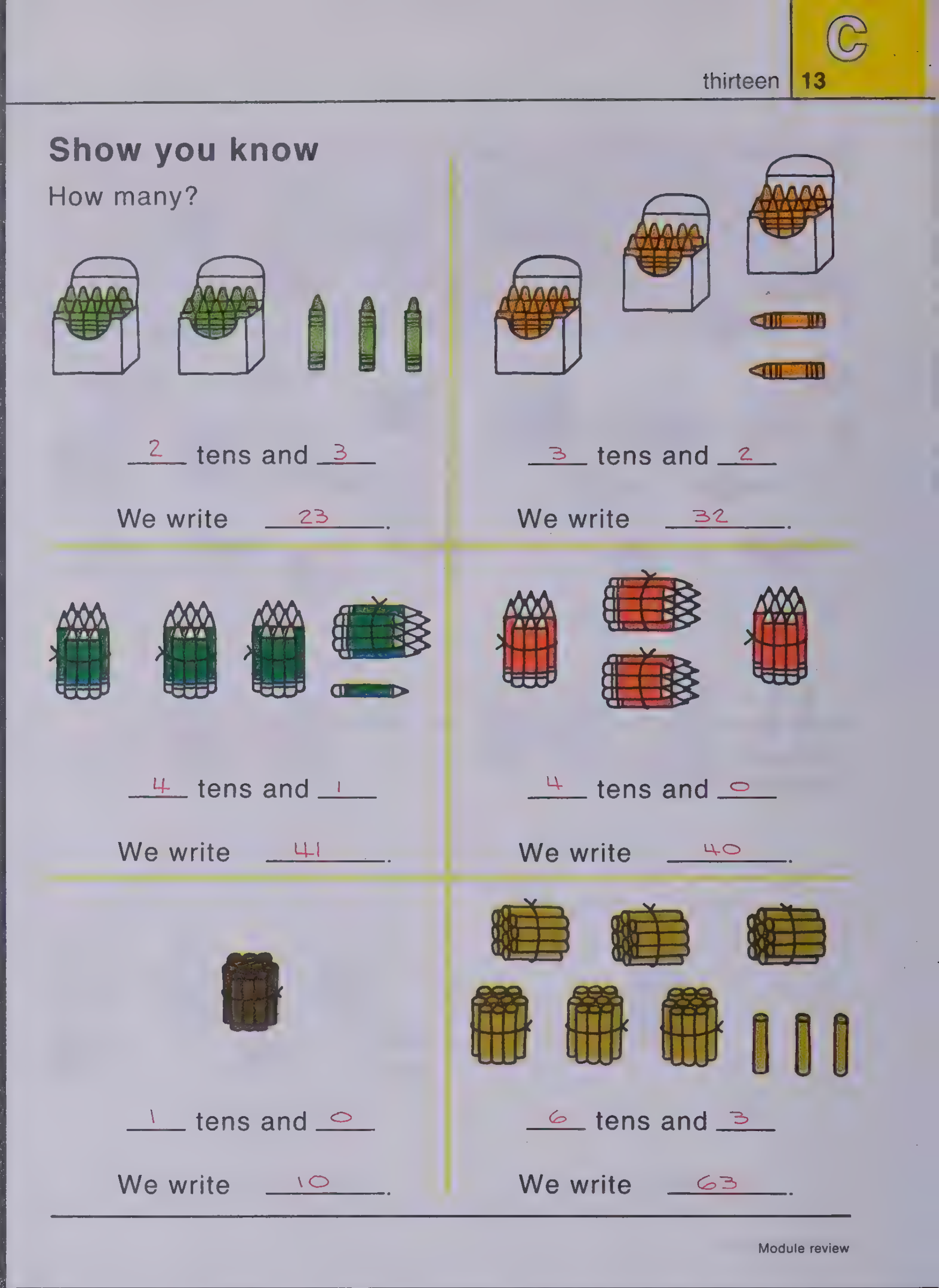
For the more capable students, the written names of the decades may also be introduced at this time. An exercise similar to the following may either be duplicated or written on the chalkboard.

Match the numeral with its name.

8	four	80	forty
4	two	40	twenty
2	six	20	sixty
6	eight	60	eighty
1	three	10	thirty
3	one	30	ten
9	seven	90	seventy
5	nine	50	ninety
7	five	70	fifty

Teaching
Page c-13

Call attention to the first frame. Since children have worked with these kinds of exercises, you might ask a child to describe what he thinks they are expected to do. Be sure that everyone understands that they are simply to tell how many objects are pictured by expressing so many tens and so many left over, and by writing the corresponding two-digit numeral.



C

thirteen 13

Show you know

How many?

2 tens and 3

We write 23.

3 tens and 2

We write 32.

4 tens and 1

We write 41.

4 tens and 0

We write 40.

1 tens and 0

We write 10.

6 tens and 3

We write 63.

Module review

Objective

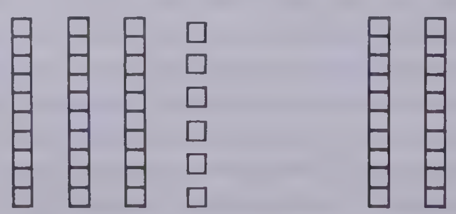
The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

Materials

strips

Give each child a set of strips. Review the number names of the strips. Stress that since the white strip is the unit, the orange strip may be thought of as ten. Then ask the children to show various numbers with their strips. If you have children work in pairs they can show numbers as great as 89; if you prefer that they work individually, keep the examples 49 or less. For example, ask them to show 36. They should lay out 3 ten (orange) strips and 6 one (white) strips. It would be helpful to include examples which contain zero.

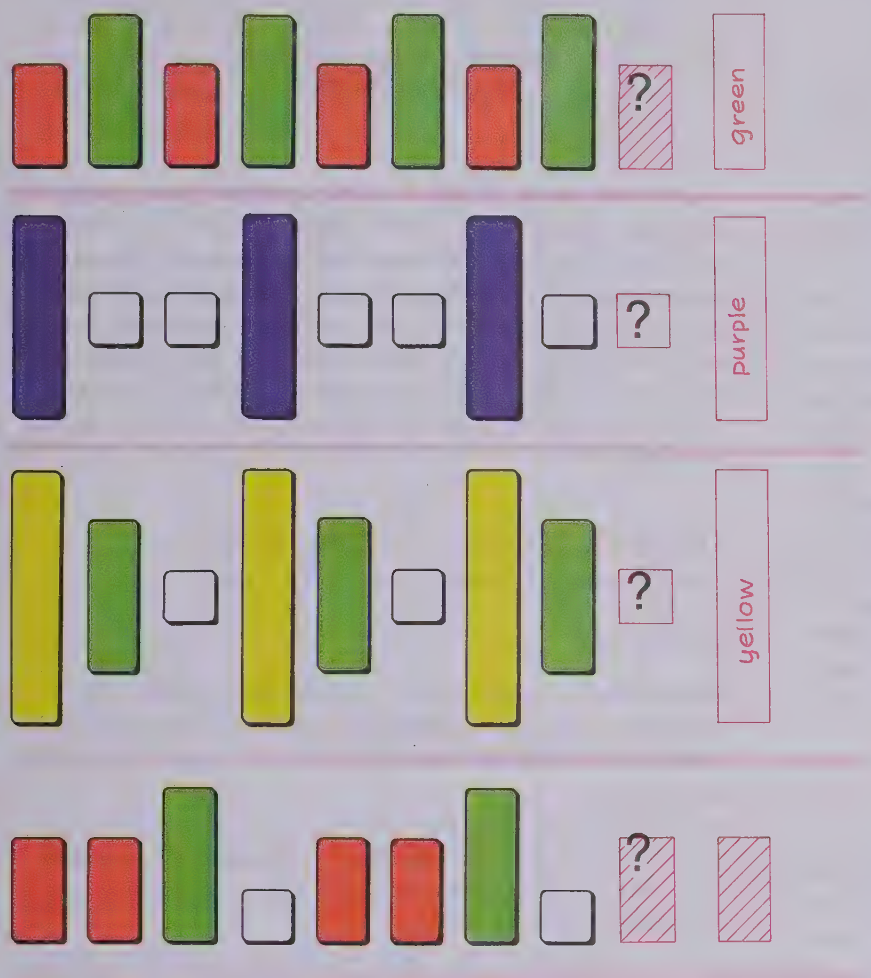


Follow-up

More capable children might enjoy making up other patterns with the strips and giving them to others to try to complete. You might give them pieces of newsprint

Let's have fun

Can you show the next two strips in each pattern?



Can you make a pattern of your own with your strips?

Teaching
Page c-14

Carefully explain the directions for this change of pace page to the children. Be sure every child has his strips. Suggest that they use them to find the strips which will complete the pattern, and then draw the shape and color of the strips they have chosen. Help the children begin by discussing the first frame with them. For example, have them read together the strips which they see: "The red strip, green strip, red strip, green strip, and so on." Ask what strip they think will come next if we keep repeating the strips in the same way. Explain that we say the strips are in a certain *pattern* because of the way they repeat themselves. Suggest that they draw the next two strips, using their strips as models. If necessary, work through other frames with the children. But, as soon as they catch on to what is expected, encourage them to work independently. Point out the question at the bottom wherein they are asked to make a pattern of their own using whichever two or three strips they choose. Distribute paper on which they might draw their patterns, or have them line up the strips for you to see.

approximately A-4 (210 mm × 297 mm) on which to trace their strips in the patterns they build.

You might challenge others to try to build patterns by matching their strips with sets of numbers you put on the chalkboard. The following examples would be suitable:

1-2-3; 1-2-3; 1-2-?
2-4-6; 2-4-6; 2-4-?
2-3-4; 2-3-4; 2-?-?
7-8-9; 7-8-9; 7-?-?

Resources for Active Learning

Patterns:

DEVELOPMENTAL MATH CARDS, $A^3$13, $B^1$9, Addison-Wesley

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Number sequences," Nos. 66–69, Responsive Environments Corp.

MATHEX: Matching and Graphing No. 1, pp. 12–15, Encyclopaedia Britannica Publications Ltd.

MATH WORKSHOP: Games and Enrichment Activities, p. 9, Encyclopaedia Britannica Educational Corp.

NOTES ON MATHEMATICS IN PRIMARY SCHOOLS, ". . . Rods," pp. 24–26; ". . . pegboard," pp. 30–33, Cambridge University Press

THINK AND COLOR "Sequential Patterns," pp. 28–37, Educational Science Consultants

WORKJOBS, pp. 36–37, 40–41, 70–71, Addison-Wesley

ORANGE MODULE, UNIT C

Counting to 99

Pages c-15 to c-26

General Objectives

To teach the order of the number 0 through 99

To continue to develop the concept of place value for two-digit numbers

To develop skill in counting from 0 to 99

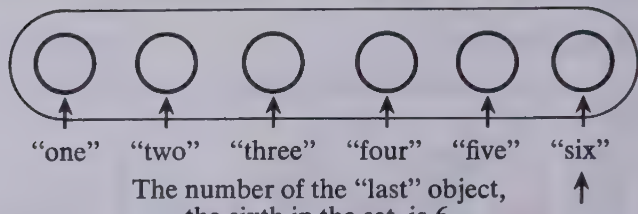
In this module the children apply their understanding of place value to order the numbers 1 through 99. An important goal is to have children write, in order, the numerals 1 through 99. An ongoing objective is that children learn the number names for each number from 1 to 99 and use them correctly in counting. Special attention is given to helping children understand spanning decades such as 19 to 20, or 29 to 30. On page c-23, the process of spanning a decade is extended to all the decades through the nineties. The module concludes with a review page which may be used for evaluation and a change of pace page.

Mathematics

A cardinal number refers to a quantitative aspect of a set of objects, whereas an ordinal number refers to the position of one object in an ordered set. The common names for the ordinal numbers are: *first, second, third,* and so on, referring specifically to position.

Cardinal-number arithmetic is the mainstream of the mathematics presented in the elementary school. However, ordinal numbers are necessary to express relative position (first, second, and so on) and as an aid in shedding light upon certain topics in cardinal-number arithmetic. For example, when we determine the cardinal number of a set by counting, we pair a *first* object in the set with 1, a *second* object in the set with 2, a *third* object with 3, and so on. To visualize this, think about counting a set of objects that are scrambled together. We would probably touch or point to each object as we count it, saying: "one, two, three, four, . . .," thus assigning a number to each object. When we have completed the counting, the number of the last object in the set is the cardinal number of the set. This relationship is pictured in the next figure.

The cardinal number of this set is 6.



The number of the "last" object,
the sixth in the set, is 6.

Children are exposed to both cardinal and ordinal concepts before they enter school. However, when counting is used to determine the number of a set, it is important for the teacher to realize that some children may not know how to use the vocabulary correctly and may need some help in doing this. It is not inherently obvious to a child that the cardinal number of a set is the last number he uses in counting the set; he must learn this.

Teaching Orange Module, Unit C

Approximate Teaching Time: 6 to 9 days

Materials

demonstration strips (these can be made from felt or construction paper with adhesive felt on the back for use with the flannelboard)

flannelboard

flash cards in multiples of ten

numeral board (a board having 10 rows and 10 columns of hooks or pegs with removable tags, spools, or discs, labelled 0 through 99)

objects or counters that can readily be grouped by tens (pencils, lollipop sticks, straws, pipe cleaners, checkers, etc.)

rubber bands, string, or pipe cleaners for bundling

Vocabulary

decade	order
dot picture	spanning a decade

Counting activities are an important part of this module. Allow children to work with objects in groups of ten and some left over as well as with the ten-strips and one-strips. Some children should be able to write the numerals to 99 without concrete materials, but the use of actual materials will strengthen their concept of place value.

Evaluation of Progress

Children's ability to write the numerals in order and count can be evaluated from their work on the written page. As children are involved in various activities in the lessons, ask a child to count from some number he is working with and allow him to count at least ten more numbers. It is also important that children be able to put the multiples of ten in order. This may be evaluated orally or on the text page.

Resources for Active Learning

General Activities:

For multi-base arithmetic blocks see "Resources for Active Learning: in Unit C, Module 1 Introduction.

MATH ACTIVITIES, Charts, pp. 46–47; Verses 2/33–40, pp. 38–40, Allyn and Bacon

WORKJOBS, "Number Dots," pp. 146–147, Addison-Wesley

Manipulative Devices:

Abacus or abacus board (Educational Teaching Aids; school supplier)

Chips (Educational Teaching Aids; Selective Educational Equipment)

Grid Kit (Scott Scientific)

Hundred number board (Hammett; Mafex Associates)

Investigation
Page c-15

If children build trains for 11, 12, and 13 during the preparation, they should not have difficulty interpreting the illustrated trains on page c-15. Read the directions with the children. Point out that we want to use not only the smallest number of strips but also as many *orange* strips as possible. For example, we would not want one orange, one brown and one purple for 22. Rather we want 2 orange and one red, thus bringing out the idea of 2 tens and 2.

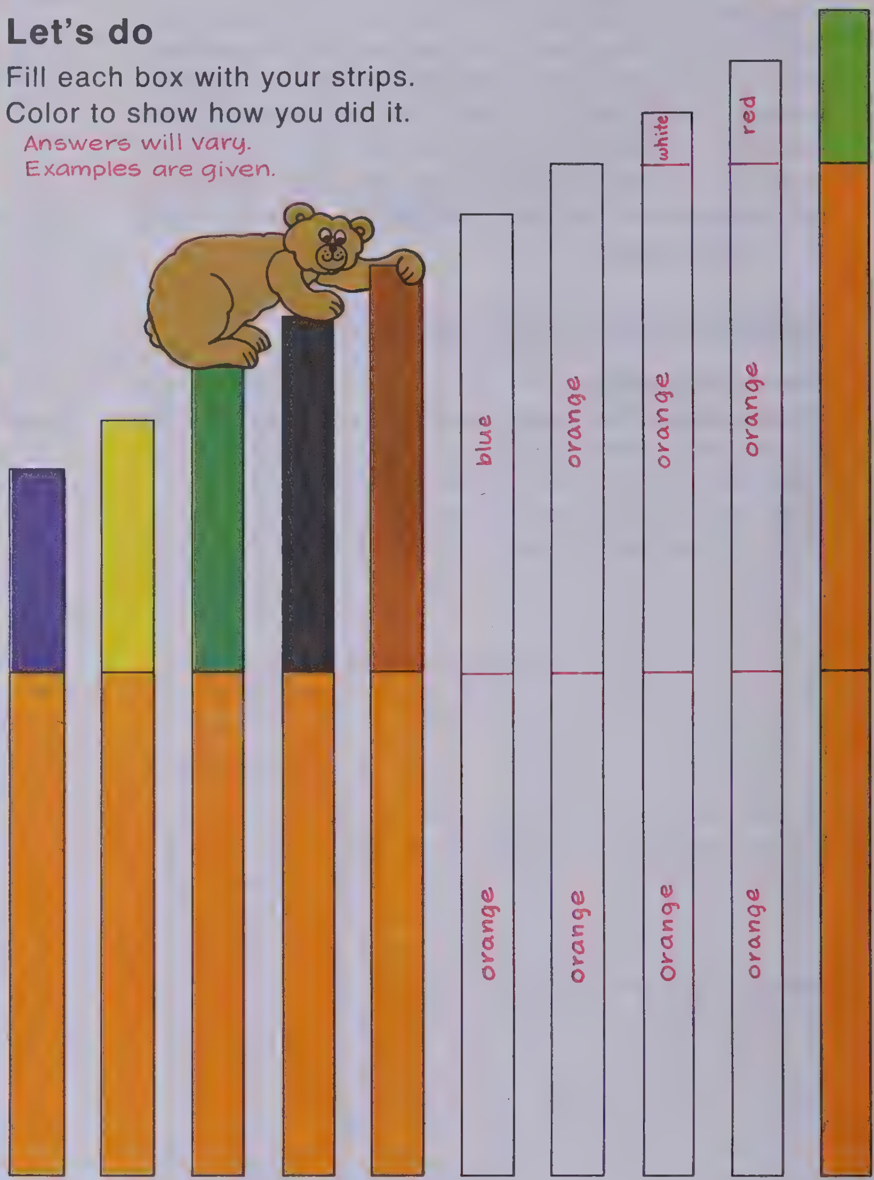
When the children have built the trains and finished the coloring, one of the main objectives of the module may be treated. Instruct the children now to put all their strips aside except for 9 white strips and the orange strips. Explain that you would like them to build trains on the outlines by using only orange and white strips. Help them begin by asking someone to describe how many of each they used to cover the orange and purple strips or the orange and yellow strips. This activity is basically a counting activity which gives children an opportunity to experience how, in counting, a multiple of ten may be expressed by replacing the 9 white strips of the preceding number with one ten-strip.

Note: The latter part of the investigation in which children build trains using only white and orange strips may be treated in a second day's session or as the pre-book activity for page c-19.

Let's do

Fill each box with your strips.
Color to show how you did it.

Answers will vary.
Examples are given.



Purpose

To provide background experiences for development of place value and counting

Preparation

Materials

set of strips for each child

After a preliminary play period with the strips, ask the children to build various trains, leading up to trains of 14, 15, and so on as shown on page c-15. For example, begin by asking them to show the strips which make them think of 8, of 9, and of 10 (if the white strip is thought of as 1). Then talk about building a train for eleven. Remind them what the symbol 11 means, namely 1 ten and 1, and suggest that they build a train to show this meaning. Have children continue to build trains for 12 and 13 using only two strips for each train. Encourage them to explain their choice of strips. If some want to use the orange strip and 2 white strips or the orange strip and 3 white strips for 12 and 13, explain that such trains are correct, but here you're asking them to explore trains with the orange strip and one other strip.

C

16 sixteen

Let's talk

Readiness for counting to 99

Discussion
Page c-16

The illustration on page c-16 should serve as an introduction to counting which includes spanning the decade. This is accomplished by showing rows of plants grouped by tens, so that the child will eventually understand that the number of full rows increases by one each time a new ten is formed. First ask the children to describe the picture. Include discussion of the fact that each full row contains 10 plants. The plants need not be counted one by one; rather it would be best to count the number of full rows and to note that there are 2 tens and 8 more.

Follow-up

A worksheet such as the following will provide children with further experience in thinking of tens.

Draw a ring around every group of ten you can find.	
XXX XXX XXX XX XXXX XX XXX XX	o o o o o o o o o o o o o o o o o o o o o o o o o o o o
XXXXXXXXXX XXXXX XXXXX XXXXX XXX	o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o o

Resources for Active Learning

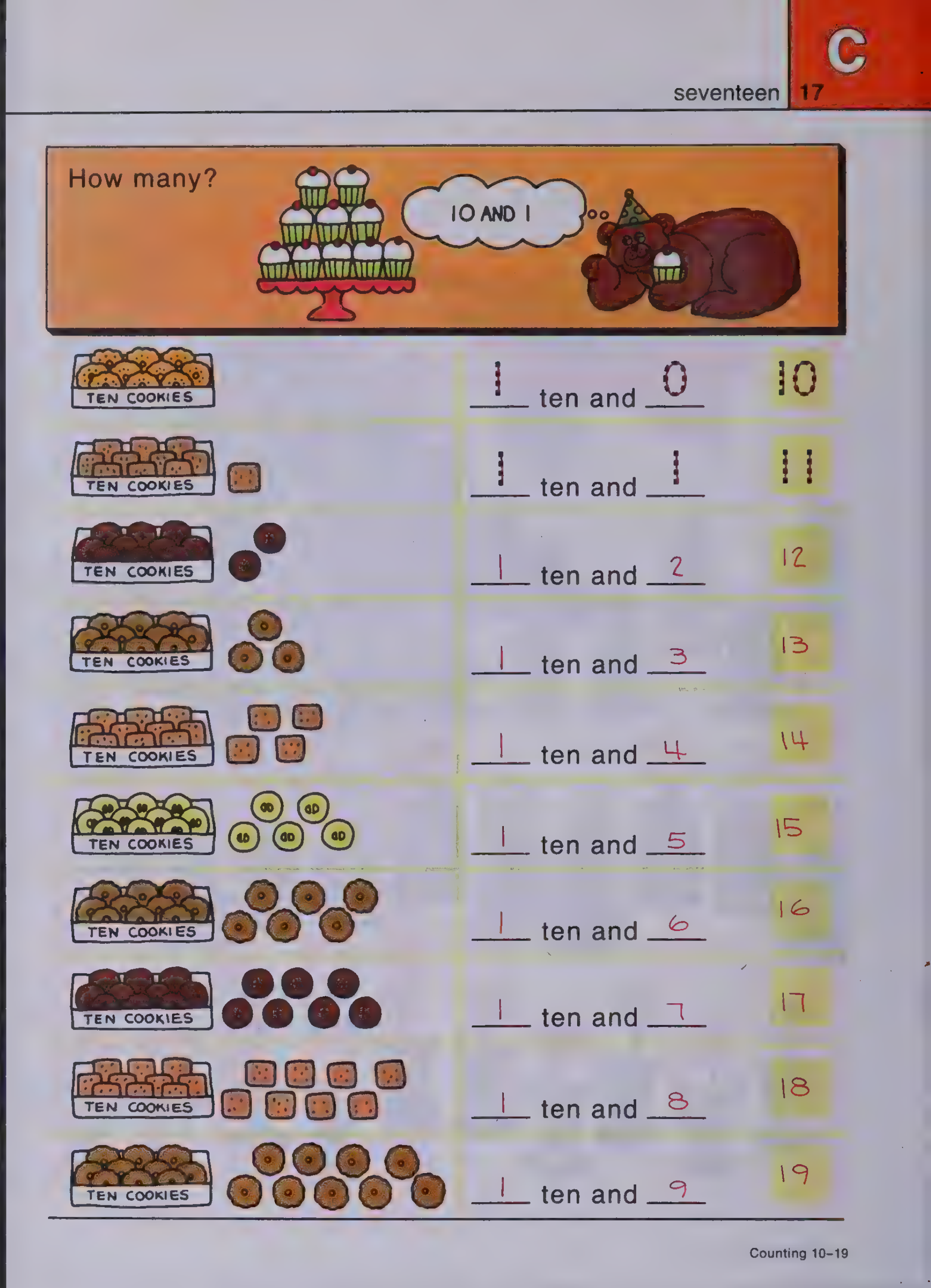
A CLOUDBURST, Vol. 1, "Let's Guess," No. 2111, Midwest Publications

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Natural Numbers," Nos. 21–22, Responsive Environments Corp.

MATH WORKSHOP: Games and Enrichment Activities, "Spatial Relations . . .," pp. 5–6, Encyclopaedia Britannica Educational Corp.

Teaching
Page c-17

Call attention to the first illustrated box of cookies and ask the children how many cookies it contains. Elicit from them that there is 1 box of ten cookies and none left over. Continue similarly with eleven: 1 box of ten and 1 more cookie. When the children have traced over the dashed numerals, encourage them to complete the page on their own.


C

seventeen 17

How many?

1 ten and 0 10

1 ten and 1 11

1 ten and 2 12

1 ten and 3 13

1 ten and 4 14

1 ten and 5 15

1 ten and 6 16

1 ten and 7 17

1 ten and 8 18

1 ten and 9 19

Counting 10–19

Objective

Given the numbers from 1 to 19, the child will be able to order them and write the numerals.

Although the child is not expected at this time to know and correctly use the number names for the teen numbers, this is an ongoing objective and these names should be used.

Pre-Book Activity

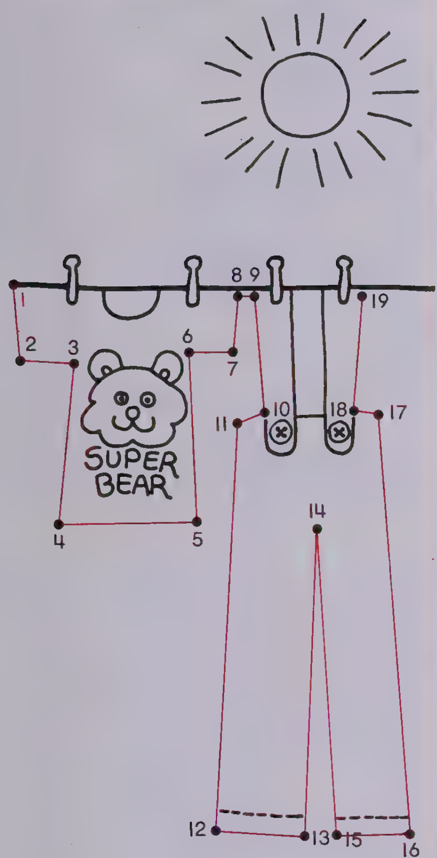
Materials

objects to demonstrate sets of 10 to 19

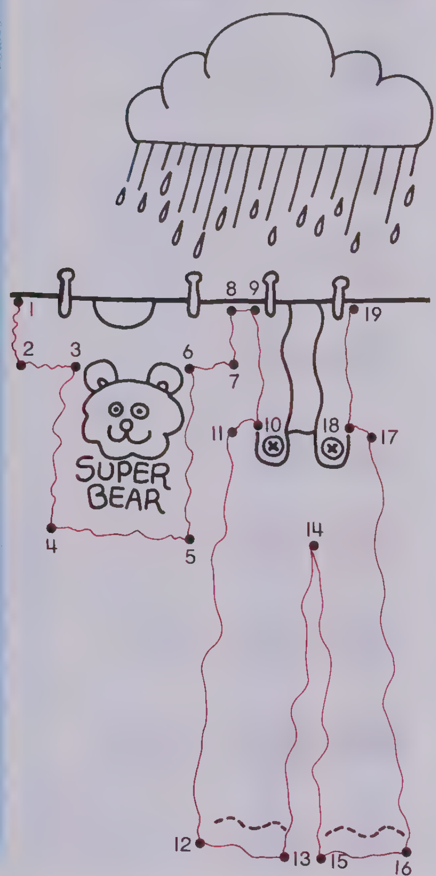
Group the counters or objects so that you have increasing sets from 10 to 19. Unless you wish to use these sets as part of a group demonstration, distribute them among the children so that every three or four children can use one set. Ask the children to arrange their set so that they have so many groups of ten and so many left over. Also suggest that they decide how to write the numeral which shows how many they have. When the children have finished, decide on a way to recognize the order of these numbers. For example, write the symbol 14 on the chalkboard. Ask someone to explain the meaning of the 1 and the 4. Then ask the group whose set contains 1 ten and 4, to place it on a table in the centre of the room. Then ask that any group whose set is larger than 14, place their set on the table and write a numeral for it on the chalkboard. Continue to ask the children to bring up sets, larger or smaller than a given set already present and arrange them on the table so that all the sets will be ordered. As this is done, list the numerals 10 to 19 in order on the chalkboard.

Connect the dots.

Draw **straight** lines.



Draw **curved** lines.



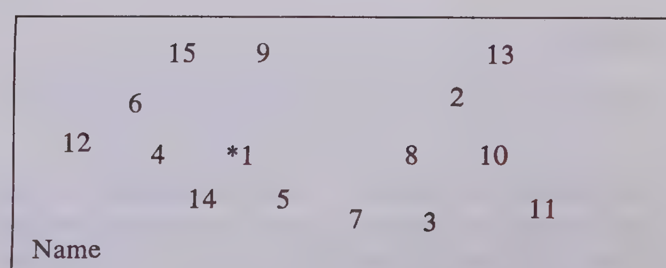
Counting 10–19

Teaching
Page c-18

Call attention to the demonstration art. Talk about how the lines might have been drawn from 1 to 2 to connect the dots in order. Also point out that one line is curved and one line is straight. Then explain that if they connect the numbers in each section in the correct numerical order, with straight lines on the left and curved lines on the right, an interesting figure will appear in each section. Some children will benefit by using a straight edge or ruler to help them draw the lines on the left. Children might enjoy coloring the figures and then displaying their papers on a bulletin board. The emphasis for this page should be on the order of the numbers. If a child has difficulty drawing the curved lines, suggest that he use straight lines. Treat the drawings with a light touch.

Follow-up

Provide each child with a large sheet of newsprint (colored, if available) and crayons. Tell the children to put a small star made of three crossed lines somewhere on their paper and write the numeral 1 next to it. Then tell them to write numerals from 2 to 15, placing them anywhere they like on their paper. When they have finished, tell them to write their names at the bottom of the paper. Collect the sheets and pass them out at random, making sure no child gets his own. Direct them to find the star beside the numeral 1 and then to find 2, and draw a line from 1 to 2. Tell them to continue until they reach 15. You might make some very simplified outlines numbered 1 through 10 for children having perceptual or motor handicaps.



Resources for Active Learning

DEVELOPMENTAL MATH CARDS, "Can You Count?" B!8, Addison-Wesley

MATH ACTIVITIES, "Number Watch," Game 2/65, pp. 51–52, Allyn and Bacon

Teaching
Page c-19

Use the illustration to introduce the page. Then call attention to the first frame. Ask a child to explain how many bags of candy and how many pieces of candy are pictured and have the children complete the numerals for 16. Be sure they realize that there are 10 pieces of candy in each bag.

Encourage most groups of children to continue this page independently. Move around the room as the children work to be sure they can span the decade from 19 to 20. If some children have difficulty, suggest that they use actual sticks as they do the page.

When they have finished the page, continue from 25 to 29 by having children match the numerals 26, 27, 28, and 29 to bundles of sticks you display. Or, work through these numbers by drawing sets on the chalkboard and having the children write each numeral you treat.

How many?

1 ten and 6	16
1 ten and 7	17
1 ten and 8	18
1 ten and 9	19
2 tens and 0	20
2 tens and 1	21
2 tens and 2	22
2 tens and 3	23
2 tens and 4	24
2 tens and 5	25

Counting—teens and twenties

Objective

The child will be able to count to 29, that is, in particular, he will be able to span the decade from 19 to 20.

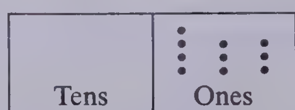
Even though the exercises in the text go only as far as 25, notice that the objective of this lesson includes numbers as far as 29. You should develop your discussion to include the numbers 26 to 29.

Pre-Book Activity

Materials

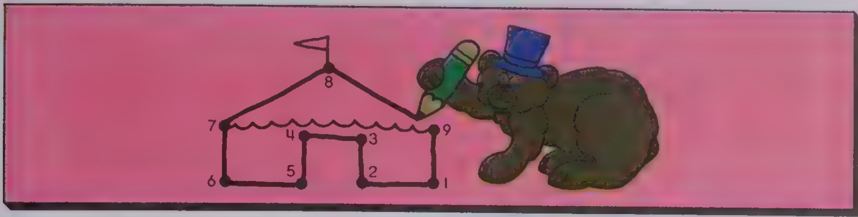
counters, 30 for each child
large newsprint, 1 sheet for each child

Direct the children to fold the newsprint in half and label the right half ONES and the other half TENS.

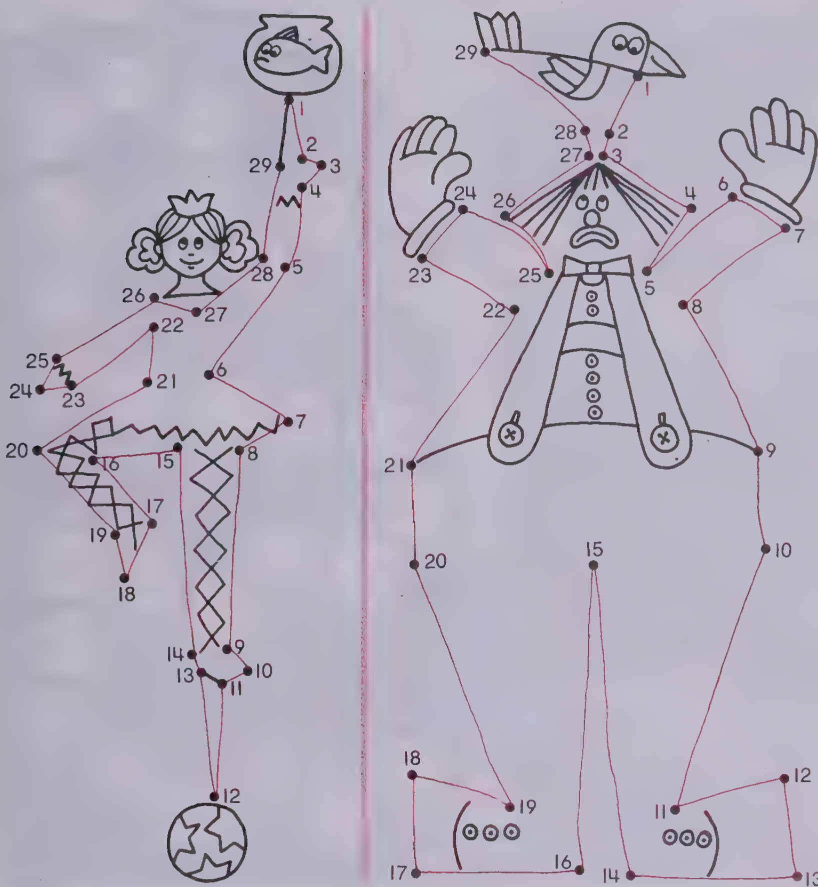


Guide them in marking the half labelled ONES with ten dots or checks as illustrated above. Then explain that you want them to count their counters and group them by tens when possible.

Point out that they will know that they have enough counters to make a group of ten when all the dots are covered. They should then place that group of ten in a stack or bundle on the TENS' section of the paper. When they have found how many counters they have, ask them to record the number of tens and ones.



Connect the dots.



Counting—teens and twenties

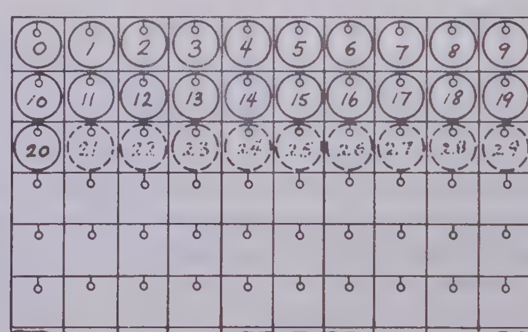
Teaching
Page c-20

Most of the children will understand what to do on this page. You should, however, give careful directions so that other pages containing dot pictures will be fairly routine. Caution the children to be very careful with this exercise so that their pictures will turn out the way they should. Direct them to place their pencils on the dot beside the 1, underneath the fish bowl. Explain that this is the first dot for the picture on the left side of the page. Then tell them to find the dot beside the 2 and to draw a line between these two dots. Next, tell them to draw a line from the dot beside the 2 to the dot beside the 3. When they have completed these two lines, instruct them to continue to the last numeral. Be sure they realize that another set of dots and numbers are shown on the right side of the page.

Assist those having difficulty in finding the dots and drawing the lines.

Follow-up

If you already have a numeral board (see orange module introduction, material list), show it to the children to illustrate the concept of decades through 20 and of order for the numbers 0 through 99. A permanent board can be made of a piece of soft wood or fiberboard about one metre square, 100 cup hooks, and 100 round identification tags. Write the numerals 0 through 99 on the tags. Leave one side of the tags blank. To begin, have the tags in order with their blank side up. Point to the top left tag (the zero tag) and ask children what they think is written on the next tag to the right if this tag has zero written on it. Develop the numbers in sequence from 0 to 9. Then focus attention on the next two rows and develop them similarly.



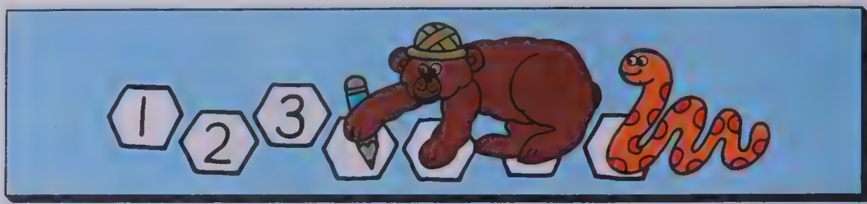
Resources for Active Learning

MATH ACTIVITIES, "Span a Place," Game 2/91, p. 69, Allyn and Bacon

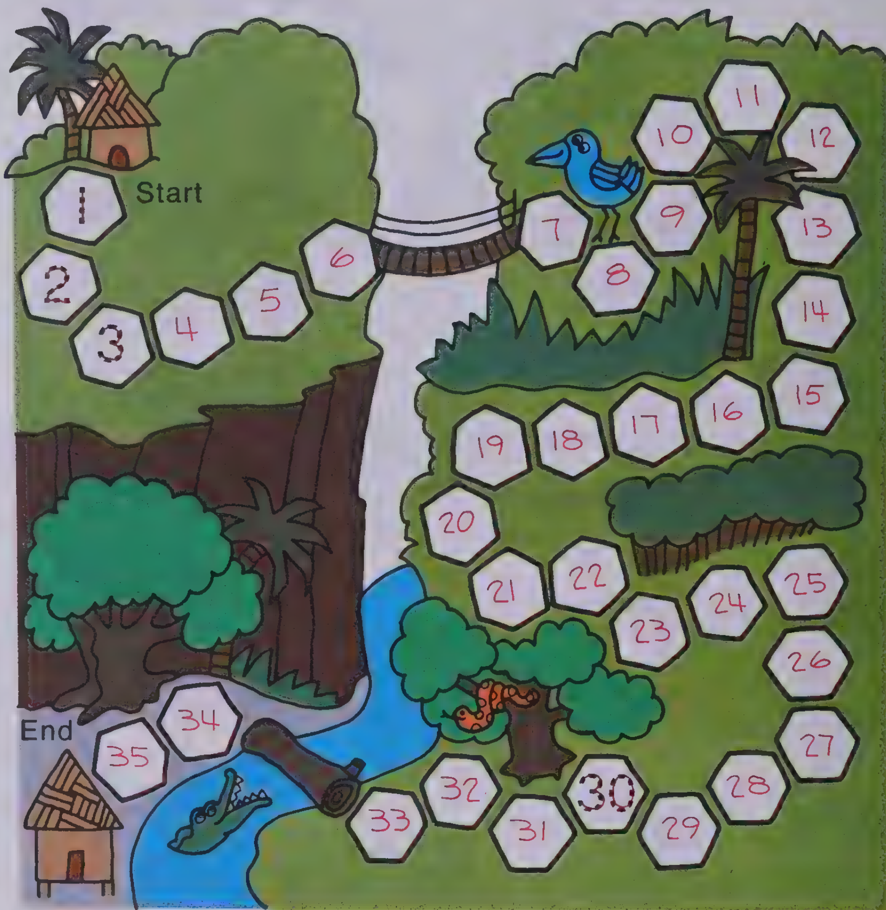
Nuffield Project: COMPUTATION AND STRUCTURE 2, "Counting . . ." pp. 42–57, Wiley

Teaching
Page c-21

Ask the children to examine the main illustration and describe the scene. Encourage suggestions for stories about the path. During this discussion, give the children time to count the stones and to label them with the correct numerals. Then use the stones to have the children count aloud to 35 with you. Also ask questions such as: "How many stones are there before the bridge?" Count aloud with them. Point out that we say "6" on the stone before the bridge so we know there must be 6 stones before the bridge. Discuss how many stones before the log in a similar fashion. Also use parts of the path to have the children count various groups of stones such as 6 to 12, or 16 to 26, or 27 to 30, and so on. Be sure children understand how to span the decade from 29 to 30.



Number the stepping stones.



Objective

The child will be able to count from 1 to 39; in particular, he will be able to span the decade from 29 to 30.

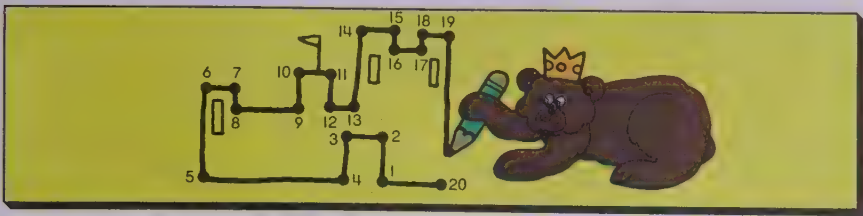
Although the primary objective of the lesson deals with counting in the thirties, the format of page c-21 may be used to stress ordinal numbers as well. For example, you could ask the children to touch the second or fourth or tenth stone.

Pre-Book Activity

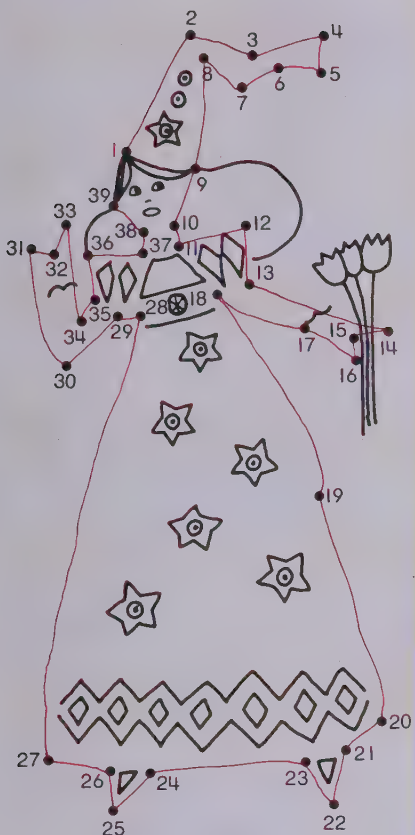
Materials

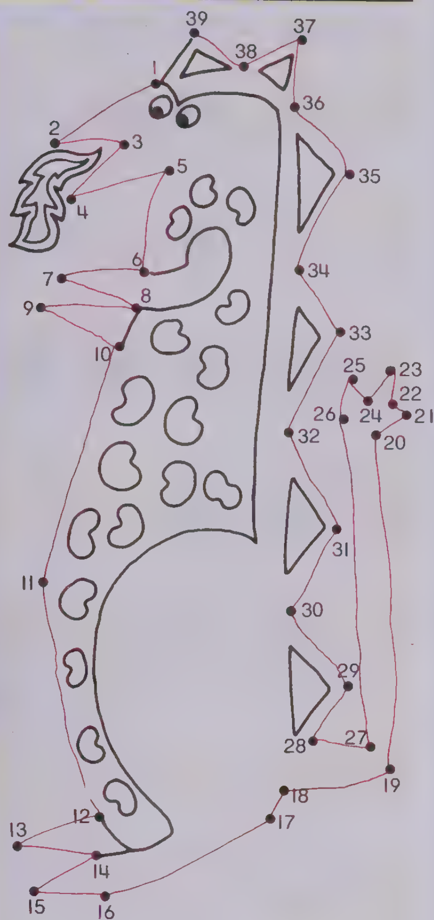
flannelboard
flannelboard demonstration strips
strips

After the children have had a short preliminary play period with the strips, ask them to show the number 26 using only orange and white strips. Use strips on the overhead projector or use demonstration strips on the flannelboard or chalkboard to have a child display how to represent 26 with the strips. Ask the children to add one more white strip to form 27. Again, have a child use the demonstration strips to show this. Continue similarly up to 29. When children add one more to 29, ask them how many tens they now have, and write the numeral 30. Elicit from them that to show 30 with the fewest possible strips, they should trade in their 10 one-strips for one orange strip, that is, for a ten-strip. Then suggest that they continue counting up to 39. As children work, encourage oral use of the number names. Your frequent oral use of the number names in order will help the children develop this skill.



Connect the dots. Start at 1.





Counting

Teaching
Page c-22

If children previously completed page c-20, most of them will understand what to do with the dot pictures on this page.

Instruct them to start at the dot beside the 1 and to follow the dots in order to the largest number. Help any children having difficulty with this exercise.

Follow-up

An exercise similar to the following might be helpful to some children.

Write in the missing numerals.

22	23	–	25	–	27	–	29
55	56	57	–	–	60	61	–
78	79	–	–	82	–	–	85
15	16	–	18	–	20	–	22
38	–	–	41	–	–	44	–
66	–	68	–	70	–	72	–

As an example of counting, show the children the calendar page for the present month. Do not insist that they learn the names of the days of the week at this time. Instead, discuss the name of the month, introduce the names of the days of the week, point out the day on which the month begins, and ask how many days there are in the month.

Give the child a duplicated sheet with the name of the month, the days of the week, and dashed numerals showing the first and last days of the month on it. Ask the children to fill in the missing numerals. They can decorate the top of the calendar according to a seasonal motif. If time permits, let them mark any special holidays by decorating the space around the numeral.

JANUARY

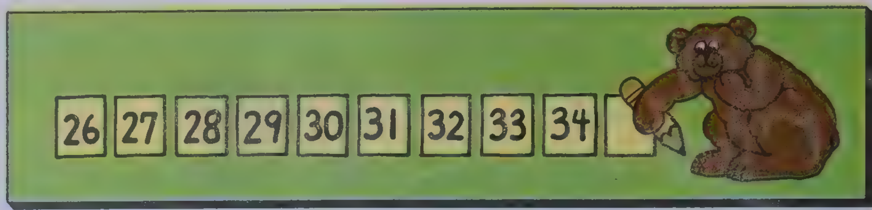
Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
	1					
			31			

Sample Calendar

Teaching
Page c-23

Direct attention to the demonstration box and ask the children if they can tell the bear what to write after 34. Then call attention to the first row of numerals. Ask the children to study it and see if they can fill in the missing numerals. Then read this row aloud with the children. When you get to 45, ask someone to give the first numeral of the next row and continue reading the numerals to 50. Encourage the children to complete the remainder of the page on their own. Children who are still not at ease in working with numerals might use sticks and bundles of sticks or ten-strips and one-strips as they write each numeral. Counting actual objects is an important activity for all children, but particularly for those who have difficulty relating to the written or oral number name.

Do not expect children to master the oral number names through 99 in one or two lessons. Continue oral activities for this purpose throughout the year. The main point of this lesson is that children write the numerals through 99, and understand their place-value meaning.



Complete each counting row.

36	37	38	39	40	41	42	43	44	45
46	47	48	49	50	51	52	53	54	55
56	57	58	59	60	61	62	63	64	65
66	67	68	69	70	71	72	73	74	75
76	77	78	79	80	81	82	83	84	85
86	87	88	89	90	91	92	93	94	95
96	97	98	99						

Objective

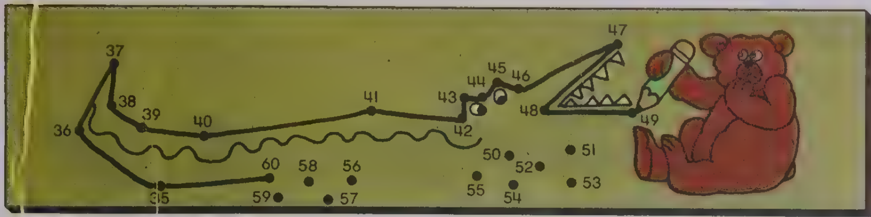
The child will be able to write the numerals in order from 1 to 99.

In this lesson the remaining decades through the nineties are treated. By now children should be accustomed to the process of spanning a decade. Notice that the number one hundred is not included. However, if the number one hundred comes up naturally during counting activities, simply accept its use but do not emphasize. Again, mastery of number names should not be expected at this time but rather treated as an ongoing objective.

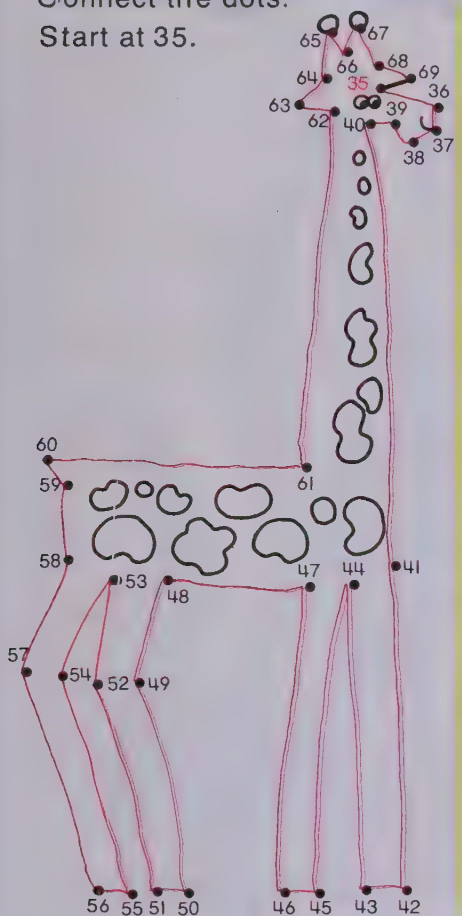
Pre-Book Activity

Exhibit a set of 3 groups of ten and 9 and ask the children: "How many?" After someone answers "thirty-nine", place 1 object with the 9 and ask: "How many now?" Show how this one more object forms a new group of 10. Thus now we have 4 tens or 40. Continue counting through 50, each time placing another object with the set and asking: "How many?" Go on to other multiples of ten and work through the decade. Then give the children an opportunity to express the meaning of various two-digit numerals.

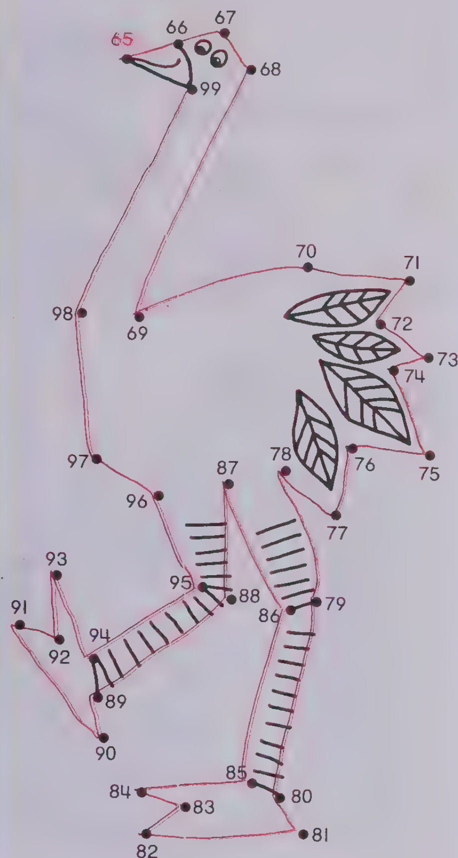
Also give the children as many concrete experiences with counting as your materials allow. If necessary, give the children turns at pointing to objects as the class counts orally. You might also distribute 99 counters, giving 3 or 4 counters to each child. Place a container in the centre of the room. Have the children walk in line by the container and drop in one of their counters as the class counts aloud. Besides teaching children to count, such an activity builds an intuitive notion for the size of 100.



Connect the dots.
Start at 35.



Start at 65.



Teaching
Page c-24

Since children should be familiar with the dot picture, give them an opportunity to complete the pictures by themselves. Before they begin, be sure they notice that their counting should begin with the numeral indicated in each picture, 35 for the left and 65 for the right.

Follow-up

As children are introduced to the higher decades, the numeral board may be used for a variety of activities. For example, fill the board up to 99, with the blank sides of the tags showing. Point to the tag for 20 and see if anyone can guess which number it is. Then show 20 and ask: "Does anyone know the next number in this row?" When someone says "twenty-one," say: "Let's turn the tag over and see." If no one correctly identifies 21 as the next number, give the children clues by saying "one more than twenty," or if one of them counts well, ask him to count to 20 out loud and tell the class what the next number will be. Continue going through the decades with special emphasis on those numbers greater than 50.

You might also have children make their own numeral board. Prepare a duplicator sheet with 10 rows of 10 columns to make 100 units. Ask the children to choose two of their favorite dark colors and make an individual chart by writing the numerals 0 through 99. Let them copy your numeral board or a large chart prepared like their sheet. Begin the first row with 0, the second row with 10, the third row with 20, and so on. Alternating the colors of each row will help the children differentiate rows.

Resources for Active Learning

MATH ACTIVITIES, "Jump Board," Game 2/28, pp. 35–36, Allyn and Bacon

MATHEX: Numeration No. 2, "Order Relations," pp. 18–21, Encyclopaedia Britannica Publications Ltd.

Teaching
Page c-25

Ask the children to look at the top row of frames and see if they can remember what to do for each picture. Be sure they understand that they should write the two-digit numeral for the number of items shown in each frame. They should also understand that in the next frame they are expected to complete each row by filling in numerals in order. Finally, in the bottom section, they should know that they are to simply connect the dots starting at 1, in order to complete the picture.

Show you know

How many?

25 14 32

Complete each row.

6	7	8	9	10	11	12	13	14	15
17	18	19	20	21	22	23	24	25	26
25	26	27	28	29	30	31	32	33	34

Connect the dots.

Module review

Objective

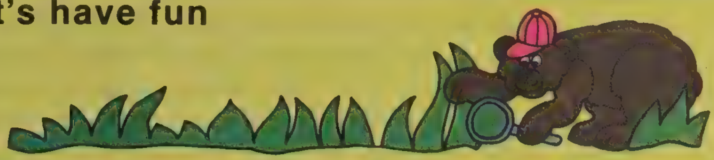
The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

Give the children an opportunity to engage in a counting activity in the classroom. For example, let the children use the "numeral chart" they copied from your demonstration board. Instruct them to choose a crayon and lightly shade in the column in which all the numerals end in zero. Ask them to read all the numerals and stress that each one is 10 more than the previous one (0, 10, 20, 30 . . . 90). Then instruct the children to find the column in which 5 is the digit in the ones' place. Direct them to shade this column in lightly, using the same color they choose for the tens. Ask them to read this column (5, 15, 25 . . .), and again point out that each is 10 more than the previous one. Instruct them to use the 5-column and the 10-column together, and count by fives. Then ask the children to find the column in which the ones' digit is 3. Try to get them to volunteer the patterns of 10 more; 13, 23, 33, and so on. Go on to the other columns.

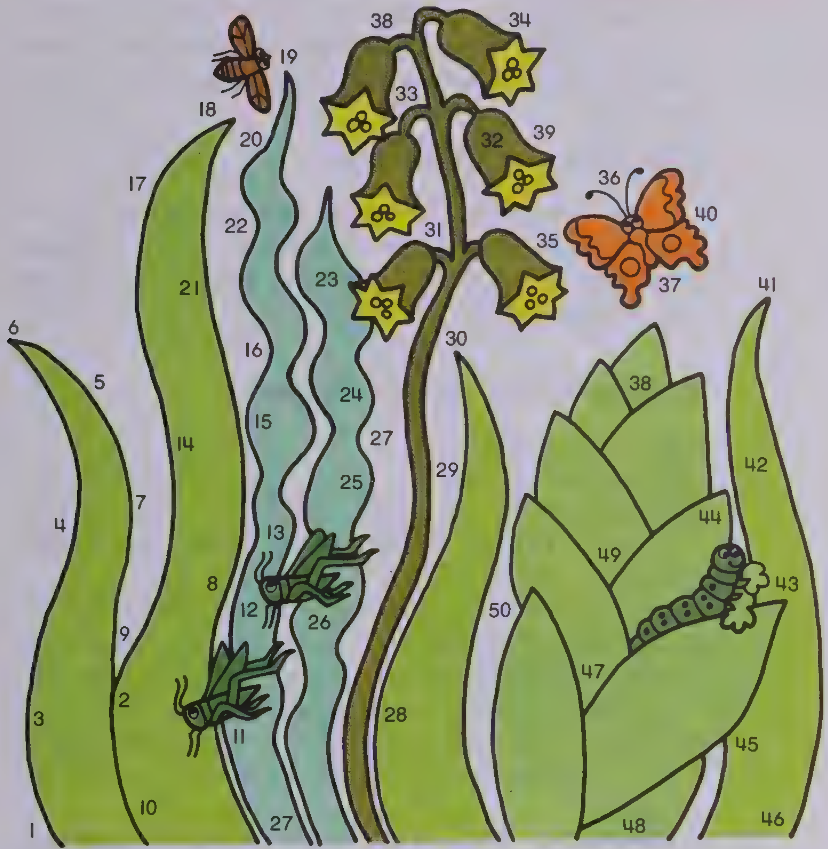
Suggest that the class look at the row beginning with 40, and ask what patterns the ones' digits make. Then inquire about the patterns that the tens' digits make in this same row. Go on to the fifties and the sixties, and try to elicit the patterns of both the ones and tens from the children. If time permits, include an oral number detective game. Start the game by telling the children that you will begin counting, and when you stop, you will call on someone to continue the sequence. That child will stop counting when you give a signal and call on someone

Let's have fun



Find the mystery numbers. Each one is in the picture twice.

Mystery numbers are 27 and 38.



Teaching
Page c-26

It is very important that children realize that this is *not* a dot picture, here they are *not* expected to connect the numerals. Rather within the illustration are numbers from 1 to 50. Two numbers, however, appear twice. Challenge the children to find these mystery numbers which are shown twice on the page. You might suggest to children who seem to be having difficulty that they circle the numbers in order from 1 to 50 so that the uncircled numbers or mystery numbers (27 and 38) will show up more clearly.

else to give the next numbers in the sequence. You may discontinue the series and begin another by saying, "new game," and then continue the game as before.

Follow-up

A chalkboard relay may be used to review what numeral comes before or after a given numeral. On the chalkboard, draw two similar tables with pairs of columns, one for each team. Copy a previously prepared list of two-digit numerals, in random order, in the left column of each table. Write *After* over the second column. Give chalk of the appropriate color to each team leader and remind them that both speed and accuracy are important. Score three points for the team completing the table first and one point for each correct answer.

When they have finished one relay, erase the numerals and directions. This time place two-digit numerals from your list in the second column and write the word *Before* over the first column. Give no points for speed if all groups play, and some children have perceptual or motor handicaps.

RED MODULE, UNIT C

Telling Time

Pages c-27 to c-34

General Objectives

To introduce telling time to the hour

To focus attention on the vocabulary of telling time

To introduce telling time to the half hour

To develop understanding of the symbols used in writing time

Teaching Red Module, Unit C

Approximate Time: 4 to 6 days and extended activities

Materials

crayons
demonstration clock
glue
paper fasteners, 1 per child
paper plates, 1 per child
punchout clock face and hands, available separately to accompany this program
scissors
strips, 1 set per child

Vocabulary

clock
half hour
hour
o'clock

Since you have probably been providing the children with activities for telling time from the beginning of the school year, consider this module as part of an ongoing objective to develop the ability to tell time. Add the activities suggested in this module to your own and continue to intersperse them throughout the remainder of the year.

If you choose, you might change from using the lesson pages in a day-to-day progression and use them periodically throughout the year. The punchout sheet that can be purchased separately provides each child with an individual clock. This can be made more sturdy by gluing it onto cardboard or a paper plate. Other lessons focus attention on recognizing the hour on a clock and also on the written symbol and the vocabulary used to express the hour. Children are then introduced to the half hour. You may wish to extend this development over a long period of time. A review page is provided at the end of the module.

Evaluation of Progress

Keep in mind that complete mastery should not be expected from the children at this time. Many will need continuing activities throughout the year to develop the skill of telling time.

Resources for Active Learning

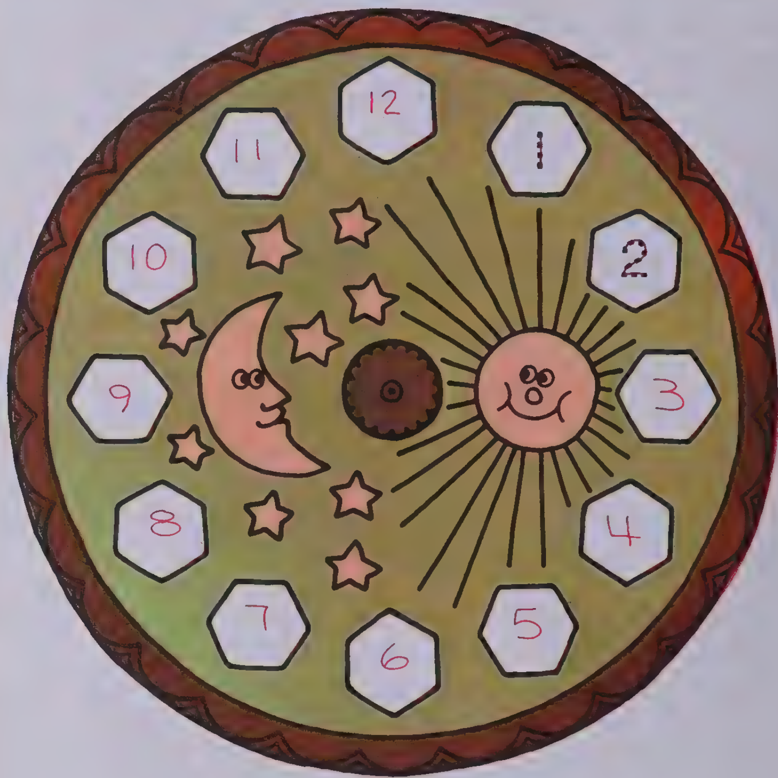
General Activities:
Measurement of time:
ELEMENTARY SCHOOL SCIENCE Primer (TE), "Time," T34–48; Book 1 (TE), "Time," pp. 48–69, Addison-Wesley
EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, pp. 28–29, Herder and Herder
MATHEX: Measurement and Estimation No. 5, pp. 35–36, Encyclopaedia Britannica Publications Ltd.
Nuffield Project: BEGINNINGS 1, pp. 92–96; COMPUTATION AND STRUCTURE 2, pp. 82–86, Wiley
WORKJOBS, "Days of the Week," pp. 74–75, Addison-Wesley
Manipulative Devices:
Egg timers (dime store)
(Racemaster) stopwatch (Edmund Scientific; Math Media)

Investigation
Page c-27

An important part of this investigation is putting together the punchout clock. Help the children attach the two hands onto the clock face with a paper fastener. To make the punchout clocks sturdier, supply each child with a paper plate and glue. Guide them in placing their clocks on the centre of the plate. If glue is not placed near the centre fastener, the fastener need not be punched through the plate. If the punchout clocks are not available, provide the children with materials and guide them in making clock faces with moveable hands. When the children have assembled their clocks introduce pages c-27 and c-28. (You might choose to discuss page c-28 before page c-27.) Ask the children to study the faces of their clocks and complete the clock face on page c-27 similarly by putting the numerals around the edge in the spaces indicated. Then ask them to show a time, such as 12 o'clock, or 4 o'clock, on their clocks. Help them with this by showing the time on a demonstration clock. Then ask them to draw the hands of the clock on page c-27 to show this time. You might also give the children other times to show on their clocks. Their responses to directions such as these will help you evaluate the children's familiarity with telling time.

Let's do

Put numerals on the clock face.



Purpose

To provide interesting activities to motivate learning to tell time

Although the skill of telling time is introduced in this particular module, this skill should be kept in mind as an ongoing objective throughout the year. Various opportunities (such as the time of arithmetic, the time of recess, or lunch, the time of dismissal, and so on) should be used to point out the time of day to the children.

Preparation

Materials

glue (optional)
paper fasteners, 1 per child
piece of cardboard or paper plate for each child (optional)
punchout clock face and hands or materials suitable for the children to make these

Use a large demonstration clock to show the children how a clock functions and how its hands tell the time. (Some clocks are made for children to manipulate. These have plastic gears and show the functioning of both hands very clearly.) Point out the two hands and explain their movement in relation to the passing of time. Emphasize that the shorter, thicker, hour hand moves from one hour to the next while, in the same time, the longer, thinner, minute hand makes one complete sweep around the clock from 12 to 12 each hour. Show various hours on the clock and stress that *o'clock* means *according to the clock*. Explain that we use this word so that everyone will know when we are speaking or writing about time.

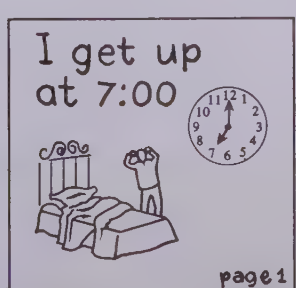
Let's talk

Discussion
Page c-28

Encourage children to look at each picture and talk about it in relation to the various times of the day. Although specific hours need not be determined, elicit from the children the approximate time of the day when they get up, when they might be at the playground or in the school cafeteria and so on. Some of the pictures, such as the playground scene, may suggest several different times of the day. As different times of the day are discussed, show on the demonstration clock a reasonable hour for the activity pictured and ask the children to show this hour on their own clocks. Point out the relative positions of the hour and minute hands. Explain that the shorter, or hour hand, points to the hour while the minute hand is on twelve. Throughout the discussion, use the phrase "o'clock" as you talk about certain hours.

Follow-up

The children might enjoy making their own "time booklets." Staple four or five sheets of A-3 (297 mm × 420 mm) newsprint together in an A-4 (210 mm × 297 mm) booklet. Suggest that the children draw pictures of their daily activities and of clocks showing the times.



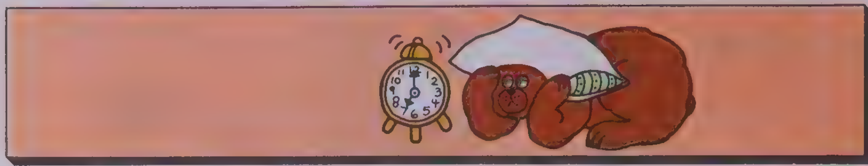
Resources for Active Learning

EXPLORATION OF SPACE AND PRACTICAL MEASUREMENT, "Games . . . Time," pp. 70–76, Herder and Herder

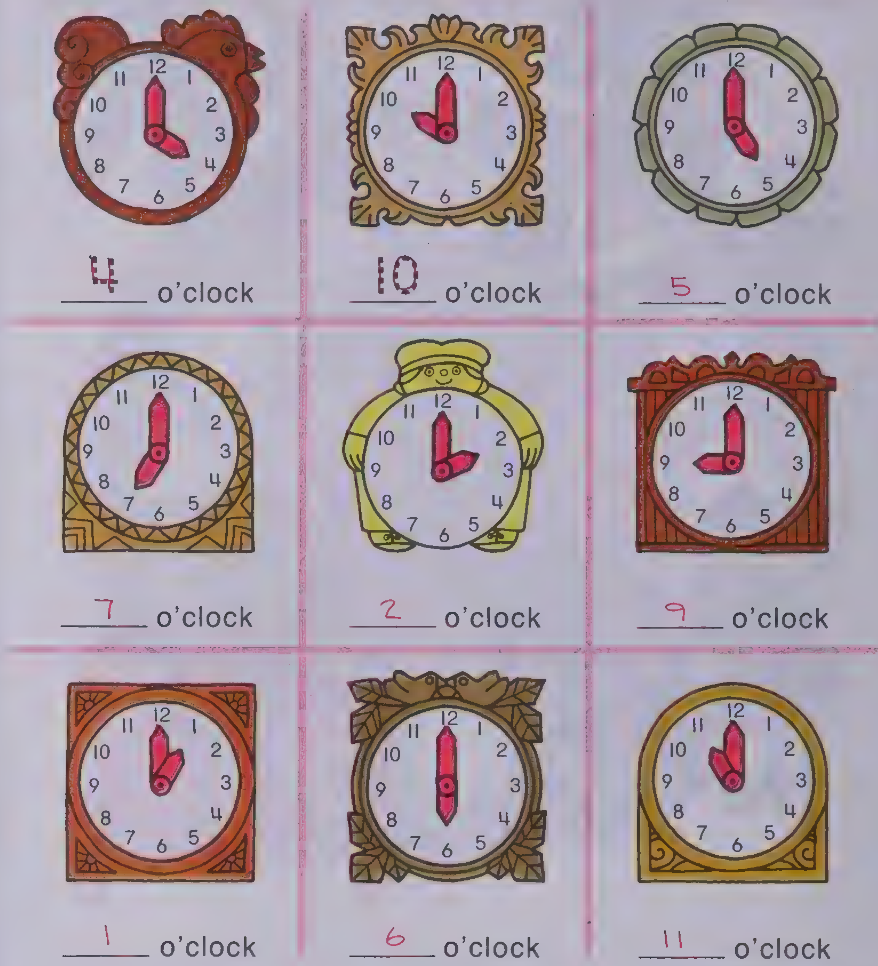
Teaching
Page c-29

Call the children's attention to the clock faces. Ask them to explain the answer given beneath the first clock by telling which hand is on the 12 and which hand is on the 4. Stress again that *4 o'clock* means that according to the clock, the time is 4. Ask them to trace over the dashed 4.

Continue similarly with the next clock. Ask the children to trace the dashed 10 when all agree that 10 o'clock is correct because the hour hand is on 10 and the minute hand is on 12. Instruct them to complete the page independently by looking at each clock carefully, deciding what time it shows, and writing the hour in the answer blank.



Give the correct time.



Telling time

Objective

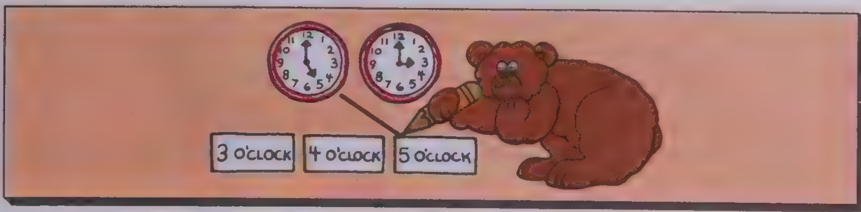
Given the face of a clock whose hands are shown at a particular hour, the child will be able to tell what time the clock pictures by using the phrase "______ o'clock."

Pre-Book Activity

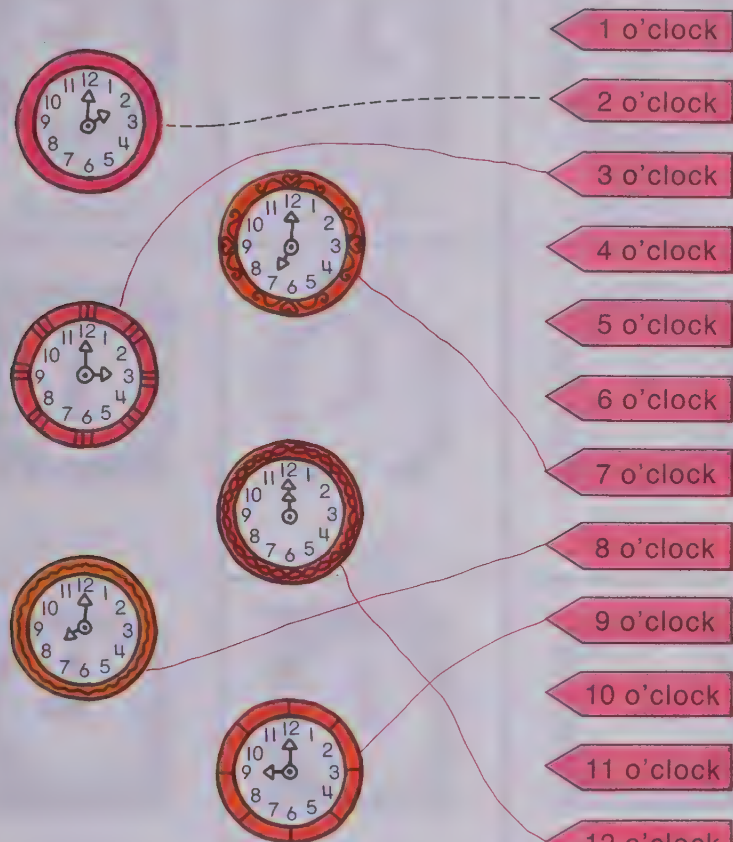
Materials

assembled clocks

Give the children various hours to show on their clocks. Then have them hold up the clocks so you can see them. During this activity continue to use the phrase "o'clock." For example, ask them to show "six o'clock," or "two o'clock," and so on. As they all show the hour on their individual clocks, ask a child to show this time on the demonstration clock. When all the children have agreed that the time on the demonstration clock is correct, have them check the position of the hands on their own clocks.



Match each clock with the correct time.



Teaching
Page c·30

Explain to the children that they are to match each clock face with the sign showing the correct time. You might use the illustration to start this activity. Ask a child to tell what time the first clock shows. When he answers "two o'clock," direct the children to trace over the dashed line from the first clock-face to the words *2 o'clock*.

Direct the children to complete the page. Help anyone who does not understand what to do.

Follow-up

Suggest that the children play a game using the clocks they made. You might draw a clock face on the board showing the hour for recess – 10:00 o'clock, for example. Instruct the children to set their individual clocks to look like the clock you drew on the board. Tell them to watch the clock in the classroom, and when it looks like their clocks, to clear their desks quietly and get ready for recess. Or you can use 2 o'clock for recess, 12 o'clock for lunch, 11 o'clock for art, and so on. This kind of activity, used frequently throughout the week, provides practical motivation for learning to tell time.

Resources for Active Learning

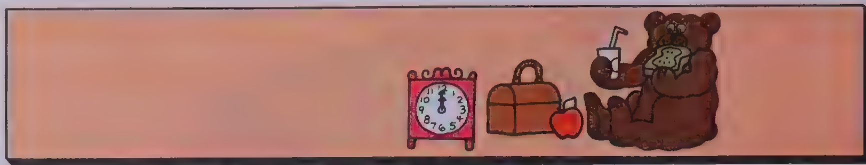
WORKJOBS, "Time," pp. 228–229, Addison-Wesley

Teaching
Page c-31

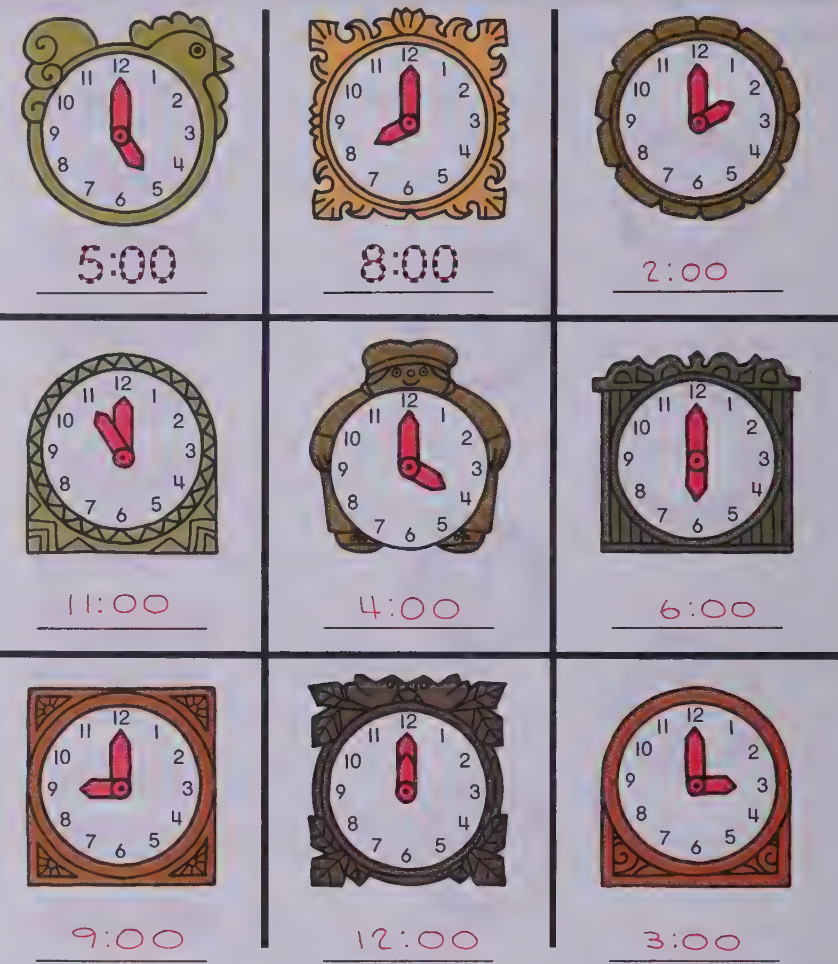
Read the directions with the children. Ask them to explain the answers in the sample problems by describing the positions of the minute and hour hands.

Direct them to trace over the dashed numerals and complete the page by using the short way to write the time under each clock.

Many children will need continuing activities to become familiar with interpreting half hour time. Teach half hour and quarter hour times when appropriate throughout the year.



Give the correct time.



Objective

Given the face of a clock whose hands are shown at a particular hour, the child will be able to express what time the clock pictures by using the conventional symbol for telling time such as, 8:00.

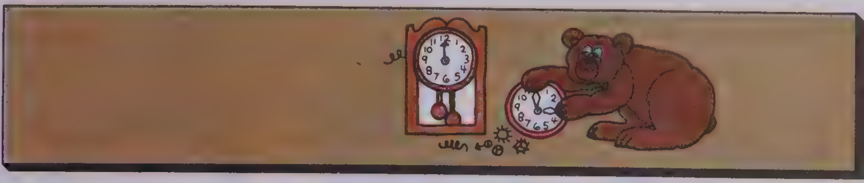
Pre-Book Activity

Divide the class into two evenly matched teams, A and B. Show a demonstration clock to the first child on Team A. If he can tell the class what time the clock shows, he scores a point for his team. If he cannot, let the first member of Team B tell the time. If he is correct, his team wins two points. To continue the game, reset the clock. Encourage the children to say o'clock to show that they are reading time from a clock.

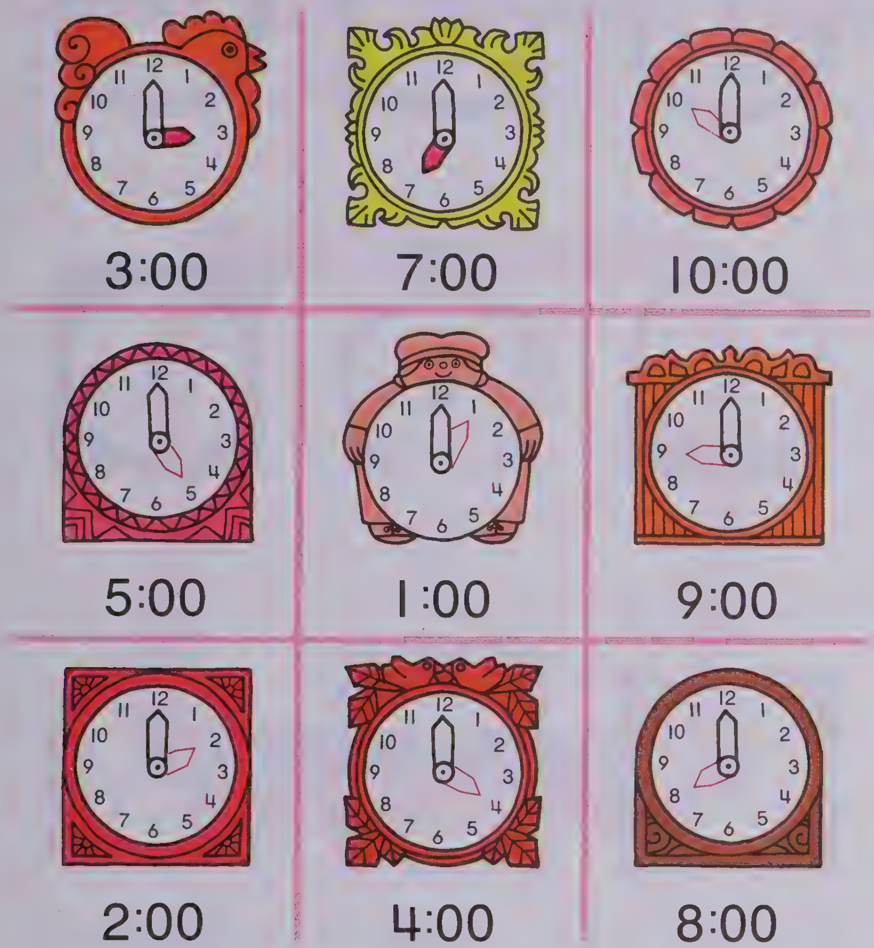
After reviewing time to the hour, show 4 o'clock on your large clock. Write 4:00 on the board as a short way to write 4 o'clock. Explain that the two zeros show that the minute hand is exactly on the 12 and that the hour hand points to the 4. The minute hand moves away from 12 for each minute after the hour. When the hand gets halfway around the clock, the time will be 4:30 (four-thirty) and 30 minutes will have passed since 4:00. In another 30 minutes, the hand will be at the 5, and 60 minutes or 1 hour will have passed since 4:00, and the time will be 5:00.

Follow-up

Take advantage of the many opportunities that arise to have the children practice telling time. For example, to combine telling time with maintaining skill in using



Put the hour hand on each clock.



Teaching
Page c-32

On this page, the children are to put the hour hand on each clock to show the time indicated below it. Call their attention to the first clock and say, "What time should this clock show?" When they answer "three o'clock," direct them to color over the hour hand to show that it points to 3.

Continue with the second example and direct the children to color over the hour hand pointing to the seven. Again emphasize that the hour hand points to the hour and that it is shorter and wider than the minute hand. Instruct the children to finish the remaining rows.

Note: Consider the problem as correct if the children place the hour hand directly on the numeral of the specified hour or any place halfway between it and the next numeral.

ordinals, give the children newsprint and instruct them to fold it into eight sections. Tell them to label the sections 1 through 8 and make a clockface in each section by tracing around a jar lid or cardboard circle. Next help them fill in the numerals. Start with 12, then adding 6, 3, and 9. The children can then follow directions such as these:

1) Third clock, show 10:00.
2) Fourth clock, show your bedtime.
3) Fifth clock, show TV time.

Teaching
Page c-33

If you think children have developed a sufficient grasp of this material, you will probably want to use this page as an evaluative instrument and assign it for children to do on their own. With some children you may choose to use it simply as review.

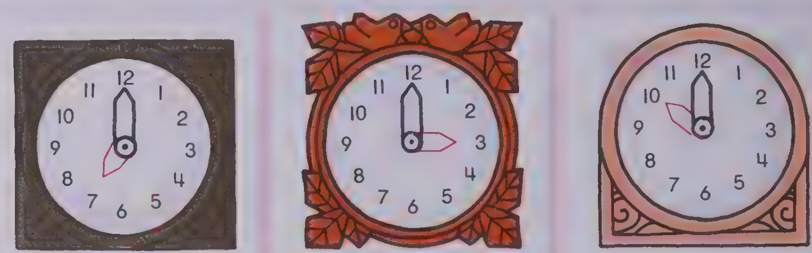
Call attention to the first frame. Have someone explain what time the clock shows and how he is able to tell. Remind the children that the shorter hand is the hour hand and when the longer or minute hand is on twelve the hour may be found by looking at the shorter hand.

Point out that in the bottom three frames they are expected to draw the shorter or hour hand to show the time indicated. When children are finished, you might want to have children use the demonstration clock to show the times of the first six clocks.

Show you know

Give the time.

Put the hour hand on each clock.



7:00 3:00 10:00

Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Since the ability to tell time is an ongoing objective for the entire year, do not limit your evaluation of a child's ability to his performance on page c-33. Telling time activities should be continued throughout the remainder of the school year. Page c-34 provides a change of pace page.

Pre-Book Activity

Materials

demonstration clock *crayons* *strips*

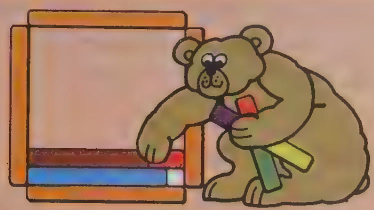
Use a demonstration clock to review telling time. For example, set the clock at 7:00. Ask someone to write the symbol for this hour on the chalkboard. Then encourage the children to tell about something they might be doing when the clock reads 7:00. Some may suggest getting up in the morning; others may speak of watching TV in the evening. Continue with other times; occasionally include settings half past the hour.

Follow-up

The worksheet in the next column may be used as enrichment depending upon the understanding of the children.

Answers will vary. An example is given.

Let's have fun



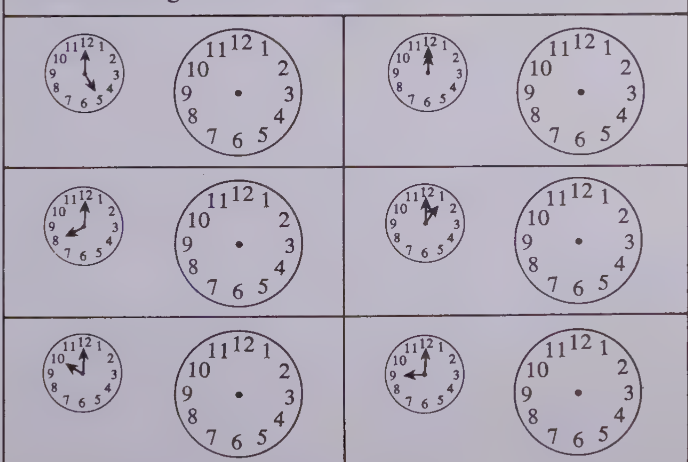
Fill the ten-square with as many different pairs of strips as you can. Color to show how you did it.

Readiness for combinations of 10

Teaching
Page c-34

Since this page is a change of pace page, children should be given much freedom in doing it. Read the directions at the top and make sure children know what they are to do. Distribute a set of strips to each child. Note that the pair light green and black is not considered the same as the pair black and light green. You might mention that they could try to form stairsteps with the strips, or begin with their favorite pair at the top. As they complete the square, children will discover that they need a repetition of one pair. Be sure they realize that this is acceptable although only one pair should be repeated. You might ask the children why this square is called a "ten-square."

Make the big clock show one hour after the little clock.



Children might also enjoy a game called "What's My Time." A child should set a time on his clock, hold it up for all to see, and ask for someone to give the time shown. Such an activity might involve a small group of four or five, or the whole class.

Resources for Active Learning

DEVELOPMENTAL MATH CARDS, "Clockline," C^{2}1, Addison-Wesley

LIGHT GREEN MODULE, UNIT C

Review of Sums and Differences to 9

Pages c-35 to c-44

General Objectives

To review the concepts of addition and subtraction

To increase skill in adding and subtracting combinations for sums equal to, or less than, 9

To prepare for work with missing addends and larger sums and differences

To encourage mastery of addition and subtraction facts for sums of 9 or less

No new mathematical concepts are introduced in this unit. Rather, the purpose of this module is to maintain and strengthen the mathematical ideas developed so far. Children should be encouraged to master the addition and subtraction facts to 9. However, it is most important that they understand the meaning of addition and subtraction so that even if they forget the basic fact, they can figure it out.

Teaching Light Green Module, Unit C

Approximate Time: 5 to 7 days

MATERIALS

centimetre rulers
counters, or other objects for sets for each child
number line for demonstration
sets of objects (pencils, paper cups, pipe cleaners, etc.) for demonstration
strips

The use of strips, objects, or counters for sets, and number lines, should accompany most of the lessons. On page c-37, children are encouraged to use centimetre rulers with the strips in order to build trains for addition. Depending on the extent to which you have used the strips in follow-up activities, you might want to encourage children to use them more freely during the study of this module. Resources for suggested activities are given in the Resources for Active Learning.

EVALUATION OF PROGRESS

A child's mastery of facts can often best be evaluated by his responses in an oral interview. During the study of the module, give individual children various sums and differences to find. This method will help children identify those facts which cause them difficulty. It will also enable you to diagnose any child who does not yet understand addition or subtraction. For example, if you ask a child: "What is $8 - 5$?" and he gives an incorrect response, ask him to use strips, counters, or a number line to figure it out. A child who cannot figure out such a problem is not ready to learn combinations for 10 or higher, or to understand missing addends. He needs much more work with concrete situations for subtraction and addition.

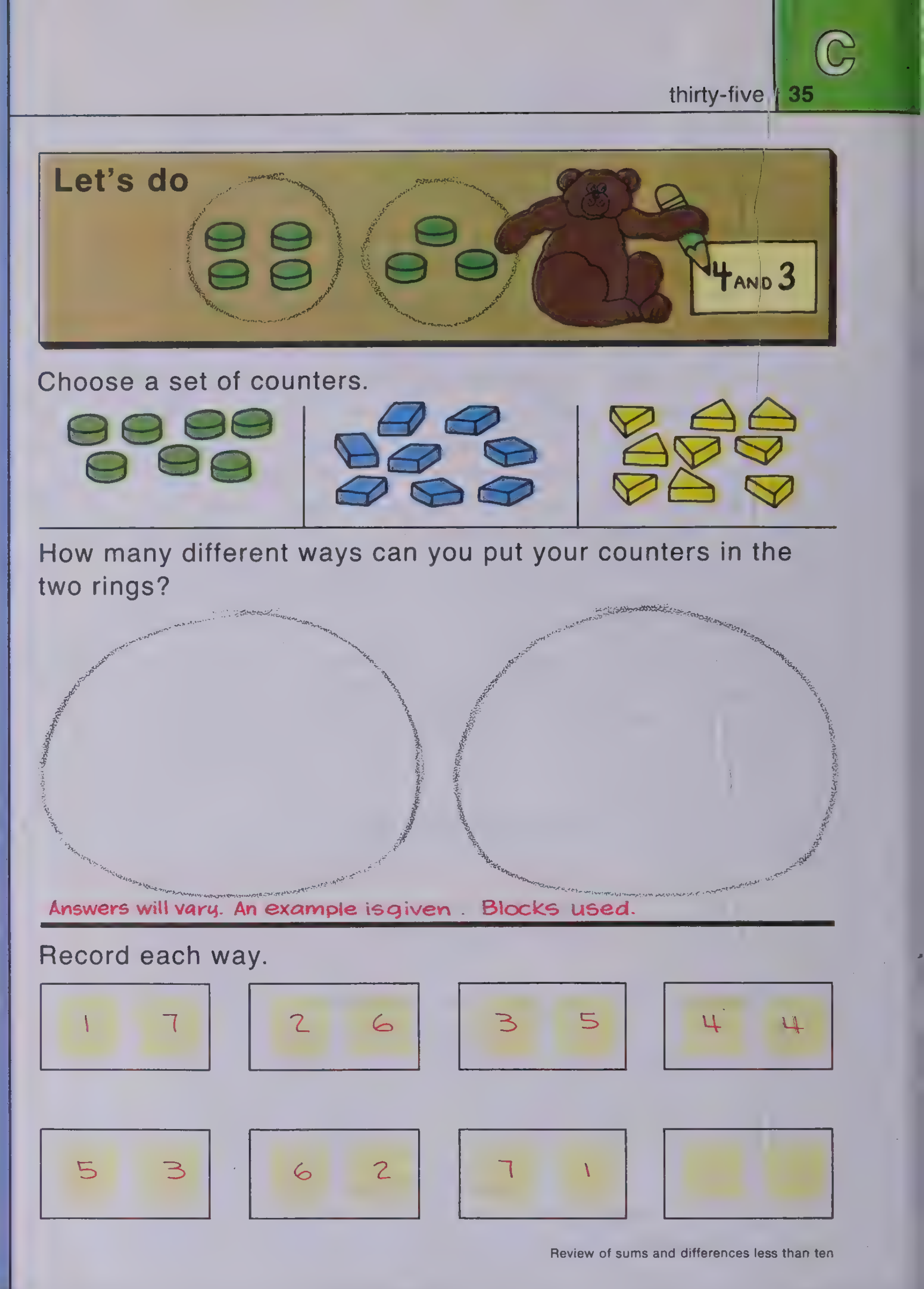
RESOURCES FOR ACTIVE LEARNING

General Activities:
Games that provide mastery of facts:
MATH ACTIVITIES, Games 3/31–68, pp. 97–114, Allyn and Bacon
MATHEX: Operations No. 3, pp. 8–9, Encyclopaedia Britannica Publications Ltd.
MATH ACTIVITIES, Activities 3/1, 2, pp. 87–88, Allyn and Bacon
Commerical Games:
Games for practicing basic facts:
Arithmecubes (Scott Foresman)
Cover-Up (Selective Educational Equipment)
Equations (Creative Publications, Wff 'N Proof)
Orbiting the Earth, Addition and Subtraction (Scott Foresman)
Sum Fun (Ideal; school supplier)
Twin-Choice (Holt, Rinehart and Winston)

Investigation
Page c-35

Be sure that every child has at least 9 counters. Then call the children's attention to the sets of circles, squares, and triangles on page c-35. Explain that they are to choose one of these sets and match it with their counters. Thus, if they choose the set of yellow squares, they will be working with 8 of their counters. When they have chosen their set, explain that they should see how many ways they can put these counters into the two rings, that is, into two groups. Point out the boxes in which they can record their sets. When the recording has been completed, ask a child to describe the number in his set and show his groupings on the chalkboard. For example, for the set of 8, a child might simply write the pairs:

8,0	5,3
1,7	6,2
·2,6	7,1
3,5	0,8
4,4	



Purposes

To review the concepts of addition and subtraction
To review addition and subtraction facts of 9 or less

Preparation

Materials

counters, about 9 per child

Ask 5 children to come to the centre of the room and group themselves in two groups. When they have done this, use a chart on the chalkboard to show the way they put 5 into two groups. Then ask them to show another way of forming two groups and again record the number in each group. Continue until your chart is similar to the following:

5	
2	3
0	5
1	4
3	2
4	1
5	0

Note that the last three pairs are simply the first three in different order. You might let the children decide if these last three should be recorded themselves or just understood from the first three. The important point is for children to see that there is more than one way in which this set of 5 can be put into two groups. They should also note how these groupings can be recorded.

Let's talk

Review of sums and differences less than 10

Discussion
Page c-36

Give children an opportunity to talk about the dart board. Elicit from them different ways to get certain scores less than ten. For example, if one dart lands on 3 and one on 6, the score is 9. This same score may be found by other combinations, such as, 2 and 7, or 5 and 4. If children suggest combinations of 10 or above, include them in the discussion but do not emphasize. Concentrate on combinations of 5, 6, 7, 8 and 9.

Follow-up

Keeping records of facts learned becomes enjoyable if each child tries to better his own record. Make a chart with each child's name on it. Announce, periodically, that this is Flash-card Day. Pair the children off. For the first trial, confine the practice to addition facts. One member of each pair should show his partner a flash card. If the partner knows the answer, he should put the card in one stack. If he does not know it, or has to make several guesses, he should put the card in a separate stack. When the pair has gone through the stack of cards, those answered correctly should be counted and the number recorded on the chart under the proper date. Then the roles are reversed, and the other member of the pair gets a chance. Again, the cards answered correctly should be counted and recorded on the chart.

Continue this activity throughout the year, alternating addition with subtraction facts and then mixing both. If the group is advanced, add the facts for sums from 11 to 18 later on, as they are developed.

Name	1/6/72	1/25/72							
Bobby A.	12	17							
Jackie C.	15	18							

Resources for Active Learning

More ideas for graphing:
DEVELOPMENTAL MATH CARDS, B²5, B¹7, 18, Addison-Wesley

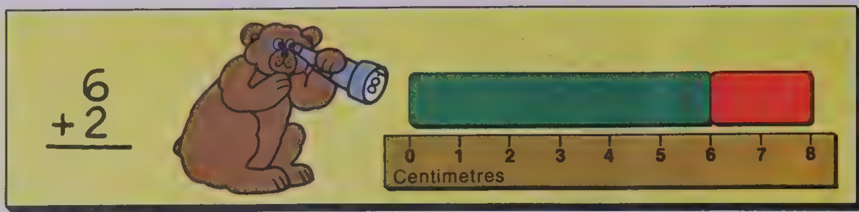
Teaching
Page c-37

As children use this page they should have access to centimetre rulers, and for those who wish, counters, and number lines should be available. Each child should also have his set of strips. However, those who are able should try to find the sums without using concrete materials.

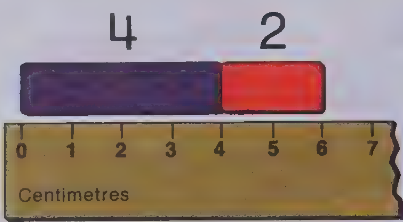
Call attention to the frames at the top of the page. Have a child explain how each picture of the strips and the ruler helps to solve the equation shown below the ruler. Encourage children to build trains on the ruler with their strips as they solve the equations. Also review how addition exercises which are written in vertical or up and down notation, may be read in the same way they are read in an equation. Thus for

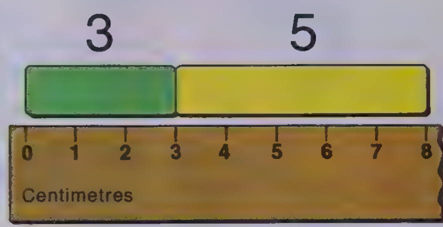
$$\begin{array}{r} 5 \\ +1 \\ \hline \end{array}$$

they simply think: "Five plus one is . . . ."



Find the sums.





$4 + 2 = \boxed{6}$ $\qquad$ $3 + 5 = \boxed{8}$

$3 + 2 = \boxed{5}$ $\qquad$ $4 + 0 = \boxed{4}$

$3 + 3 = \boxed{6}$ $\qquad$ $2 + 5 = \boxed{7}$

$5 + 4 = \boxed{9}$ $\qquad$ $2 + 7 = \boxed{9}$

$$\begin{array}{r} 5 \\ +1 \\ \hline 6 \end{array} \quad \begin{array}{r} 5 \\ +3 \\ \hline 8 \end{array} \quad \begin{array}{r} 0 \\ +5 \\ \hline 5 \end{array} \quad \begin{array}{r} 3 \\ +6 \\ \hline 9 \end{array} \quad \begin{array}{r} 1 \\ +6 \\ \hline 7 \end{array} \quad \begin{array}{r} 2 \\ +6 \\ \hline 8 \end{array}$$

Objective

Given addition equations for sums of 9 or less, the child will be able to find the sum.

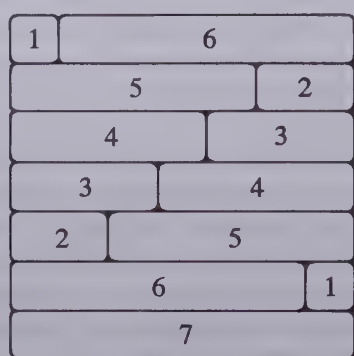
Pre-Book Activity

Materials

centimetre rulers
strips

Be sure each child has a complete set of strips. After a brief time for preliminary free play with the strips, explain to the children that you would like them to try to build a set of number trains to match the 5, 6, 7, 8, or 9 strip. For example, take the 7-strip and ask the children how many different trains of two strips they can find.

You may or may not choose to use trains of reverse order such as, | 3 | 4 | and | 4 | 3 |. Encourage children to build as many sets of trains as time permits. The following is a possible arrangement for 7.



Find the sums.

$4 + 3 = \boxed{7}$	$4 + 1 = \boxed{5}$
$2 + 3 = \boxed{5}$	$8 + 1 = \boxed{9}$
$6 + 2 = \boxed{8}$	$3 + 3 = \boxed{6}$
$0 + 3 = \boxed{3}$	$2 + 2 = \boxed{4}$
$5 + 4 = \boxed{9}$	$5 + 2 = \boxed{7}$
$4 + 2 = \boxed{6}$	$4 + 4 = \boxed{8}$

$\begin{array}{r} 4 \\ +5 \\ \hline 9 \end{array}$	$\begin{array}{r} 2 \\ +4 \\ \hline 6 \end{array}$	$\begin{array}{r} 2 \\ +6 \\ \hline 8 \end{array}$	$\begin{array}{r} 2 \\ +1 \\ \hline 3 \end{array}$	$\begin{array}{r} 6 \\ +1 \\ \hline 7 \end{array}$	$\begin{array}{r} 3 \\ +2 \\ \hline 5 \end{array}$
$\begin{array}{r} 3 \\ +1 \\ \hline 4 \end{array}$	$\begin{array}{r} 6 \\ +3 \\ \hline 9 \end{array}$	$\begin{array}{r} 1 \\ +4 \\ \hline 5 \end{array}$	$\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$	$\begin{array}{r} 3 \\ +5 \\ \hline 8 \end{array}$	$\begin{array}{r} 1 \\ +5 \\ \hline 6 \end{array}$

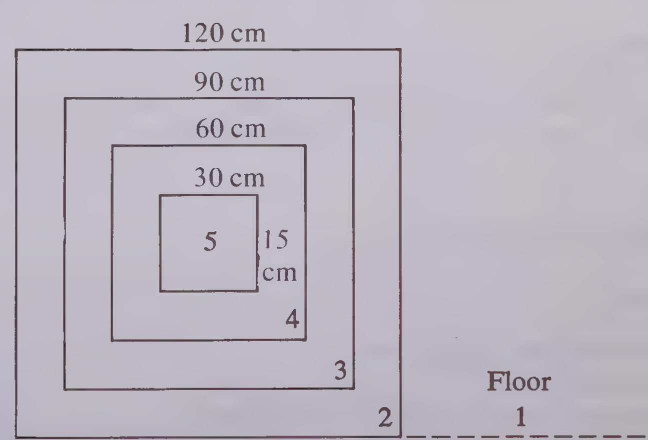
Sums less than 10-number line

Teaching
Page c-38

Observe with the children that the blank number line at the top of the page is provided for their use as they try to find the sums. If children prefer another material, such as counters or strips, encourage them to use the aid they prefer. However, encourage those who are able to work without these materials.

Follow-up

Prepare a target board for a bean-bag game. If possible have two or three target boards and two bean-bags for each board. Number the target board from 1 to 5 to avoid combinations of 10 or above. (Most children will have no difficulty with 5 and 5.) Divide the class into an appropriate number of teams. Have each person record his own score for his two throws, but encourage all the team members to check each score. On the chalkboard, give each team a space for recording their scores. When all members have had a turn, you might find the grand total of each team's scores to see which team has the highest score. Otherwise, simply declare each child who had the highest score in a round, the winner. In the next round, you might declare that the lowest score wins.

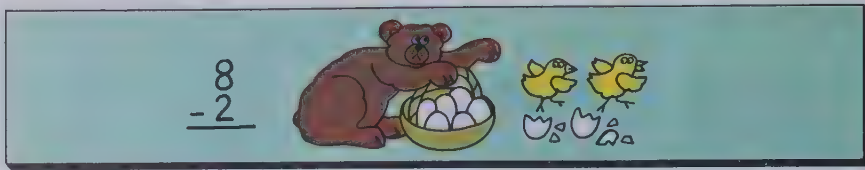


A throwing line approximately 1.5 metres from the target should also be marked off with chalk or tape. Notice that in this target, 1 point is allowed if the bean-bag lands off the target in back of the dashed line.

Teaching
Page c-39

Ask the children to describe each of the sets in the top frames and tell how they fit the equations. Be sure they discuss that in each picture some number of animals appears to be leaving the whole group.

Explain that they might solve the equations with or without concrete materials. Counters, strips, centimetre rulers, and number lines should all be available; but encourage those who are able, to gradually work the exercises without dependence on the materials. You might suggest to many that they first try the exercises without any materials and then use materials to check those they were not sure of.



Find the differences.

$6 - 2 = \boxed{4}$ $7 - 4 = \boxed{3}$

$8 - 5 = \boxed{3}$ $9 - 6 = \boxed{3}$

$7 - 2 = \boxed{5}$ $7 - 0 = \boxed{7}$

$9 - 2 = \boxed{7}$ $6 - 4 = \boxed{2}$

$6 - 5 = \boxed{1}$ $9 - 4 = \boxed{5}$

$7 - 6 = \boxed{1}$ $8 - 3 = \boxed{5}$

8	5	7	9	8	6
−4	−3	−4	−3	−2	−3
4	2	3	6	6	3

Differences related to sums less than 10

Objective

Given subtraction equations related to sums of 9 or less, the child will be able to find the differences.

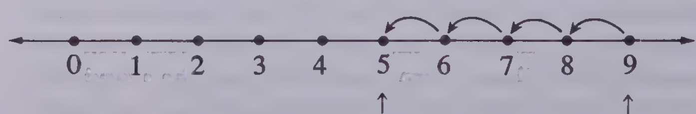
Pre-Book Activity

Materials

demonstration number line
flannelboard
objects or counters for set demonstrations

Display the set materials and ask children to suggest ways of showing subtraction. For example, one child may circle 8 felt objects with yarn on the flannelboard and then remove 2 of the objects from the inside to the outside of the yarn. Another child might pass out 7 sticks and then take 3 of those away from 3 of the children holding them. Also encourage children to use the demonstration number line to show subtraction. For this you might suggest an equation and ask for a volunteer to show it on the number line.

$9 - 4 = \square$



$9 - 4 = 5$

Find the differences.

$4 - 1 = \boxed{3}$ $5 - 2 = \boxed{3}$

$7 - 3 = \boxed{4}$ $9 - 6 = \boxed{3}$

$9 - 3 = \boxed{6}$ $3 - 2 = \boxed{1}$

$5 - 4 = \boxed{1}$ $6 - 5 = \boxed{1}$

$8 - 4 = \boxed{4}$ $8 - 2 = \boxed{6}$

$6 - 2 = \boxed{4}$ $7 - 6 = \boxed{1}$

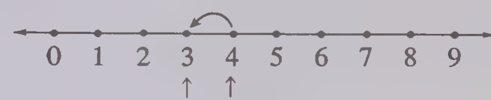
2	8	6	5	7	4
−2	−3	−4	−3	−5	−3
0	5	2	2	2	1

9	6	5	9	7	8
−7	−3	−1	−8	−4	−5
2	3	4	1	3	3

Differences related to sums less than 10-number line

Teaching
Page c-40

Since the format of this page is similar to that of c-38, children should have no difficulty in understanding what to do. However, you might want to show one or two examples of subtraction on the number line. Remind the children that they begin at the number given first in the equation and then count to the left as many places as the second number in the equation indicates.



$4 - 1 = \square$

Follow-up

To challenge more capable children, you might suggest that they try to figure out how to use the strips for subtraction. Encourage creative thinking. Examples are given below. However, if children do not think of these examples, do not teach them.

$8 - 5 = \square$

1) Show the eight-strip, line up 8 one-strips beneath it. Now take 5 away. The 3 remaining one-strips show the difference.
2) Show the eight-strip. Line up a five-strip beneath it. Find the strip which is needed to make a train with the five-strip that is as long as the eight-strip.

Teaching
Page c-41

Point out to the children that the problems on this page are both addition *and* subtraction problems. Encourage them to do the page independently, using materials if they want, but seeing how much they can do without concrete objects.

Find the sums and differences.

$4 + 2 = \boxed{6}$	$7 - 2 = \boxed{5}$
$2 + 3 = \boxed{5}$	$5 - 4 = \boxed{1}$
$4 + 4 = \boxed{8}$	$8 - 6 = \boxed{2}$
$3 + 6 = \boxed{9}$	$4 - 3 = \boxed{1}$
$4 + 3 = \boxed{7}$	$9 - 5 = \boxed{4}$
$2 + 2 = \boxed{4}$	$6 - 1 = \boxed{5}$

$\begin{array}{r} 2 \\ +6 \\ \hline 8 \end{array}$	$\begin{array}{r} 4 \\ +1 \\ \hline 5 \end{array}$	$\begin{array}{r} 1 \\ +3 \\ \hline 4 \end{array}$	$\begin{array}{r} 3 \\ +3 \\ \hline 6 \end{array}$	$\begin{array}{r} 7 \\ +2 \\ \hline 9 \end{array}$	$\begin{array}{r} 2 \\ +5 \\ \hline 7 \end{array}$
$\begin{array}{r} 4 \\ -2 \\ \hline 2 \end{array}$	$\begin{array}{r} 8 \\ -5 \\ \hline 3 \end{array}$	$\begin{array}{r} 6 \\ -4 \\ \hline 2 \end{array}$	$\begin{array}{r} 7 \\ -4 \\ \hline 3 \end{array}$	$\begin{array}{r} 5 \\ -3 \\ \hline 2 \end{array}$	$\begin{array}{r} 9 \\ -8 \\ \hline 1 \end{array}$

Review addition and subtraction

Objective

Given addition and subtraction equations for sums of 9 or less, the child will be able to find the sums and differences.

After the review of the previous pages, it would be appropriate to begin stressing the mastery of basic facts.

Pre-Book Activity

Oral problems given in sequence stimulate children to work through the facts mentally. Say, for example, "Five plus one [pause] plus three." Call on a child for the answer. Go on with another problem: "Nine minus five [pause] plus two." Work up to problems having three or four steps, but keep the speed of the problem within the capability of the children. For the present, keep the sums under 9.

Another oral review game is called "Cross Over the Bridge." Direct two teams of children to line up on opposite sides of the room. Suggest that the children imagine a bridge across the open space at the front of the room. The first two members of each team compete against each other in answering flash cards showing addition and subtraction combinations to 9. When a flash card is shown, the first one to give a correct answer may stay on his side of the river. The other person must "cross over the bridge" and go to the end of the line on the other team. At the end of the game, the team with the most members is the winner.

Find the sums and differences.

$2 + 4 = \boxed{6}$	$8 - 2 = \boxed{6}$
$7 - 6 = \boxed{1}$	$3 + 4 = \boxed{7}$
$2 + 7 = \boxed{9}$	$9 - 3 = \boxed{6}$
$5 + 3 = \boxed{8}$	$4 - 0 = \boxed{4}$
$5 - 4 = \boxed{1}$	$3 + 3 = \boxed{6}$
$2 + 2 = \boxed{4}$	$6 - 3 = \boxed{3}$

$\begin{array}{r} 5 \\ +2 \\ \hline 7 \end{array}$ $\begin{array}{r} 8 \\ -6 \\ \hline 2 \end{array}$ $\begin{array}{r} 6 \\ -4 \\ \hline 2 \end{array}$ $\begin{array}{r} 7 \\ -4 \\ \hline 3 \end{array}$ $\begin{array}{r} 2 \\ +6 \\ \hline 8 \end{array}$ $\begin{array}{r} 5 \\ -2 \\ \hline 3 \end{array}$

$\begin{array}{r} 7 \\ +2 \\ \hline 9 \end{array}$ $\begin{array}{r} 1 \\ +5 \\ \hline 6 \end{array}$ $\begin{array}{r} 9 \\ -5 \\ \hline 4 \end{array}$ $\begin{array}{r} 8 \\ -3 \\ \hline 5 \end{array}$ $\begin{array}{r} 4 \\ +1 \\ \hline 5 \end{array}$ $\begin{array}{r} 4 \\ +5 \\ \hline 9 \end{array}$

$\begin{array}{r} 3 \\ -2 \\ \hline 1 \end{array}$ $\begin{array}{r} 7 \\ -5 \\ \hline 2 \end{array}$ $\begin{array}{r} 4 \\ +4 \\ \hline 8 \end{array}$ $\begin{array}{r} 9 \\ -6 \\ \hline 3 \end{array}$ $\begin{array}{r} 6 \\ -2 \\ \hline 4 \end{array}$ $\begin{array}{r} 6 \\ +1 \\ \hline 7 \end{array}$

Review addition and subtraction

Teaching
Page c-42

Children should know what to do on this page, but be sure they realize that the addition and subtraction problems are mixed together. Stress that they must carefully look for the + or − signs to tell them what to do. Emphasize that it is more important to read the signs correctly and carefully than to rush through the page.

Follow-up

Duplicate tables that have seven rows and seven columns to be used as addition tables, as shown below. Use the overhead projector to help children understand how to use the table. You might use directions such as: "Begin with a number on the left; add a number from the top; place the sum in the box where the row and column of your two numbers meet." Help the children fill in boxes until they understand the procedure. Then ask them to finish the remaining sums on their own.

+	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

When children understand how to use such a table, you might give them tables similar to those below to practice sums of 9 and less.

+	4	7	2	5
1				
7				
4				
6				

+	6	3	2	4
4				
5				
2				
1				

(Shaded boxes should be included. They cover boxes of sums children have not yet studied.)

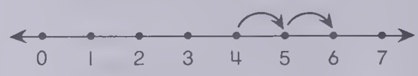
Teaching
Page c-43

Since children have previously worked with the format of each of the frames, ask them to explain what they are expected to do. Again, remind them to watch the signs carefully. You will probably want the children to do all of these exercises independently.

Show you know

Find the sums and differences.

$5 + 3 = \boxed{8}$ $\qquad$ $6 - 4 = \boxed{2}$



$4 + 2 = \boxed{6}$ $\qquad$ $7 - 3 = \boxed{4}$

$5 - 2 = \boxed{3}$ $\qquad$ $3 + 3 = \boxed{6}$

$8 - 6 = \boxed{2}$ $\qquad$ $7 + 2 = \boxed{9}$

$7 - 3 = \boxed{4}$ $\qquad$ $3 + 5 = \boxed{8}$

$\begin{array}{r} 4 \\ +3 \\ \hline 7 \end{array}$ $\quad$ $\begin{array}{r} 1 \\ +4 \\ \hline 5 \end{array}$ $\quad$ $\begin{array}{r} 5 \\ +4 \\ \hline 9 \end{array}$ $\quad$ $\begin{array}{r} 9 \\ -4 \\ \hline 5 \end{array}$ $\quad$ $\begin{array}{r} 8 \\ -3 \\ \hline 5 \end{array}$ $\quad$ $\begin{array}{r} 6 \\ -6 \\ \hline 0 \end{array}$ $\quad$ $\begin{array}{r} 4 \\ +4 \\ \hline 8 \end{array}$

Objective

The child will demonstrate his ability to work with the concepts presented in this module, in particular, he will be able to find the sums and differences of 9 or less.

Although mastery of the facts up to 9 should be considered an ongoing objective extending through the remainder of the year, it is hoped that children have a fairly good mastery at this time. A minimum ability to find the sums or differences of 9 or less should be expected before children are allowed to study missing addends in the blue module. The greater a child's mastery of the facts, of course, the greater will be his skill in thinking of missing addends.

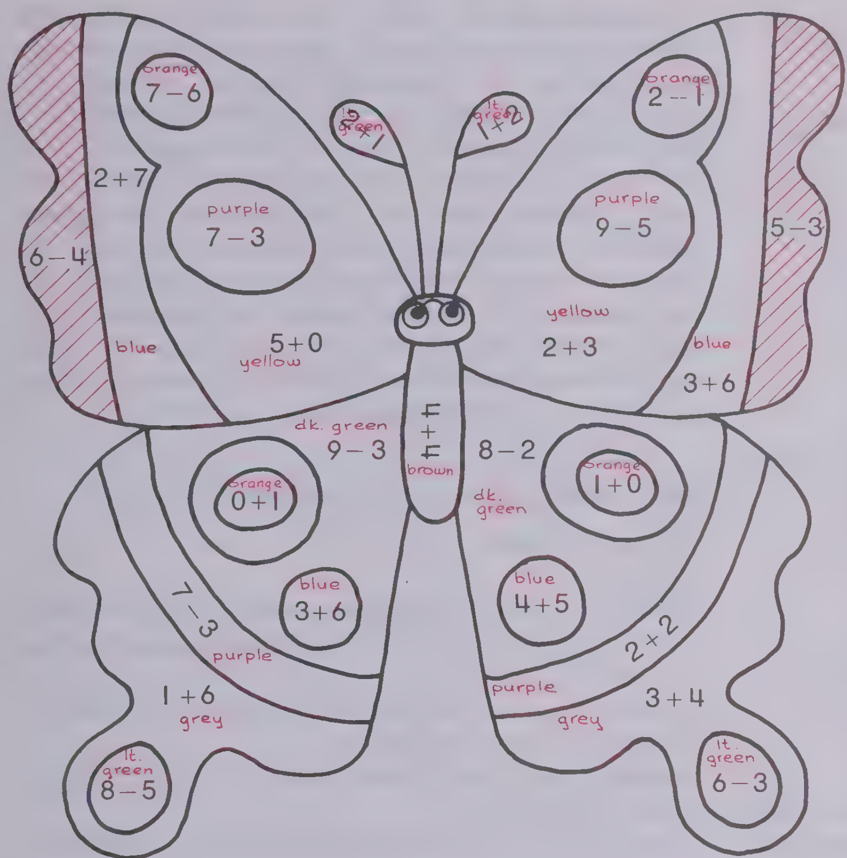
Pre-Book Activity

Draw a long double column on the left-hand side of the chalkboard. At the top of this double column, write "Add 1" and go down the first column, writing the digits 0 through 6 in order. Then (still moving down the column) repeat the digits 0 through 6 at random. Leave the second column blank. Reproduce these columns on the right-hand side of the board.

Next, organize a "Blue" team and a "Green" team. Give the leader of each team the appropriate color chalk. At a signal, the first member of each team should write the answer in the right column, give the chalk to the next child in his line, and go to the end of the line. The game continues until the players reach the end of the columns of figures. Give three points to the team that finishes first and one point for each correct answer. The rest of

Let's have fun

Color by the numbers.



Addition and subtraction combinations

Teaching
Page c-44

Have the children observe that the colors at the top match the colors of their strips. Then point out the combinations which are printed in the various parts of the picture. Explain that they are to figure out the sum or difference in each part and then color that part by matching their sum or difference with the color given at the top. Children will enjoy seeing their finished pictures displayed.

the class may sit as a jury to determine the correct answers.

Add 1	
0	
1	
2	
3	
4	
5	
6	
4	
5	
1	
2	

Follow-up

A simple cross-number puzzle may be used as a review of basic facts:

1	2		3	4	
5				6	
			7		8
9		10			11
	12			13	
14		15		16	

1. $2+3$
2. $5-4$
3. $7+2$
4. $8-5$
5. $5+5$
6. $6-3$
7. $7-4$
8. $9-5$
9. $3+4$
10. $2+5$
11. $7-2$
12. $2+4$
13. $8-4$
14. $5-1$
15. $6-2$
16. $3-1$

DARK GREEN MODULE, UNIT C

Money

Pages c-45 to c-52

General Objectives

To introduce the symbol ¢ for cent

To develop skill in working with collections of pennies, nickels, and dimes

To provide opportunities for children to count coins in buying and selling situations

Teaching Dark Green Module, Unit C

Approximate Time: 4 to 6 days

Materials

real or play coins for each child
large cotton boxes
empty boxes, bottles, and other suitable items to be bought and sold at a "store"
real coins for demonstration and discussion

Vocabulary

cents	nickel
coin	penny
dime	quarter

The goals of this module will best be achieved by providing the child with many actual situations in which he deals with coins. There is no substitute for working with real coins, but since to do so is often not possible, play coins should be used extensively. Many children will already be familiar with coins and their value. However, those children who are not, should not be expected to learn the values of the coins from the printed page. These children, in particular, need many situations which reproduce life situations involving the use of coins. You should teach these children the value of the coins and then provide opportunities for them to work with them. For this reason it is strongly recommended that you build with the children a "play store."

If this module is used after the children have studied place value, you might extend the discussion of coins to include place-value concepts. Suggestions for such an extension of this module are given on page c-52.

Evaluation of Progress

Understanding the value of coins and developing the ability to use them in buying, selling, and making change should be considered ongoing objectives. The progress a child makes in this area is often dependent on many factors outside of his school life. Thus, some children will have an advanced understanding and ability to use money, whereas some may have had very few opportunities to develop this skill up to this point. Page c-51 may give an indication of a child's understanding of the coin values, but a truer evaluation will come from your observation of the children as they participate in situations which simulate actual life use of money, such as buying, selling, and making change at the "store."

Resources for Active Learning

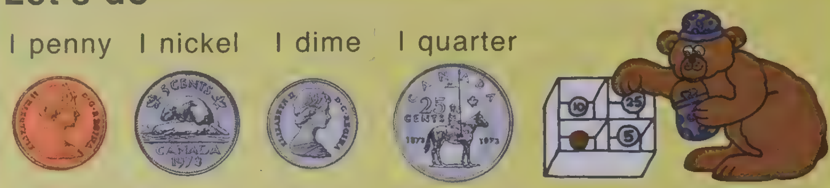
General Activities:
Money:
MATHEX: MEASUREMENT AND ESTIMATION NO. 5, pp. 40–44, Encyclopaedia Britannica Publications Ltd.
Nuffield Project: COMPUTATION AND STRUCTURE 2, pp. 91–102, Wiley
Nuffield Project: BEGINNINGS 1, "Imitative Play," pp. 24–35, Wiley
TEACHING AIDS FOR ELEMENTARY MATHEMATICS, "Toytown," pp. 112–113, Holt, Rinehart and Winston
WORKJOBS, "Piggy Banks," pp. 160–161; "The Store," pp. 226–227, Addison-Wesley
Manipulative Devices:
Store play materials (CCM School Materials; school supplier)
Commercial Games:
Count Your Change Game (Lakeshore; school supplier)

Investigation
Page c-45

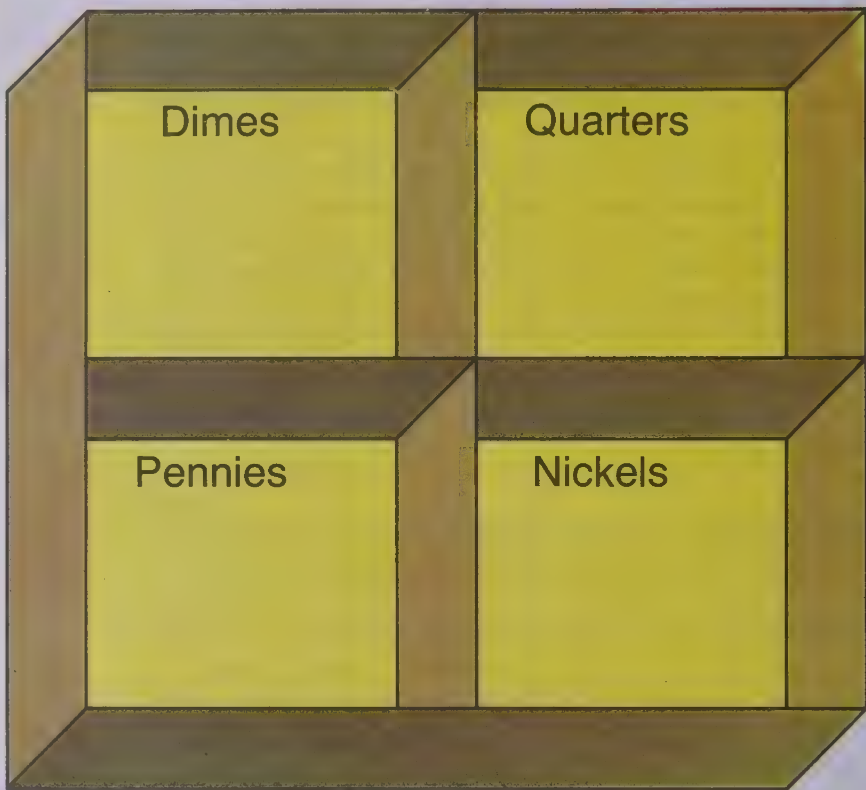
Each child should have an assortment of pennies, nickels, dimes, and quarters. Explain that they should think of the sections on the centre of page c-45 as parts of a drawer for coins. They should pretend that they are putting each of their coins into the proper section of the drawer. Since most children have some familiarity with coins and their value, this activity should not cause difficulty. Even those with little experience with coins should be able to match the coins with the illustrations. This investigation should be considered an introduction to a module in which most "learning" will take place *before* using the printed page. If children do not already have a familiarity with coins and their value, they will need some explanation either from you or from other children. They should not be expected to learn coin values from material on the printed page. Have them record the number of coins in each section of the drawer.

Let's do

I penny	I nickel	I dime	I quarter
I cent	5 cents	10 cents	25 cents
I ¢	5¢	10¢	25¢



Sort your coins. Record how many of each. Answers may vary.



Readiness for work with money

Purposes

To introduce the penny, nickel, dime, and quarter

To give the child experiences in sorting pennies, nickels, dimes, and quarters

Preparation

Materials

coins

Set up a play store in the classroom. A large carton and a stack of primary blocks can be made into a store and furnished with empty boxes and bottles from home. Use prices less than $1.00 for items in the "store." Write the prices with the cent sign (6¢, 15¢, 29¢). Encourage the children to bring items to supplement those you use. Later they should help price new items and keep up the store. Explain to the children that they will be able to buy and sell at the store with coins (play or real) that you will supply.

Follow-up

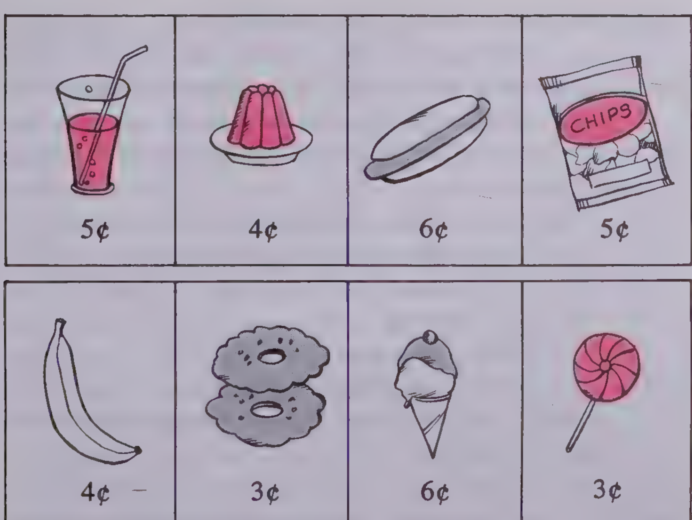
Encourage children to start activities at the play store. Ask those not so familiar with money to sort out nine play (or real) pennies. Then give them a large sheet of newsprint, crayons, and paste. Make a chart on the chalkboard (or tagboard) containing pictures of various things the children could buy to eat. The items should be labelled with the prices in cents. Discuss the symbol for "cent." Ask each child to draw and label the food he wishes to buy and trace underneath it enough coins to buy it.

Let's talk

Discussion
Page c-46

The illustration on page c-46 will serve as an introduction to keeping store. Counting should be stressed throughout discussion of the prices and items on page c-46, as well as during children's actual "play store" situations. For example, point out the 8¢ lollipops. Ask children to pretend that they wish to buy one. They can count eight pennies, "One cent, two cents, . . . eight cents." Or, they can use 1 nickel and 3 pennies, *counting* "five cents, six cents, seven cents, eight cents." Some children may think $5 + 3 = 8$ and then associate this combination with money, but this should not be stressed. Also discuss the possibility of using a dime and getting 2¢ change. In general, stress the importance of counting as coins are used to match prices.

Also use this page to review the value of the penny, the nickel, the dime, and the quarter. Teach the words for each as you discuss them.



Resources for Active Learning

MATHEX: Measurement and Estimation No. 5, "Sorting," p. 39, Encyclopaedia Britannica Publications Ltd.
WORKJOBS, "Money Game," pp. 224–225, Addison-Wesley

Teaching
Page c-47

Use the bear's collection to introduce this page by discussing the kinds of coins falling out of his bank. Direct the children to look at the first frame. Make up a story or have a child tell a story for it. Conclude by asking a child to count the coins to find how much money the first frame shows. As children work through frames, you might want them to count some collections of coins aloud. For example, 2 nickels and 4 pennies would be counted five cents, ten cents, eleven cents, twelve cents, thirteen cents, fourteen cents. If some children experience difficulty in counting by fives for the nickels, suggest that they use their coins, build a matching set and then trade in one of the nickels for five pennies. Encourage this type of exchange until children are able to count the nickels and, if necessary, also the dimes.

Find the value of each coin collection.

Counting amounts of money—pennies, nickels, and dimes

Objective

Given a collection of dimes, nickels, and pennies whose value is less than one dollar, the child will be able to give the value in cent notation by counting the values of the coins.

Pre-Book Activity

Materials

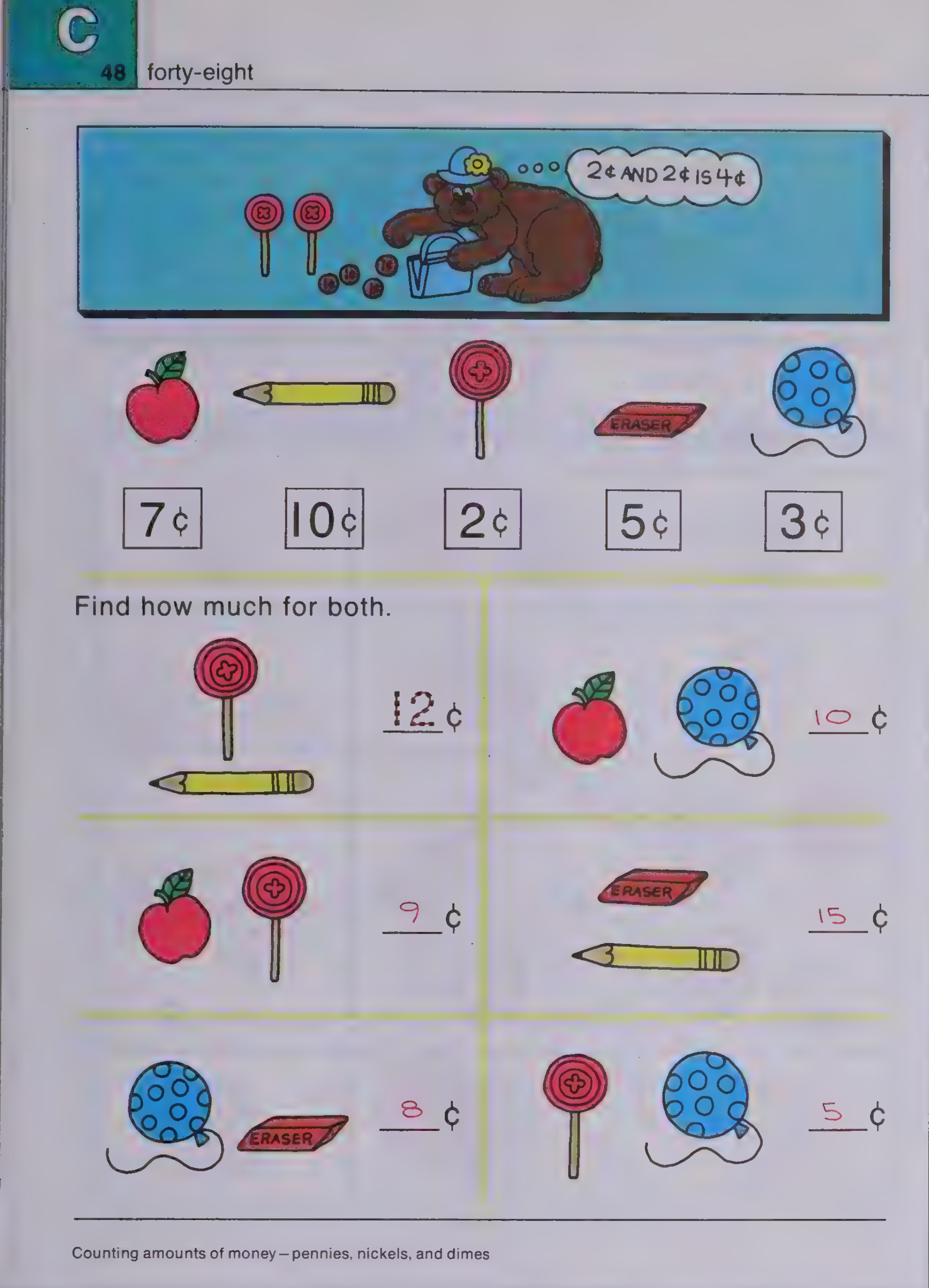
coins

Provide the children with many opportunities for counting money. For example, ask the children to show different ways of counting out coins to find amounts that you write on the chalkboard. If you write 9¢, one child may show 9 pennies and count by ones. Another child might show a nickel and 4 pennies and count: "five cents, six cents, seven cents, eight cents, nine cents." Give guidance as needed, but help the children to learn from their actual work with the coins. Sample amounts appropriate at this time and possible sets of coins are given for your convenience.

17¢ 1 dime, 1 nickel, 2 pennies
"ten cents, fifteen cents, sixteen cents, seventeen cents."

21¢ 2 dimes, 1 penny
"10 cents, 20 cents, 21 cents."

13¢ "2 nickels, 3 pennies"
"five cents, ten cents, eleven cents, twelve cents, thirteen cents."



Teaching
Page c-48

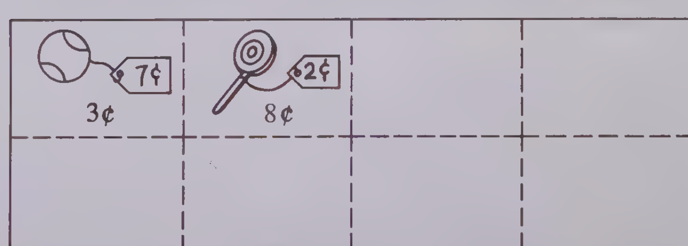
Ask the children to look at the items on the page and have them tell how much each costs. Then read the direction line with them and work through the first problem. Ask, "If a pencil costs ten cents and a lollipop costs two cents, how much would it cost to buy both?" Stress that they can count the coins, "10¢, 11¢, 12¢" instead of adding 10 and 2.

Encourage children to use their coins to figure out how much they need. If necessary, suggest that they count out enough money for each single item of a pair and then count this collection of coins together. Some children would benefit from working in pairs. Discuss their answers when all have finished.

Follow-up

Make a chart showing pieces of fruit, toys, and school supplies that are easy to identify. Label each item with a price of 20 cents or less. Tell the children to try to imagine that they each have 20 cents to spend and to choose the items that they would like to buy. Encourage children to work in pairs. If, for example, Sally chose a lollipop and a balloon as the two things she would like, she can ask Timmy to show a collection of coins she might use. Sally can then compare her choice of coins to those that Timmy chose. Also allow groups of children to continue buying and selling activities at the store.

The children may also use your chart or the list in the book in the following way: Direct the children to fold sheets of newsprint to make eight sections. Now ask them to pretend that they have 10 cents each. Tell them to choose one thing from the chart that they might like to buy and to draw it in the first space. Then ask them to write how much they would have left if they bought it.



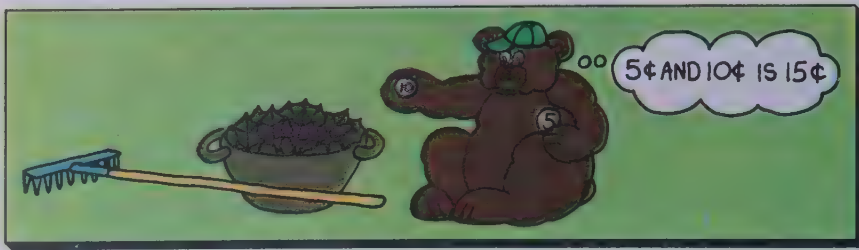
Resources for Active Learning

MATHEX: Measurement and Estimation No. 5, "Matching," p. 40, Encyclopaedia Britannica Publications Ltd.

Teaching
Page c-49

Use the illustration to begin discussion. Suggest that the bear has just finished raking a yard and earned some money. Then suggest that the children make up stories for each frame. For the first frame, tell a story such as: "Jill had a dime in her piggy bank. Then she earned 6 cents for doing an errand for a neighbor. How much money does Jill have now?" Have a child describe how much he thinks Jill has. Then ask him to display the coins and have the class count the amount with him.

Ask the children to tell stories for the second and third problems. Continue to help children who need help; emphasize that each child who earned some money will add coins to what he had. Encourage those who are able to finish the remaining frames independently.



Complete the table.

Had	Earned	Have
		16 ¢
		10 ¢
		16 ¢
		25 ¢
		30 ¢

Objective

Given a collection of coins and an amount spent or earned, the child will be able to give the final amount by counting.

Pre-Book Activity

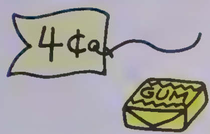
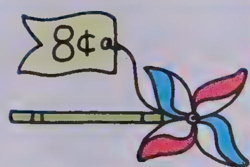
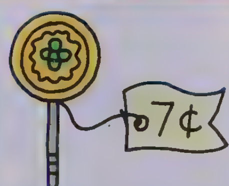
Materials

coins

Develop many stories which the children can act out as demonstrations of earning and spending money. For example, "Sue had 1 dime and 1 nickel. She helped her neighbor take in her groceries and earned a nickel. How much money does Sue have now?" Have the children use their individual sets of coins to show the action. Ask a child who has counted to the correct amount to explain his collection and how he counted it. Sample stories follow:

1) Patty had a nickel. She found 2 pennies under the television set. Mother said she could keep them. How much does Patty have now?
2) Sally only had a penny; her brother, Jim, gave her a nickel for her birthday. How much does Sally have now?
3) Peter had 15 cents, but he spent 8 cents at the candy counter. How much does Peter have left?
4) Jack went to the store with his father. He wanted some licorice that cost 3 cents. Jack had a nickel and a penny. Can Jack have the licorice? Will he have any money left? How much? Will he have any nickels left?

Find the change.

Had	Bought	Change
		4 ¢
		1 ¢
		2 ¢
		3 ¢
		2 ¢

Teaching
Page c-50

Direct the children to look at the top row while you tell a story. Emphasize that this time, the children are spending money. Try to get the class to discover that instead of adding more coins, the children in these stories will be subtracting because they spent some coins.

If children have difficulty figuring out how much they have left, tell them to exchange some of the coins in the "Had" column for pennies.

Follow-up

Encourage children to start giving change during their activities at the store. However, their coins should be limited to quarters, dimes, nickels, and pennies.

Give the children a group of old toy catalogs or magazines and small file cards. Ask them to cut out small toys and paste them on the file cards. Give them felt-tipped pens, and tell them to label the cost of each article.

These may be placed together on a counter as a special part of the store, or the children might exchange cards and draw collections of coins which they might use to buy these items.

Teaching
Page c-51

Since all of the exercises on this page are not in the same format, help children to understand the directions before they begin. Since they have done similar exercises in previous lessons, they should not have difficulty with the first four frames, after you read the directions for them. Also read the directions on the bottom of the page for them, but encourage them to figure out the amounts independently.

Allow them to use their coins while they work the problems. Remind them that the best way for them to be sure of the amount is to count the values of the coins.

Show you know

Find the value of each collection.

Give the final amount.

Had	Earned	Have
		16 ¢

Had	Bought	Change
		4 ¢

Module review

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

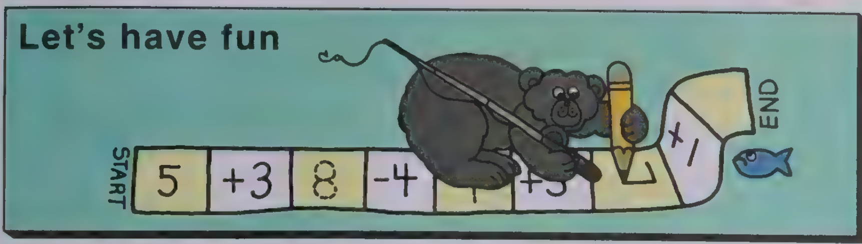
Pre-Book Activity

Children would benefit from continued practice with stories in which they must figure out how much they have left after earning or spending some amount. However, another activity would be to play a guessing game of "What coins am I holding?" Pick up some coins and hide them in your hand (after you know which they are). Tell the children what the coins in your hand are worth, such as 23¢, and ask them to guess which coins you are holding. Explain that you will only answer questions with a "Yes," or "No." Each time they guess they should try to guess your coins with the fewest number of questions. You might keep a record for each set of coins to see what is the fewest number of guesses they use.

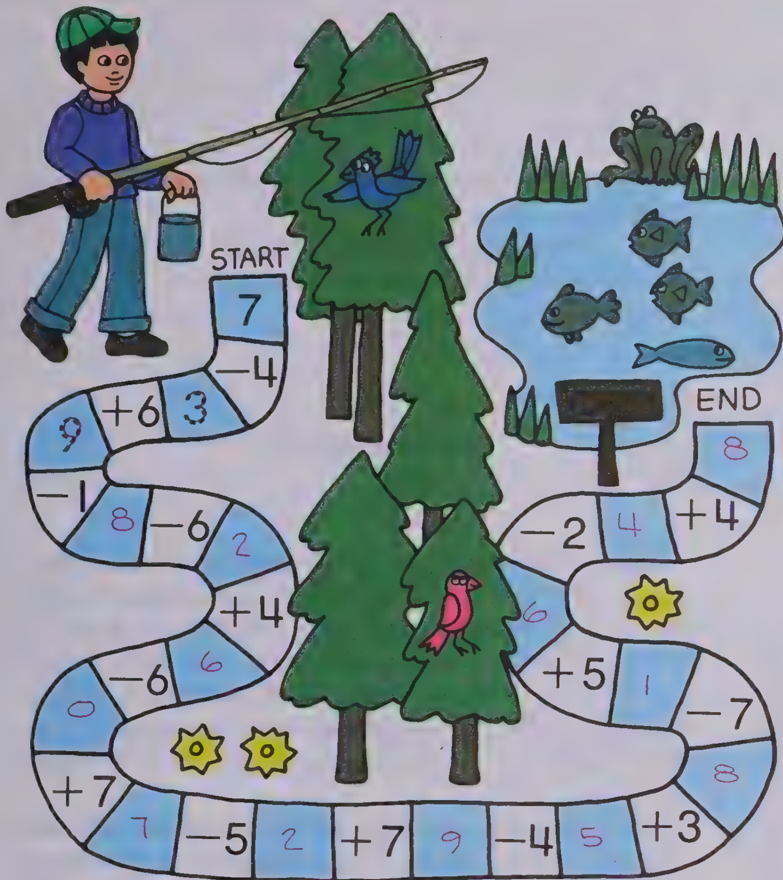
Follow-up

As an extension of this module, relate the use of dimes to children's work with place value. Prepare 18 10-cm flannel circles. Write 10 on nine of them and 1 on the other nine. Use these on the flannelboard as dimes and pennies. Use the hundred board and turn all the tags to the blank side except the tag showing 10. Ask the children to count aloud by ten as you turn over the tags for the multiples of 10. Then ask them to count again, saying one dime is 10 cents, 2 dimes 20 cents, 3 dimes 30 cents, and so on. Next, return all the tags to the blank side and expose the tag for 14. Now ask the children to think of

Let's have fun



Follow the trail. You should end with 8.



Addition, subtraction trail

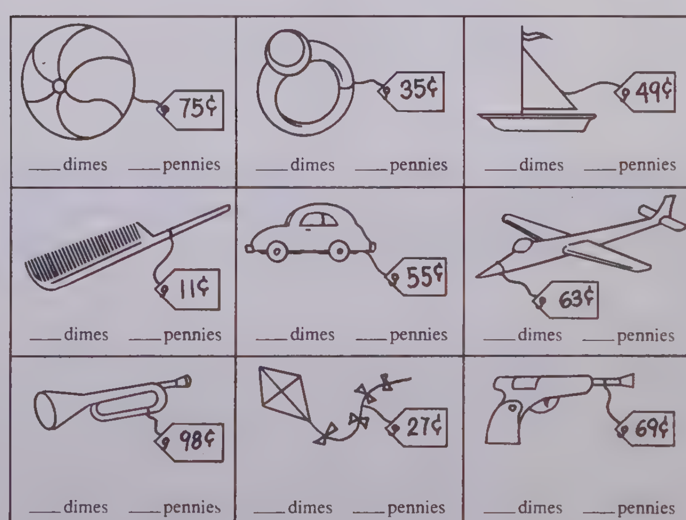
TEACHING
Page c-52

This change of pace page contains a review of the addition and subtraction combinations of 9 or less. Show the children where to start on the trail and where to put their first answer, for 7 − 4. Then explain that they use that 3 to add to the 6. Work with them until they catch on to what to do.

money and tell how many pennies they needed to represent 14 cents.

Continue to expose numbers on the hundred board, 37, for example, and relate this to money by asking for the greatest number of dimes and least number of pennies needed to make 37 cents. Children might use the felt circles on the flannelboard to show the number in dimes and pennies. In stressing place value, do not use nickels or more than 9 pennies to represent any given value.

You might also give the children a duplicated page similar to the one in the next column. Encourage them to finish it by themselves and use their coins if necessary.



BLUE MODULE, UNIT C

Inverses and Missing Addends

Pages c-53- to c-62

General Objectives

To develop the understanding that the differences in a subtraction equation may be thought of as a missing addend in a related addition equation

To increase the child's understanding of addition and subtraction

To develop skill in finding missing addends

The investigation to this module introduces the child, in an informal way, to the notation of missing addends. The concept itself is treated in the discussion, page c-54. The correlation between missing addend and difference is treated in written notation only after children have dealt with this concept by using sets of objects and strips. Finally, the fact that four related addition and subtraction equations may be written for each combination (of 9 or less) is explored. The module concludes with the usual evaluative page and a change of pace.

Mathematics

Children have been introduced to subtraction by manipulating sets, mostly through the idea of removing part of a set from a given set. In this module, the relationship between addition and subtraction is emphasized in preparation for eventual development of finding differences through thinking about missing addends. Finding missing addends is an extension of the inverse relation concept between addition and subtraction. It can also be thought of as a direct application of the definition in terms of addition. When we define subtraction in terms of addition, we merely define each addend as a particular difference. For example, if we have

$$a + b = c$$

Then we say that the addend a is

$$c - b$$

and the addend b is

$$c - a$$

Note that in this definition, each addend, a and b, is defined as a particular difference. We lead the children to this idea by asking them to identify a particular missing addend by drawing on their knowledge of addition. From this we lead them to see that when they have found the given missing addend, they have found the particular difference. For example, in the equation.

$$3 + \square = 7$$

When the children find that the number 4 is the correct solution, it is important that they see also that 4 is the same as 7 minus 3.

Teaching Blue Module, Unit C

Approximate Time: 5 to 7 days

Materials

counters or other objects for individual work with sets
flannelboard
small squares of paper or tagboard, 4 per child
objects for use in set demonstrations
strips

Vocabulary

addend	missing addend
difference	sum

Most of the terms in the vocabulary list are already known by the children. However, be sure the meanings are clear. It would be helpful to introduce each term as if it were new to assure adequate understanding. Correct use of these terms by the children themselves will develop gradually, but it is most important that they know what numbers you are referring to as you discuss equations.

Set illustrations are used carefully to show partitioned sets. Since the concept of this module is quite abstract, the use of concrete objects must be guided carefully. Missing addends are studied here as a prelude to finding differences later; the inverse relation of addition and subtraction will be studied and will be further developed in Book 2.

Oral examples accompanied with demonstrations should help you guide the children to develop thinking which will help them eventually solve subtraction equations. The more informal opportunities a child is given to see the relationship between addition and subtraction, the easier it will be later to use the missing addend concept to find differences.

Evaluation of Progress

Since this is an enrichment module, do not expect children to master all the concepts introduced. Many should be able to complete addition equations that have a missing addend, and think of a missing addend when they try to find the difference in a subtraction equation. They should also be able to see that for any given partitioning of a set into two subsets, they can think about two addition and two subtraction equations.

Resources for Active Learning

General Activities:

MATH ACTIVITIES, "Which Was It?" Game 2/30, pp. 36–37, Allyn and Bacon

Mathex: Operations No. 3, "Placeholders," pp. 26–27, Encyclopaedia Britannica Publications Ltd.

Manipulative Devices:

"Invicta" Math Balance (Math Media; Selective Educational Equipment)

Sigma Chips (Scott Scientific)

Investigation
Page c-53

This is actually a very simple investigation and might even be presented as a game. Read the directions for the children. Explain that they should cover one blue numeral in each equation and then try to remember the numeral that has been hidden. You might have children work in pairs for this investigation, so that one child covers a numeral and the other must guess which numeral has been covered.

During this investigation, ask questions for a particular equation such as: "Which number is the sum?" Point out that the numeral they are covering is an addend. They are not seeking the sum of two numbers here, but rather a hidden or *missing* addend.

Let's do

$\square + 2 = 4 \quad 3 + \square = 7$

In each equation, cover one of the blue numerals with a square. Can you remember what is under each square?

$$2 + 2 = 4$$

$$4 + 3 = 7$$

$$3 + 2 = 5$$

$$2 + 4 = 6$$

Readiness for missing addends

Purpose

To introduce missing addends

Preparation

Materials

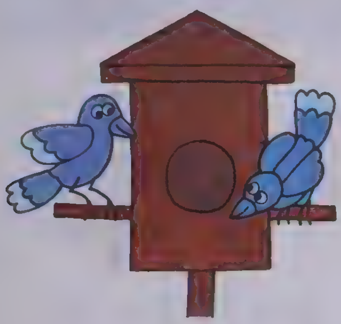
small squares or discs to cover numerals in the text, 4 per child

Unless you wish to start the math period with a short game to review addition combinations and equations, you might begin the investigation in the text immediately.

Let's talk

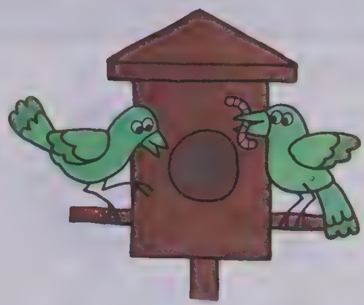
How many birds are in their house?
Solve the "birdhouse" equations.

3 birds



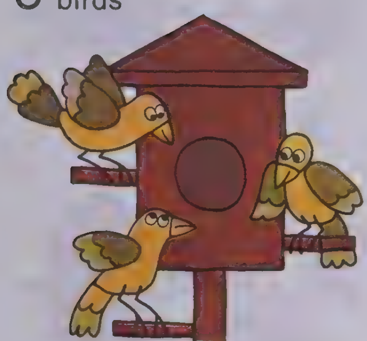
$2 + \boxed{1} = 3$

4 birds



$2 + \boxed{2} = 4$

5 birds



$3 + \boxed{2} = 5$

6 birds

$1 + \boxed{5} = 6$

Readiness for missing addends

Discussion
Page c-54

The basic idea of missing addend is presented on page c-54 in a semi-concrete setting. This type of activity is generally easier for children than working with the written notation. Call attention to the first frame. Explain that the phrase "3 birds" tells us that altogether there are 3 birds. Ask the children how many birds they see. When someone responds "2," ask how many more there should be if there are 3 altogether. Ask where they think the other bird is. When they realize that the other bird is hidden in the bird house, relate the frame to the equation by writing 2 + [birdhouse] = 3 on the chalkboard. Ask children why they think the place for the missing addend is pictured as a bird house. Then have them complete the equation. Discuss the other frames in a similar manner. Help children realize that the equation for a picture has been "solved" as soon as they discover the number of birds hidden in the bird house.

Follow-up

Distribute five or six strips of paper or tagboard approximately 20 centimetres long and 6 centimetres wide to each child. Suggest that children write an addition equation on each strip. Allow them to use their flash cards to recall combinations. When they have written five or six equations, have them choose partners and then take turns holding up the equations, with one addend covered, for their partner to guess the missing addend.

$\boxed{3 + \text{[hand]} = 8}$

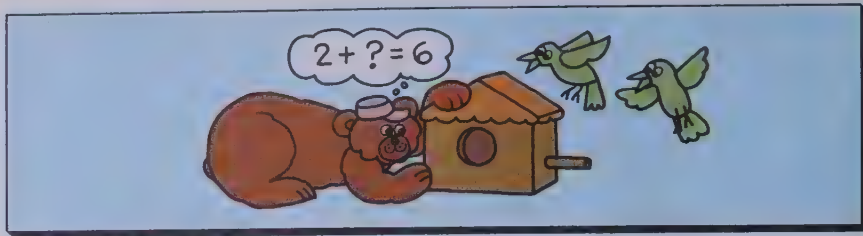
Can you guess my hidden addend?

Resources for Active Learning

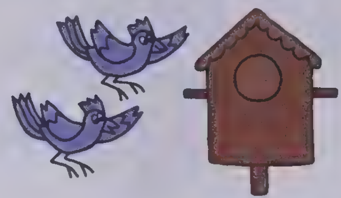
WORKJOBS, "Hide 'n' Go Seek," pp. 198–199, Addison-Wesley

Teaching
Page c-55

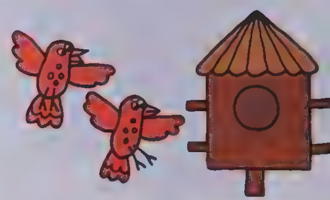
Ask children to look at the first frame and describe how many birds they see and why they think the birdhouse is pictured. Help them realize that they should think of some number of birds hidden in the bird house. Then relate the equation to the picture. *Stress that 5 is the total number of birds*. Thus the first number, 2, represents the number of birds not in the house, the  represents the number of birds hidden in the birdhouse, and the sum, 5, represents the number of birds in all. Have children explain the remaining pictures and complete the equations. Point out that they must know the total number, or sum, and one of the addends, that is one of the numbers to be added, before they can figure out the missing addend. You might use phrases such as the following during the discussion, "2 plus what number is 5?" or "If you have 4 and you want 6, how many should you add to 4?"



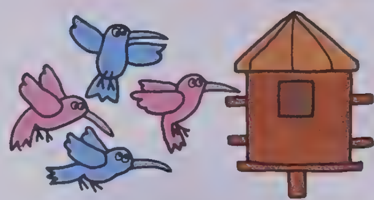
Solve the equations.



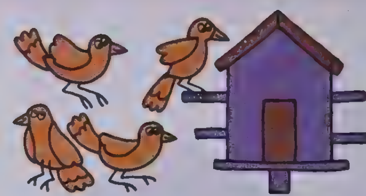
$2 + \boxed{3} = 5$



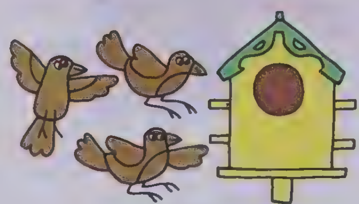
$2 + \boxed{5} = 7$



$4 + \boxed{4} = 8$



$4 + \boxed{2} = 6$



$3 + \boxed{3} = 6$

$4 + \boxed{3} = 7$

Objective

Given an addition equation with a missing addend, the child will be able to find the missing addend by associating the equations with related illustrations or by working with the strips.

Pre-Book Activity

Materials

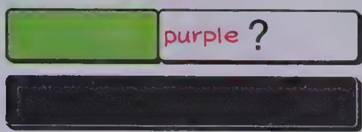
set of strips for each child

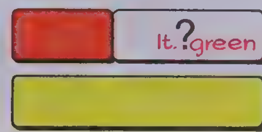
Ask children to take their red strip and place it above the black strip. Then ask them to try to find the strip that will fit next to the red strip and complete the train to match the black strip. Ask children to identify the numbers the strips represent. Relate the strips to the equations $2 + 5 = 7$ by writing $2 + \square = 7$. Ask the children: "What number strip did we have to add to the two-strip to match the seven-strip?" Have children work through several other examples in a similar manner.

Follow-up

A duplicated worksheet such as the one in the next column will help children become more familiar with finding missing addends. Point out to the children the number given at the top. Ask them to complete each picture so it has that many objects. Then they should complete each equation by filling in the missing addend.

Find the missing strips. Solve the equations.





$3 + \boxed{4} = 7$ | $2 + \boxed{3} = 5$

$2 + \boxed{2} = 4$ | $1 + \boxed{3} = 4$

$1 + \boxed{3} = 4$ | $7 + \boxed{1} = 8$

$3 + \boxed{2} = 5$ | $3 + \boxed{1} = 4$

$2 + \boxed{4} = 6$ | $4 + \boxed{1} = 5$

$3 + \boxed{1} = 4$ | $5 + \boxed{0} = 5$

$6 + \boxed{1} = 7$ | $2 + \boxed{2} = 4$

$2 + \boxed{1} = 3$ | $7 + \boxed{2} = 9$

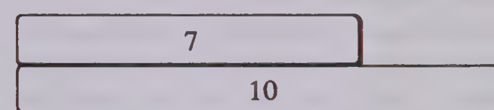
$1 + \boxed{4} = 5$ | $3 + \boxed{5} = 8$

Introduction to missing addends—strips

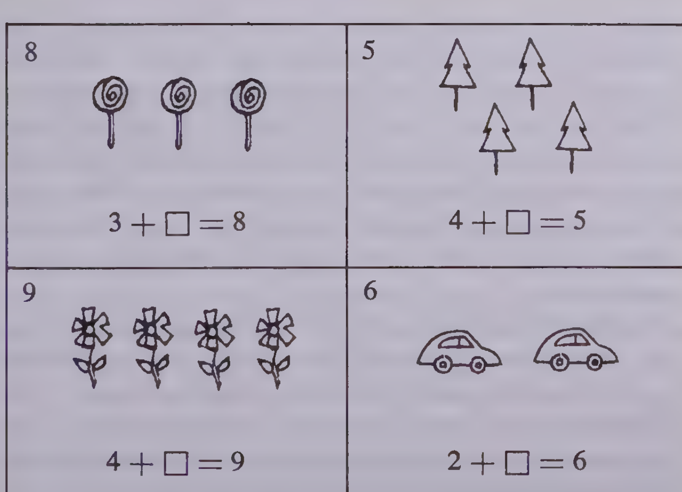
Teaching
Page c-56

If the pre-book activity suggested for this lesson was used, children should be able to interpret the strips in the top frames. Work through both equations, asking: "What strip is needed to complete a train that will match the black strip? (or the yellow strip?) Relate the strips to the equation by asking: "Three plus what number is seven?" or "What number must be added to three to get seven?"

Encourage children to complete the remaining equations on their own, but to use the strips so they can find a strip to complete the train that matches the sum.



Some children may need help in doing this. Give guidance when necessary, but if a child appears to be having much difficulty suggest that he choose another kind of material, such as counters, to help him.



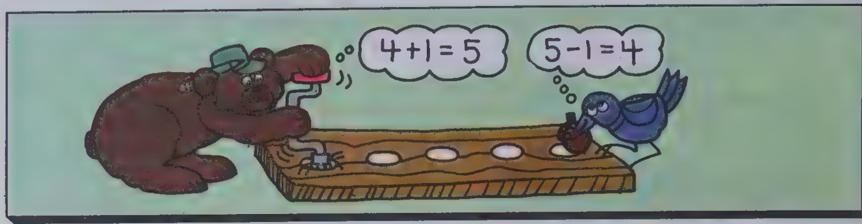
Resources for Active Learning

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Addition," No. 30, Responsive Environments Corp.

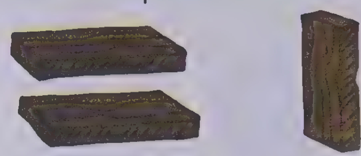
MATH WORKSHOP: Games and Enrichment Activities, "Missing Addend and Sum Games," pp. 43–44, Encyclopaedia Britannica Educational Corp.

Teaching
Page c-57

Call the children's attention to the first frame at the top of the page and to the pair of equations, 2 + □ = 3 and 3 − 2 = □. Point out that there are three pieces of wood in all and that they are shown in two sets. Direct them to cover one of the sets so they see only 2 pieces of wood. Read the equation saying: "Two plus what number is three?" Immediately refer to the equation 3 − 2 = □ and show the children how it may be read not only as: "Three minus two equals what number?" but also *in the opposite direction,* that is, right to left, it may be read as: "What number can be added to two to get three?" Work through several of the pairs of equations in this way. If necessary, use sets of objects to demonstrate each pair of equations. Note that the last two pairs are not accompanied by an illustration. Some children might draw their own picture for each of these frames or simply use counters or strips as aids.

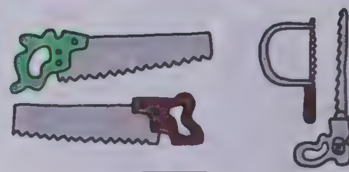


Solve the equations.



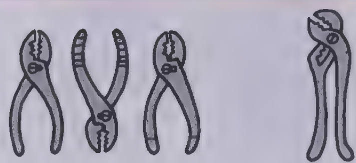
$2 + \boxed{1} = 3$

$3 - 2 = \boxed{1}$



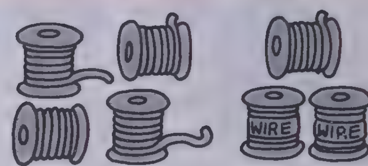
$2 + \boxed{2} = 4$

$4 - 2 = \boxed{2}$



$3 + \boxed{1} = 4$

$4 - 3 = \boxed{1}$



$4 + \boxed{3} = 7$

$7 - 4 = \boxed{3}$

$1 + \boxed{5} = 6$

$6 - 1 = \boxed{5}$

$2 + \boxed{3} = 5$

$5 - 2 = \boxed{3}$

Objective

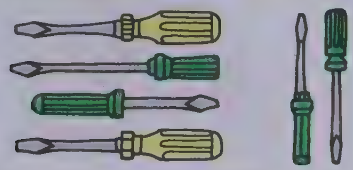
Given a pair of related addition and subtraction equations with a missing addend, the child will be able to find the difference by thinking of the missing addend.

Pre-Book Activity

Many children will benefit from stories accompanied by demonstrations. For example, tell the children that you have two sets, one in each hand, and that you are going to show them one set. Display one set, such as a set of four circles, on the flannelboard or chalkboard. Ask the children to close their eyes. Combine your second set with the first to show the total, for example, six, on the flannelboard. Ask the children if they can figure out how many were in your second set. Relate the equation 4 + □ = 6 to this action saying: "What number did I add to 4 to get 6?" Show how this may be related to 6 − 4 = □ by having the children think of the question: "What number did I add to 4 to get 6?" while you point from right to left under the equation. Work through several examples of this type. In all cases relate the story or demonstration action to the appropriate addition and subtraction equations. The following stories may serve as examples:

1) Anne brought 6 cookies to school in her lunch. She gave some of them to Jane and found that she had 3 left. How many did she give to Jane?
2) Billy gave Tom 2 baseball cards. Joe gave Tom some more. When Tom counted his baseball cards, he had 5 in all. How many did Joe give him?
3) Jane had some balloons, but 3 of them broke. She had 2 left. How many balloons did she start with?

Solve the equations.



$4 + \boxed{2} = 6$	$5 + \boxed{4} = 9$
$6 - 4 = \boxed{2}$	$9 - 5 = \boxed{4}$
$2 + \boxed{2} = 4$	$3 + \boxed{1} = 4$
$4 - 2 = \boxed{2}$	$4 - 3 = \boxed{1}$
$3 + \boxed{3} = 6$	$3 + \boxed{4} = 7$
$6 - 3 = \boxed{3}$	$7 - 3 = \boxed{4}$
$4 + \boxed{4} = 8$	$4 + \boxed{5} = 9$
$8 - 4 = \boxed{4}$	$9 - 4 = \boxed{5}$

Relation between addition and subtraction

TEACHING
Page c-58

Direct the children's attention to the first frame and elicit from them the relationship between the pictured sets and the equations. Point out that it is helpful to know the missing addend when you are looking for the difference. Since they know that $4 + 2 = 6$, they can think: "What number added to 4 gives 6," as they solve $6 - 4 = \square$. Work through equations $5 + \square = 9$ and $9 - 5 = \square$ together with the children. Then encourage them to complete the remaining equations independently. When they finish, ask them to explain how they found their answers.

FOLLOW-UP

A game called "Heads Up" is helpful in the study of missing addends.

Place 8 paper cups upside down on a demonstration table. Ask the children, "How many?" Everyone should respond "Eight." Tell them that they are going to play a detective game called "Heads Up." They must fold their arms on their desks, close their eyes, and bury their heads in their arms. While they are not looking remove some cups; for example, remove 5, and then call "Heads Up!" When they look up, tell those who know how many cups are missing to fold their arms over their chests. When someone you call on answers correctly, you might say "Yes, there must be 5 missing because we started with 8, and 3 and 5 make 8." Continue this activity by removing different numbers of cups each time.

A worksheet such as the following could be used to emphasize that knowing the missing addends helps in finding the missing differences.

Find the missing addends.

$9 - 6 = 3$ because $3 + 6 = 9$

$8 - 7 =$ __ because __ $+ 7 = 8$

$9 - 5 =$ __ because __ $+ 5 = 9$

$6 - 2 =$ __ because __ $+ 2 = 6$

$9 - 8 =$ __ because __ $+ 8 = 9$

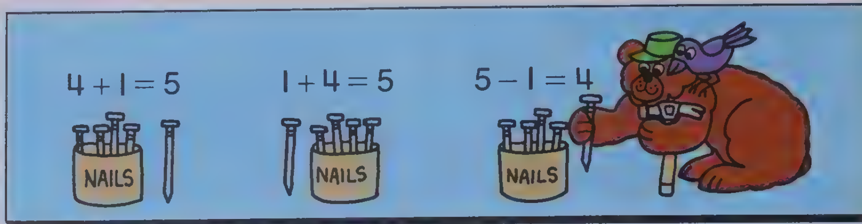
$7 - 3 =$ __ because __ $+ 3 = 7$

$8 - 4 =$ __ because __ $+ 4 = 8$

$5 - 1 =$ __ because __ $+ 1 = 5$

Teaching
Page c-59

Tell the children to look at the set of paint brushes and solve all the equations that describe this set. Work through the first frame with them. Discuss each equation as you go. Continue with the set of paint cans. If the children need help, the equations could be read aloud with them. Otherwise, encourage them to finish the page by themselves.



Solve the equations.

$3 + 1 = \boxed{4}$

$1 + 3 = \boxed{4}$

$4 - 1 = \boxed{3}$

$4 - 3 = \boxed{1}$

$3 + 2 = \boxed{5}$

$2 + 3 = \boxed{5}$

$5 - 2 = \boxed{3}$

$5 - 3 = \boxed{2}$

$1 + 2 = \boxed{3}$

$2 + 1 = \boxed{3}$

$3 - 2 = \boxed{1}$

$3 - 1 = \boxed{2}$

$0 + 2 = \boxed{2}$

$2 + 0 = \boxed{2}$

$2 - 2 = \boxed{0}$

$2 - 0 = \boxed{2}$

Objective

Given a partitioning of a set of 9 or less, the child will be able to solve two addition and two subtraction equations related to the given set.

Pre-Book Activity

Materials

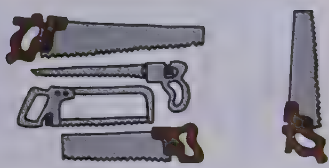
objects for set demonstrations

Display a set of 5 objects clearly divided into subsets of 3 and 2. Ask a child to come up to the chalkboard and write an addition equation for this set. Tell another child to write a second addition equation for this set. Cover up the set of 2, and ask a child to write the resulting subtraction equation. Now cover up the set of 3, and ask a fourth child to write this subtraction equation. Emphasize that for one set, we can write two addition and two subtraction equations. Continue this activity for several sets until each child has had an opportunity to participate.

Follow-up

If children work well with the strips, have them build a train to match a single strip and write two addition and two subtraction equations for their train. An example is shown in the next column.

Solve the equations.

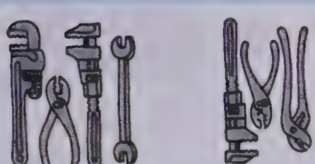


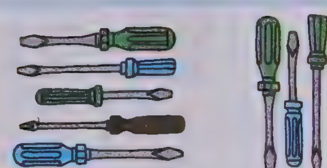
$4 + 1 = \boxed{5}$ $4 + 2 = \boxed{6}$

$1 + 4 = \boxed{5}$ $2 + 4 = \boxed{6}$

$5 - 1 = \boxed{4}$ $6 - 2 = \boxed{4}$

$5 - 4 = \boxed{1}$ $6 - 4 = \boxed{2}$





$4 + 3 = \boxed{7}$ $5 + 3 = \boxed{8}$

$3 + 4 = \boxed{7}$ $3 + 5 = \boxed{8}$

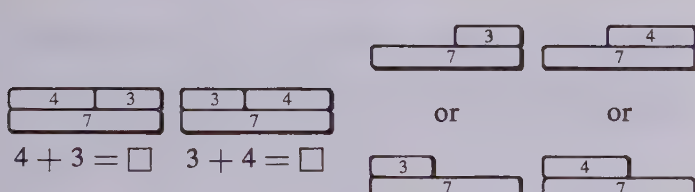
$7 - 3 = \boxed{4}$ $8 - 3 = \boxed{5}$

$7 - 4 = \boxed{3}$ $8 - 5 = \boxed{3}$

Relation between addition and subtraction

Teaching
Page c-60

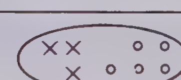
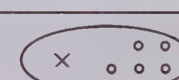
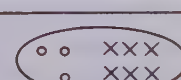
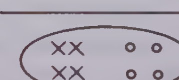
Since this page follows the same format as page c-59, you might have the children continue on to it from the other side. Help any children who are having difficulty; you might encourage these children to use counters or other objects to build sets similar to those in the picture. However, expect most children to work from the printed page, since illustrated sets are provided.



$4 + 3 = \square$ $3 + 4 = \square$ $7 - 3 = \square$ $7 - 4 = \square$

Encourage children to experiment with ways of positioning the strips. It would be helpful, however, for you to list groups of three strips for them to experiment with. Examples: 3, 4, 7; 2, 6, 8; 4, 5, 9; 1, 2, 3; 2, 3, 5; 4, 1, 5; 4, 4, 8; 7, 2, 9;

A worksheet such as the following would also benefit some children.

Write 4 equations for each set.	
 — + — = □ — + — = □ — − — = □ — − — = □	 — + — = □ — + — = □ — − — = □ — − — = □
 — + — = □ — + — = □ — − — = □ — − — = □	 — + — = □ — + — = □ — − — = □ — − — = □

Teaching
Page c-61

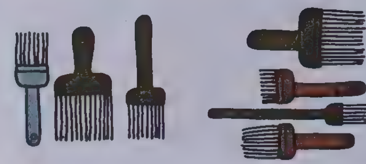
Call attention to the top two frames. Remind children that the illustrations are provided to help them solve the related addition and subtraction equations. Then point out that the next frames simply have equations to solve, without any accompanying illustrations. Suggest that they use counters or strips if they want to. Finally, explain that the illustration at the bottom of the page is related to each of the four equations to the right and might help them solve these equations. Encourage the children to work independently, but use their answers for discussion when they have finished.

Show you know

Solve the equations.

$1 + \boxed{2} = 3$

$3 - 1 = \boxed{2}$



$3 + \boxed{4} = 7$

$7 - 3 = \boxed{4}$

$2 + \boxed{2} = 4$

$4 - 2 = \boxed{2}$

$3 + \boxed{3} = 6$

$6 - 3 = \boxed{3}$

$1 + \boxed{7} = 8$

$8 - 1 = \boxed{7}$

$2 + \boxed{3} = 5$

$5 - 2 = \boxed{3}$

$2 + 4 = \boxed{6}$

$4 + 2 = \boxed{6}$

$6 - 4 = \boxed{2}$

$6 - 2 = \boxed{4}$

Module review

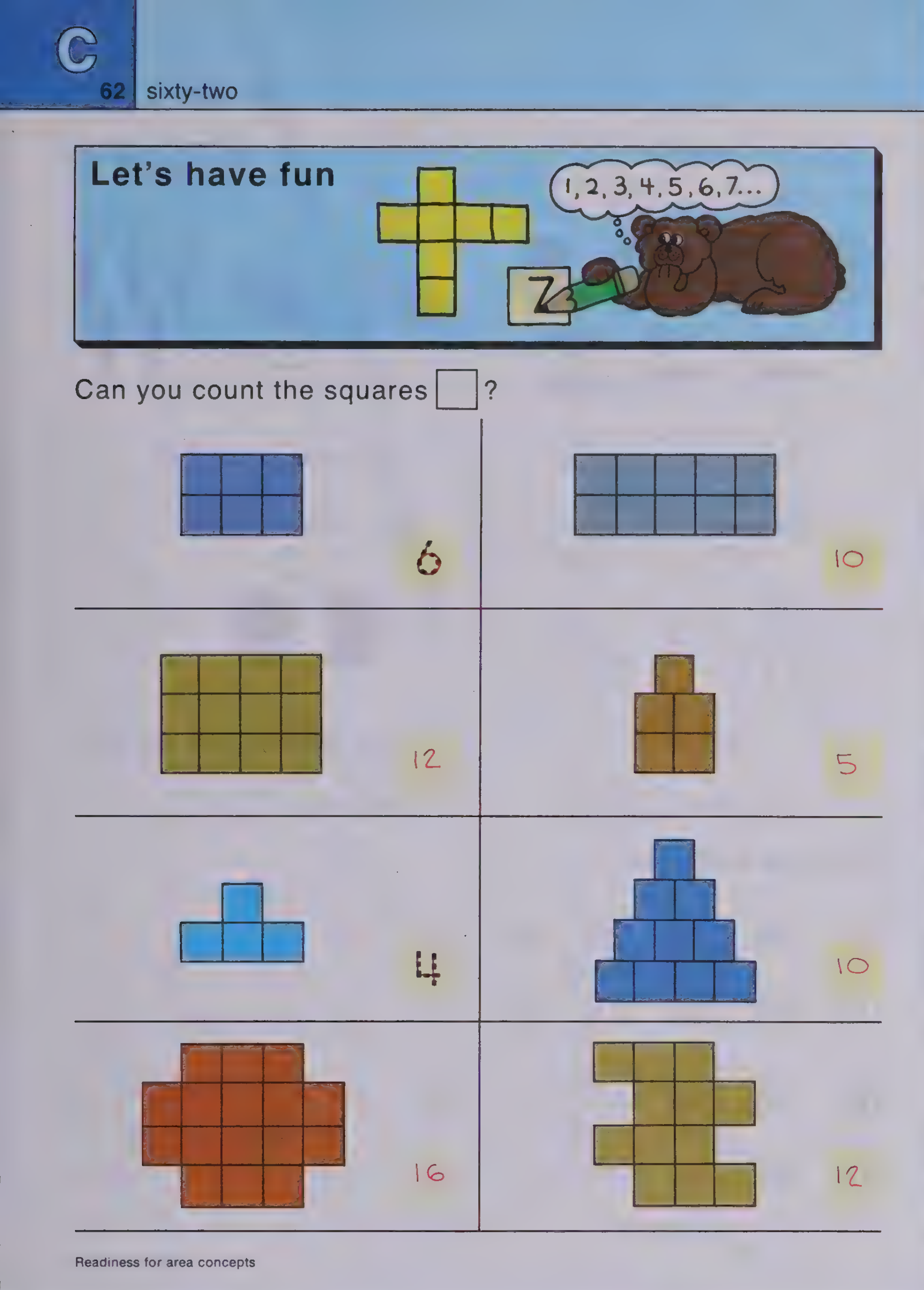
Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

Give the children a short oral drill to review finding a missing addend. For example, say: "I am thinking of a number. When I add my number to 5, I get 7. What's my number?" or "I am thinking of a number. When I add my number to 4, I get 8. What is my number?" and so on. If such a completely oral drill seems too difficult, accompany your examples with displays of sets of objects.

Alternately, you might simply hold a set in each hand. Show one set, giving the children its number and show the total set, giving the total number. Then ask children to figure out how many you added to the first set to get the final set.


C
62 sixty-two

Let's have fun

Can you count the squares ☐?

6	10
12	5
4	10
16	12

Readiness for area concepts

Teaching
Page c-62

Explain to the children that this page simply contains counting exercises. Each frame shows some number of squares. They should write how many in each answer box.

Follow-up

Give the children exercises like the following ones, on duplicated sheets, for additional practice in finding missing addends.

Solve.

$$\begin{array}{r} 7 \\ +\square \\ \hline 8 \end{array} \quad \begin{array}{r} \square \\ +2 \\ \hline 7 \end{array} \quad \begin{array}{r} 1 \\ +\square \\ \hline 5 \end{array} \quad \begin{array}{r} \square \\ +0 \\ \hline 2 \end{array} \quad \begin{array}{r} 5 \\ +\square \\ \hline 8 \end{array} \quad \begin{array}{r} 0 \\ +\square \\ \hline 0 \end{array}$$

$$\begin{array}{r} \square \\ +2 \\ \hline 8 \end{array} \quad \begin{array}{r} 6 \\ +\square \\ \hline 7 \end{array} \quad \begin{array}{r} 2 \\ +\square \\ \hline 8 \end{array} \quad \begin{array}{r} \square \\ +3 \\ \hline 7 \end{array} \quad \begin{array}{r} \square \\ +1 \\ \hline 8 \end{array} \quad \begin{array}{r} 3 \\ +\square \\ \hline 5 \end{array}$$

Resources for Active Learning

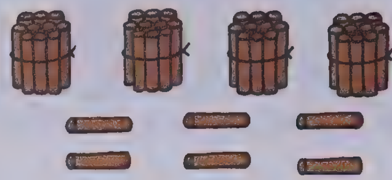
MATHEX: Operations No. 3, pp. 5–8, Encyclopaedia Britannica Publications Ltd.

Teaching
Page c-63

Read the directions at the top of the page with the children. Be sure they realize that they are expected to write both the number of tens and ones as well as the two-digit numeral each phrase indicates. Explain that in the bottom section of the page they should simply complete each row.

Looking back

How many?

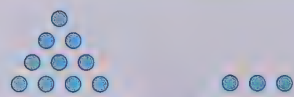


4 tens and 6

We write 46.

2 tens and 5

We write 25.



1 tens and 3

We write 13.

2 tens and 0

We write 20.

Complete each row.

27	28	29	30	31	32	33	34	35	36
42	43	44	45	46	47	48	49	50	51
64	65	66	67	68	69	70	71	72	73
83	84	85	86	87	88	89	90	91	92

Cumulative review

Objective

The child will demonstrate his ability to work with the concepts presented throughout Unit C.

Pre-Book Activity

Review the concepts of two-digit numerals and some basic facts. For example, ask children to count off in groups of ten. Ask a child to record how many tens and how many left over. Ask another child to write the two-digit numeral for this number. You might then give the children some two-digit numerals to write, using phrases such as, 7 tens and 6, or 9 tens and 0, and so on.

It would also be helpful to review some of the basic combinations to ten so that children become accustomed to finding them without relying on materials.

Find the sums.

0 1 2 3 4 5 6 7 8 9

$5 + 2 = \boxed{7}$ $3 + 4 = \boxed{7}$

$6 + 0 = \boxed{6}$ $5 + 3 = \boxed{8}$

$8 + 1 = \boxed{9}$ $7 + 2 = \boxed{9}$

6	0	1	4	5	7
+2	+3	+1	+4	+4	+0
8	3	2	8	9	7

Find the differences.

$6 - 2 = \boxed{4}$ $9 - 5 = \boxed{4}$

$9 - 6 = \boxed{3}$ $6 - 0 = \boxed{6}$

$7 - 3 = \boxed{4}$ $8 - 1 = \boxed{7}$

5	8	9	6	7	9
−4	−2	−7	−6	−5	−2
1	6	2	0	2	7

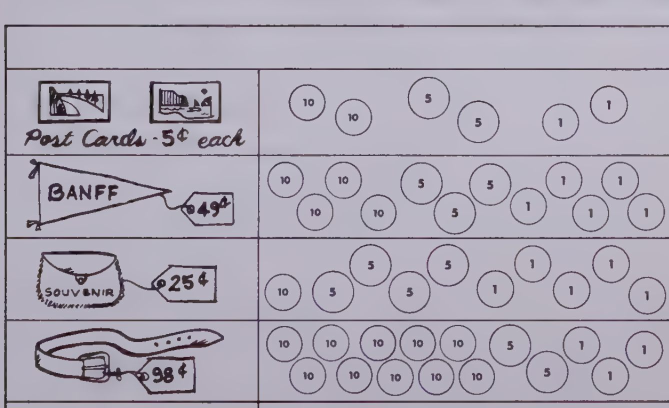
Cumulative review

Teaching
Page c-64

Point out the number line at the top of the page. Stress that it is shown so that it may be used by those who wish. Also be sure that they realize that the top part of the page requires that they find the sums and the bottom section that they find the differences.

Follow-up

Duplicate an exercise similar to this one. *Note:* Trace real coins for actual size; write in the value of each.



Make up stories to fit the pictures about children taking vacations and buying souvenirs. For example:

Jim bought postcards while he was on a trip to the mountains. How many postcards did Jim buy? How much did he pay for them? What coins did he use?

Tell the children to mark the coins shown on the right "to pay" for the different souvenirs. Undoubtedly, the children will mark different combinations of coins for the same amount.

The more able pupils may enjoy finding out how much money would be left after paying for each souvenir.

YELLOW MODULE, UNIT D

Sums and Differences Through 10

Pages d-1 to d-12

General Objectives

To introduce addition and subtraction combinations of 10

To maintain and sharpen skills with combinations through 10 in preparation for finding sums 11 to 18 and related differences

Since the symbol 10 has a specific place-value meaning, the study of combinations of 10 is undertaken after children have been introduced to place value in Unit C. Skill in finding the combinations of 10 is of particular importance for those who will be studying the sums 11 through 18 in the dark green extension module. In the dark green module sums such as $8 + 6$ will be reduced to problems involving sums of 10 or less.

Mathematics

Special emphasis is given to sums of 10 in preparation for the use of the associative principle in finding sums 11 to 18. The following example illustrates this idea.

$$\begin{aligned} 8 + 7 &= 8 + (2 + 5) \\ &= (8 + 2) + 5 \\ &= 10 + 5 \\ &= 15 \end{aligned}$$

Familiarity with sums of 10 is required for this mathematical skill.

Teaching Yellow Module, Unit D

Approximate Time: 6 to 8 days

Materials

counters, at least 10 per child
bean bags, four
eight sets of geometric shapes, 10 of each shape, in contrasting colors
sets of 10 objects for demonstration (blocks, empty plastic bottles, tenpins from a toy bowling game, paper cups)
flannelboard and felt objects and symbols
demonstration number line
strips
flash cards for sums and differences of 10 or less
10 small bowling pins or 10 cardboard rolls, and a small ball

Strips, counters, and the number line should all be made available to children as they solve the equations for combinations of 10. However, after each of these aids has been used to introduce the combinations, children should be encouraged to gradually discontinue using them. Some children might be inclined to continue using an aid when they are really capable of remembering the facts. Thus, the materials should be available, but children should be encouraged to master the facts through 10.

Evaluation of Progress

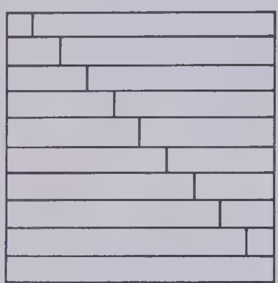
Proficiency in basic combinations can be measured on pages d-9 through d-11. However, the best evaluation of a child's progress will come from your daily observations and questions asked individual children.

Resources for Active Learning

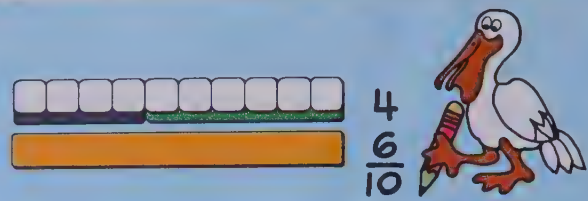
General Activities:
Games that provide mastery of facts:
MATH ACTIVITIES, Games 3/31–68, pp. 97–114, Allyn and Bacon
MATHEX: Operations No. 3, pp. 8–9, Encyclopaedia Britannica Publications Ltd.
MATH ACTIVITIES, Activities 3/1, 2, pp. 87–88, Allyn and Bacon
TEACHING AIDS FOR ELEMENTARY MATHEMATICS, "Little Puzzles for Beginners," pp. 72–73, Holt, Rinehart and Winston
Commercial Games:
Games for practicing basic facts:
Arithmecubes (Scott Foresman)
Cover-Up (Selective Educational Equipment)
Equations (Creative Publications; Wff 'N Proof)
Orbiting the Earth, Addition and Subtraction (Scott Foresman)
Sum Fun (Ideal; school supplier)
Twin Choice (Holt, Rinehart and Winston)

Investigation
Page d-1

Explain the directions to the children. Although they are instructed to cover each strip with white strips, those children who are certain of the number names of the strips may realize that they do not need to actually cover the strips to know how many white strips cover each strip. Be sure that any children who do use white strips push the white strips as close together as possible or they are apt to get the wrong number. You might want many children to extend the investigation by having them build a square to show combinations of 10 and recording each combination.



Let's do



Cover the strips with white strips.
Write how many for each strip.

Purpose

To introduce combinations of ten

Preparation

Materials

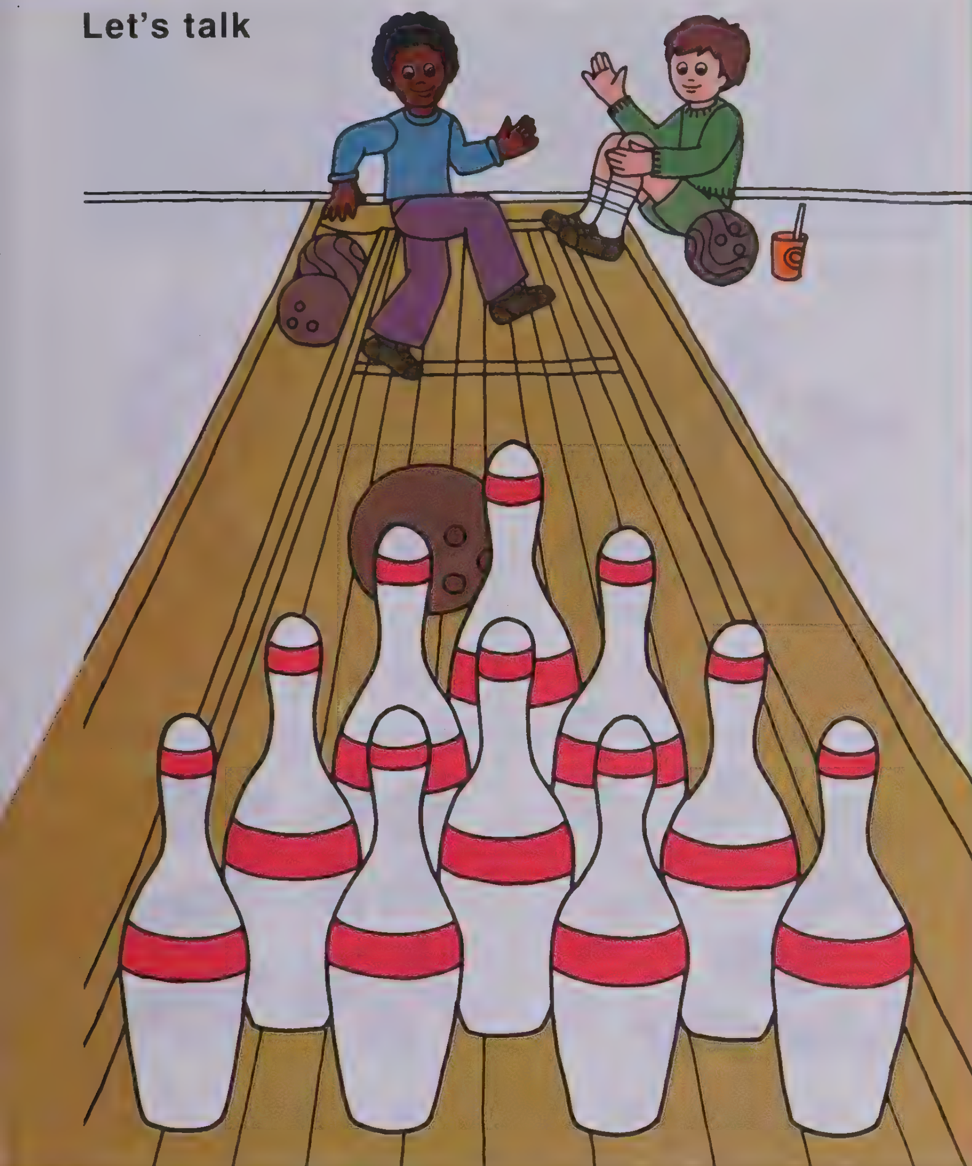
set of strips for each child

Encourage the children to begin the lesson with free play with the strips. For example, suggest that they build a school, or a barn, and use at least one of every color strip. You might also review the number names for the strips.

d

2 two

Let's talk



Readiness for addition combinations of ten

Discussion
Page d-2

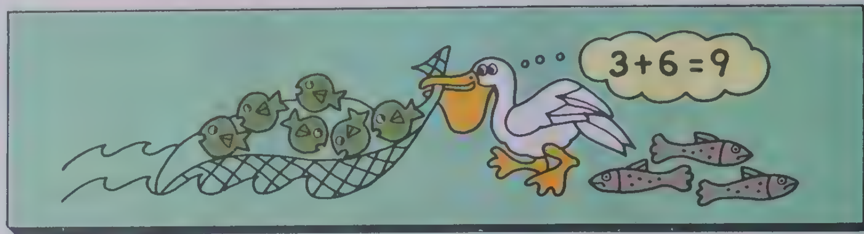
The main purpose of page d-2 is to provide material for a discussion of the combinations of ten. Help children describe the picture. Make sure that everyone understands the goal of the bowling game. Then elicit from them various possibilities for how many pins might go down and how many would remain standing when the ball is thrown. Thus, if 7 are knocked down, how many are still standing. Or if there are 5 standing how many have been knocked down.

Follow-up

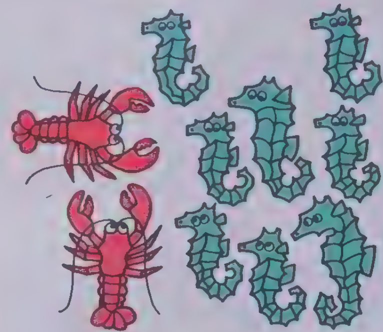
The discussion would very well extend into a bowling activity in which the children actually play a type of bowling game. Commercial equipment for bowling is available, although homemade materials may be used. For example, ten cardboard paper rolls or even ten heavy sheets of paper rolled and taped so that they stand might be used for bowling pins. Children may take turns rolling a ball which is about 15 or 20 centimetres in diameter, from a reasonable distance to try to knock down the pins. Throughout the activity, stress the various combinations of ten.

Teaching
Page d-3

Use the illustration to review that in the equation, $3 + 6 = 9$, the answer, 9, is the sum. Then call attention to the first frame. Have children describe how many in each set and relate these numbers to the equation. Explain that for each frame they should study the sets and find the sum for the given equation. Encourage them to complete the page on their own. Allow those who wish to use counters to do so.

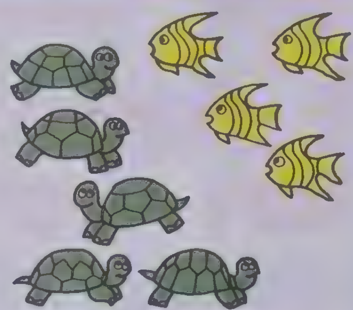


Find the sums.



$2 + 8 = \boxed{10}$

$4 + 6 = \boxed{10}$



$5 + 4 = \boxed{9}$

$7 + 3 = \boxed{10}$

Addition combinations for sums of 10 or less

Objective

Given an addition equation with two addends whose sum is 10, the child will be able to find the sum using a related set illustration.

Pre-Book Activity

Materials

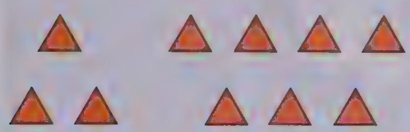
10 counters per child

Direct the children to count their counters and make sure they each have 10. Tell them that you are thinking of two sets which when combined give ten. Explain how many one of your sets contains and ask children to find the other set needed to have 10 altogether. For example, say: "I am thinking of a set of 3. How many more counters do I need to make 10?" Then elicit from the children the related addition equation, $3 + 7 = 10$, and write it on the chalkboard. Work through several of the following examples:

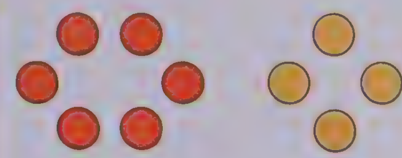
$1 + 9 = 10$	$9 + 1 = 10$
$2 + 8 = 10$	$8 + 2 = 10$
$3 + 7 = 10$	$7 + 3 = 10$
$4 + 6 = 10$	$6 + 4 = 10$

$5 + 5 = 10$

Find the sums.



$3 + 7 = \boxed{10}$

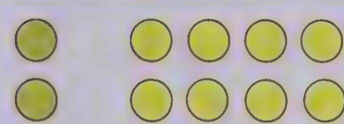


$6 + 4 = \boxed{10}$

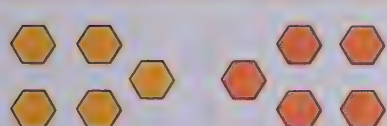
$4 + 3 = \boxed{7}$

$9 + 1 = \boxed{10}$

$7 + 2 = \boxed{9}$



$2 + 8 = \boxed{10}$



$5 + 5 = \boxed{10}$

$3 + 5 = \boxed{8}$

Addition combinations for sums of 10 or less—sets

Teaching
Page d-4

Have the children observe the sets of triangles in the first frame and ask them first to identify the number of each subset of triangles. Then call their attention to the number of triangles in all. Ask them to write the sum in the box and read: "Three plus seven equals ten." Explain that they should continue to study each frame similarly and write the sum in the box for each equation. When they have finished, it would be helpful to discuss the sets and the related equations to the sets.

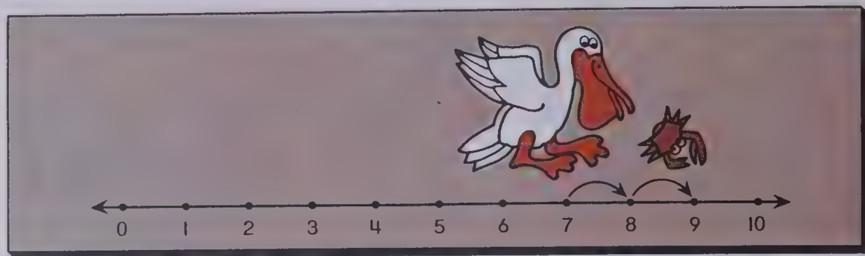
Follow-up

Ask the children to make the fact cards for the addition combinations listed in the pre-book activity. If possible, have children make these of the same material they used for the fact cards for the combinations they studied earlier.

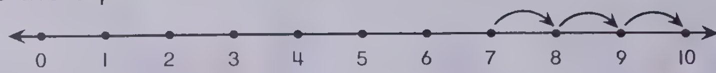
Teaching
Page d-5

Call the children's attention to the number line in the illustration and relate some number line stories to it and to the equation $7 + 2 = 9$. Be sure children realize that the number line is labelled to include 10. You might point out how the other numbers that they have studied could also be shown because we can think of the number line as going on and on.

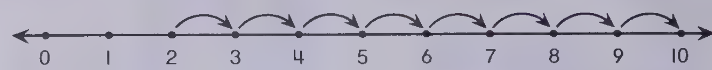
Then ask the children to study the number line above the equation $7 + 3 = \square$. Elicit from them that the number line can be used as an aid in solving the equation. Point out that they can find the point for 7 and then count 3 spaces to the right to find $7 + 3$. Use the following number line and equation, $2 + 8 = \square$, similarly. Then encourage the children to solve the remaining equations by using the number lines provided on page d-6. When the children are finished have them use a demonstration number line to show how they solved the equations.



Solve the equations.



$$7 + 3 = \boxed{10}$$



$$2 + 8 = \boxed{10}$$

$5 + 5 = \boxed{10}$	$9 + 1 = \boxed{10}$
$4 + 5 = \boxed{9}$	$6 + 3 = \boxed{9}$
$6 + 4 = \boxed{10}$	$10 + 0 = \boxed{10}$
$2 + 5 = \boxed{7}$	$8 + 2 = \boxed{10}$
$3 + 7 = \boxed{10}$	$4 + 3 = \boxed{7}$
$4 + 4 = \boxed{8}$	$3 + 7 = \boxed{10}$

Addition combinations for less than 11-number line

Objective

Given addition equations for sums of 10 and a number line labelled from 0 to 10, the child will be able to find the sums by referring to the number line.

Pre-Book Activity

Use a demonstration number line and make up stories about animals or insects jumping along the line. As you tell a story, ask the children to figure out the equation which records the story. Although you will want most of the combinations you use to be for sums of 10, occasionally use a sum of 9 or less to keep the children alert. It would be helpful to have the children use individual symbol cards to show the story in equation form.

Encourage them not only to build the equations for stories you make up, but also to take turns making up stories themselves.

Find the sums.

0 1 2 3 4 5 6 7 8 9 10

$1 + 9 = \boxed{10}$	$5 + 2 = \boxed{7}$
$6 + 4 = \boxed{10}$	$3 + 6 = \boxed{9}$
$4 + 2 = \boxed{6}$	$7 + 3 = \boxed{10}$
$5 + 5 = \boxed{10}$	$4 + 5 = \boxed{9}$
$8 + 2 = \boxed{10}$	$4 + 6 = \boxed{10}$
$3 + 4 = \boxed{7}$	$2 + 6 = \boxed{8}$

$$\begin{array}{r}7\\+3\\\hline 10\end{array}\quad\begin{array}{r}2\\+8\\\hline 10\end{array}\quad\begin{array}{r}5\\+4\\\hline 9\end{array}\quad\begin{array}{r}3\\+5\\\hline 8\end{array}\quad\begin{array}{r}4\\+6\\\hline 10\end{array}\quad\begin{array}{r}9\\+0\\\hline 9\end{array}$$

$$\begin{array}{r}6\\+2\\\hline 8\end{array}\quad\begin{array}{r}4\\+3\\\hline 7\end{array}\quad\begin{array}{r}5\\+5\\\hline 10\end{array}\quad\begin{array}{r}7\\+2\\\hline 9\end{array}\quad\begin{array}{r}1\\+9\\\hline 10\end{array}\quad\begin{array}{r}3\\+3\\\hline 6\end{array}$$

Addition combinations for less than 11-number line

Teaching
Page d-6

Call attention to the number line at the top of the page. Explain to the children that they may use it as they solve the equations. If they use the eraser end of their pencil to jump along the number line they can use it for all the problems.

Point out the vertical notation used for the exercises at the bottom section of the page. Remind children that they may think through these in the same way they do equations.

For $\begin{array}{r}7\\+3\\\hline\end{array}$

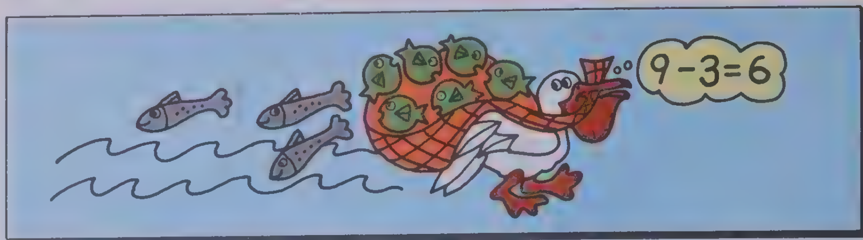
they may think: "seven plus three is ten."

Follow-up

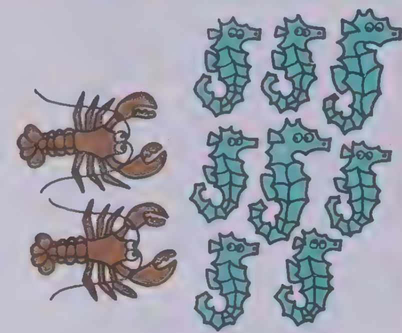
Roll out along the classroom floor a piece of paper about 50 centimetres wide and 4 metres long, on which you have marked an area for each number from 0 to 10. Have previously prepared sums written on cards and placed in a pile or box. The children can "fish" for a sum. If they pull, for example, 3 + 6, they should start at 3, then jump 6 steps to land on 9. The child who gets closest to 10 after two turns wins. Children can make up their own rules, or you might decide that the sum the child pulls on his second turn must be less than or equal to the amount needed to get to 10. Thus if a child's first sum was 3 + 6, and his second 2 + 3, he would have to stay on 9. But if a child's first sum was 2 + 3 and his second was 4 + 0, his first move would be to 5 and his second to 9.

Teaching
Page d-7

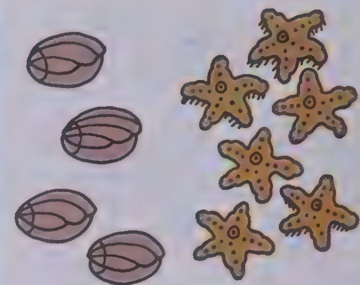
Again use the illustration to introduce page d-7. Call the children's attention to the first frame. Have them figure out how many objects there are in all and that each set may be thought of as two groups or subsets. Also discuss any action the illustrated subsets make them think of. Relate the subsets to the equation $10 - 8 = \square$. When the children have completed the equation, direct them to finish the remaining exercises independently.



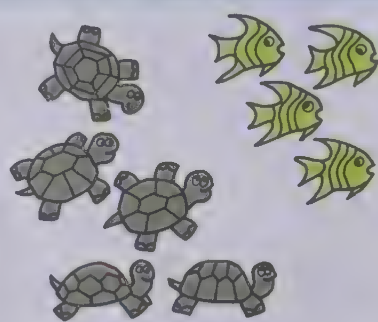
Find the differences.



$10 - 8 = \boxed{2}$



$10 - 6 = \boxed{4}$



$9 - 4 = \boxed{5}$

$10 - 3 = \boxed{7}$

Objective

Given subtraction equations related to combinations of ten, the child will be able to find the differences by studying a related set illustration.

Pre-Book Activity

Materials

counters, at least 10 for each child
11 cards, numbered from 0 to 10

Ask the children to place a set of 10 counters on their desks or table tops. Show them a set of cards numbered from 0 to 10 and ask a child to pick one. Explain that for each number that is picked you want them to subtract that number from their set of 10. Thus, if the first child picks 8, they should all subtract 8 from 10. As they do this, ask a child to write the subtraction equation on the chalkboard, $10 - 8 = 2$. Then have a child pick another card from the pile and subtract that number from 10. Continue similarly until all the cards have been used.

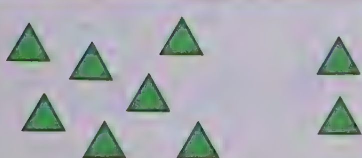
Find the differences.

$10 - 7 = \boxed{3}$

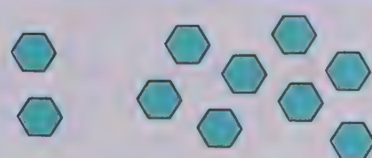
$10 - 4 = \boxed{6}$

$7 - 3 = \boxed{4}$

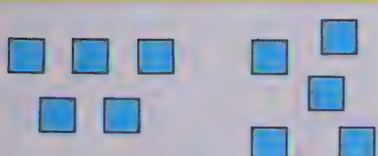
$10 - 1 = \boxed{9}$



$9 - 2 = \boxed{7}$



$10 - 8 = \boxed{2}$



$10 - 5 = \boxed{5}$

$8 - 5 = \boxed{3}$

Differences associated with sums of less than 11 – sets

Teaching
Page d-8

Since children should not have difficulty knowing what is expected of them on this page, ask a child to explain what he thinks they should do. Then encourage them to do the exercises independently. Note that a few frames review combinations other than those of ten. If some children wish to use counters, allow them to do so. However, most will be able to use the pictured sets as an aid in solving the equations.

Follow-up

You might instruct the children to make subtraction fact cards related to sums of 10. Remind them to use the same color of crayon they previously chose for subtraction facts so that these cards may be added to their growing set of individual fact cards.

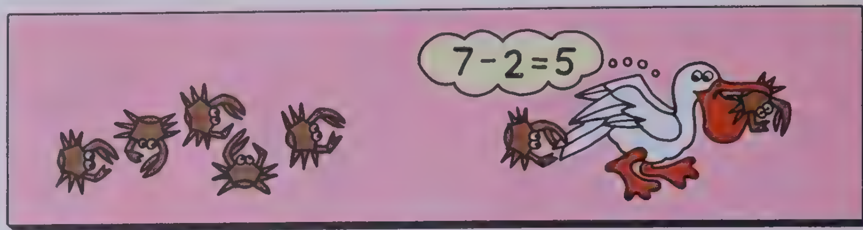
To give the children more practice on the basic combinations, duplicate an exercise sheet similar to the following:

Find the pattern and solve the problems.

$\begin{array}{r}1\\+9\\\hline\end{array}$ $\begin{array}{r}10\\-9\\\hline\end{array}$	$\begin{array}{r}3\\+5\\\hline\end{array}$ $\begin{array}{r}8\\-5\\\hline\end{array}$	$\begin{array}{r}4\\+5\\\hline\end{array}$ $\begin{array}{r}9\\-5\\\hline\end{array}$
$\begin{array}{r}6\\+4\\\hline\end{array}$ $\begin{array}{r}10\\-4\\\hline\end{array}$	$\begin{array}{r}1\\+7\\\hline\end{array}$ $\begin{array}{r}8\\-7\\\hline\end{array}$	$\begin{array}{r}7\\+3\\\hline\end{array}$ $\begin{array}{r}10\\-3\\\hline\end{array}$
$\begin{array}{r}2\\+7\\\hline\end{array}$ $\begin{array}{r}9\\-7\\\hline\end{array}$	$\begin{array}{r}2\\+3\\\hline\end{array}$ $\begin{array}{r}5\\-3\\\hline\end{array}$	$\begin{array}{r}8\\+2\\\hline\end{array}$ $\begin{array}{r}10\\-2\\\hline\end{array}$

Teaching
Page d-9

Many children will be able to do this page without the aid of counters, a number line, or illustrated sets. However, children should be made to understand that they may use any of these materials if they need them. For those who want to use materials you might suggest that they first try to do the equations without using the materials and that they then use the materials to check their answers. To introduce the page help children relate the demonstration art to the equation $7-2=5$. When the children have finished, check several of the exercises with them.



Find the differences.

$10-3=\boxed{7}$	$9-3=\boxed{6}$
$10-7=\boxed{3}$	$10-1=\boxed{9}$
$9-4=\boxed{5}$	$6-4=\boxed{2}$
$10-5=\boxed{5}$	$7-4=\boxed{3}$
$8-4=\boxed{4}$	$10-4=\boxed{6}$

$$\begin{array}{r}10\\-2\\\hline 8\end{array}\quad\begin{array}{r}5\\-0\\\hline 5\end{array}\quad\begin{array}{r}9\\-5\\\hline 4\end{array}\quad\begin{array}{r}10\\-6\\\hline 4\end{array}\quad\begin{array}{r}6\\-5\\\hline 1\end{array}\quad\begin{array}{r}8\\-5\\\hline 3\end{array}$$

$$\begin{array}{r}7\\-5\\\hline 2\end{array}\quad\begin{array}{r}10\\-8\\\hline 2\end{array}\quad\begin{array}{r}9\\-6\\\hline 3\end{array}\quad\begin{array}{r}10\\-9\\\hline 1\end{array}\quad\begin{array}{r}7\\-7\\\hline 0\end{array}\quad\begin{array}{r}8\\-3\\\hline 5\end{array}$$

Objective

Given addition or subtraction equations for combinations of 10 or less, the child will be able to find the sums or differences.

Pre-Book Activity

Plan an oral warm-up game by giving combinations and asking the children whether your sum or difference is correct. For example, say: "Five plus three equals eight. Correct?" Continue, mixing incorrect with correct equations to keep the children alert. Then say: "Let's see how well you can do a group of combinations in your head. Remember each answer, and use it in the next part of the problem." To start, give the children a number, like 4, and say: "Plus three, [pause] minus two, [pause] plus five." Tell them to write their answers on a piece of paper.

Call on a child to give his final answer, and ask the class to agree or disagree. (Hint: A previously prepared list of such sequential activities will save your trying to recall the precise problem, which becomes difficult when problems become lengthy and succeed one another rapidly.)

Find the sums and differences.

$6 + 3 = \boxed{9}$	$9 - 4 = \boxed{5}$
$5 + 5 = \boxed{10}$	$10 - 4 = \boxed{6}$
$2 + 7 = \boxed{9}$	$8 - 6 = \boxed{2}$
$3 + 7 = \boxed{10}$	$10 - 2 = \boxed{8}$
$10 - 5 = \boxed{5}$	$4 + 6 = \boxed{10}$
$8 + 2 = \boxed{10}$	$10 - 3 = \boxed{7}$
$8 - 3 = \boxed{5}$	$7 - 4 = \boxed{3}$
$5 + 4 = \boxed{9}$	$7 + 3 = \boxed{10}$

$\begin{array}{r} 6 \\ +4 \\ \hline 10 \end{array}$	$\begin{array}{r} 8 \\ -4 \\ \hline 4 \end{array}$	$\begin{array}{r} 10 \\ -9 \\ \hline 1 \end{array}$	$\begin{array}{r} 2 \\ +8 \\ \hline 10 \end{array}$	$\begin{array}{r} 9 \\ +1 \\ \hline 10 \end{array}$	$\begin{array}{r} 10 \\ -2 \\ \hline 8 \end{array}$
$\begin{array}{r} 10 \\ -3 \\ \hline 7 \end{array}$	$\begin{array}{r} 2 \\ +6 \\ \hline 8 \end{array}$	$\begin{array}{r} 3 \\ +4 \\ \hline 7 \end{array}$	$\begin{array}{r} 9 \\ -2 \\ \hline 7 \end{array}$	$\begin{array}{r} 5 \\ +5 \\ \hline 10 \end{array}$	$\begin{array}{r} 10 \\ -1 \\ \hline 9 \end{array}$

Teaching
Page d-10

Call the children's attention to the first frame and ask them what type of equations (addition or subtraction) these four are. Then have them observe that the next frame contains only subtraction equations. Finally point out the mixture of addition and subtraction equations in the remaining exercises. Stress how important it is for them to watch the signs as they try to solve these equations. Again, when they are finished, check several of the exercises with them.

Follow-up

For additional practice on facts, duplicate, or write on the chalkboard, an exercise similar to the following.

Subtract 3

10		8	
6		4	
5		7	
3		9	

Subtract 4

10		9	
8		7	
6		5	
4		10	

Add 5

5		0	
2		3	
1		4	

Add 2

8		6	
5		4	
7		1	

Resources for Active Learning

Using a math balance:

A CLOUDBURST, Vol. 1, No. 1511, Midwest Publications

MATHEX: Numeration No. 2, pp. 22–25, Encyclopaedia Britannica Publications Ltd.

Nuffield Project: COMPUTATION AND STRUCTURE (2), pp. 65, Wiley

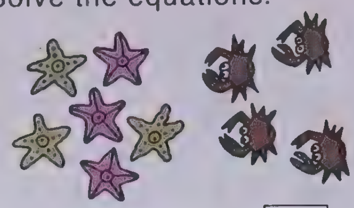
USING . . . MATHEMATICAL BALANCE, Math Media

Teaching
Page d-11

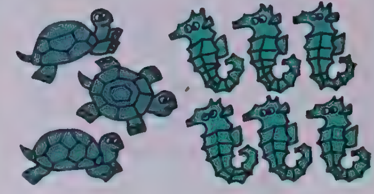
Although three sets of directions are given on this page children should not have difficulty in understanding what to do. In all of the frames they are to find the sums or differences as the signs indicate. Point out that the illustrations provide sets which they should study to help them solve the related equations. They should try to work the other exercises without dependence on materials, but if they wish to check their answers with counters, the number line, or strips, they should feel free to do so.

Show you know

Solve the equations.

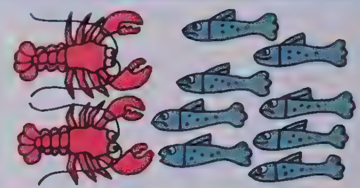


$6 + 4 = \boxed{10}$



$3 + 6 = \boxed{9}$

$7 + 3 = \boxed{10}$



$2 + 8 = \boxed{10}$

Find the sums.

$4 + 6 = \boxed{10}$

$5 + 4 = \boxed{9}$

$1 + 9 = \boxed{10}$

$\begin{array}{r} 5 \\ +5 \\ \hline 10 \end{array} \quad \begin{array}{r} 6 \\ +2 \\ \hline 8 \end{array} \quad \begin{array}{r} 9 \\ +1 \\ \hline 10 \end{array}$

Find the differences.

$10 - 4 = \boxed{6}$

$9 - 3 = \boxed{6}$

$10 - 7 = \boxed{3}$

$\begin{array}{r} 10 \\ -5 \\ \hline 5 \end{array} \quad \begin{array}{r} 8 \\ -3 \\ \hline 5 \end{array} \quad \begin{array}{r} 10 \\ -6 \\ \hline 4 \end{array}$

Objective

The child will demonstrate his ability to work with the concepts presented in this module by finding the sums or differences for combinations of 10 or less.

Pre-Book Activity

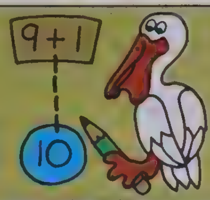
An oral-review game such as "Cross Over the Bridge" would be appropriate before children begin the text pages. Direct two teams of children to line up on opposite sides of the room. Suggest that the children imagine a bridge across the open space at the front of the room. The first two members of each team compete against each other to correctly answer a flash card which you display showing an addition or subtraction combination to 10. When a flash card is shown, the first one to give a correct answer may stay on his side. The other person must "cross over the bridge." At the end of the game, the team with the most members wins.

You might also place a set of 10 objects on the flannel-board. Show two readily recognizable subsets, such as 6 orange circles and 4 blue circles. Ask a child to write an addition equation on the board to go with the set presented. Ask another child for a second addition equation and other volunteers for the two related subtraction equations.

Follow-up

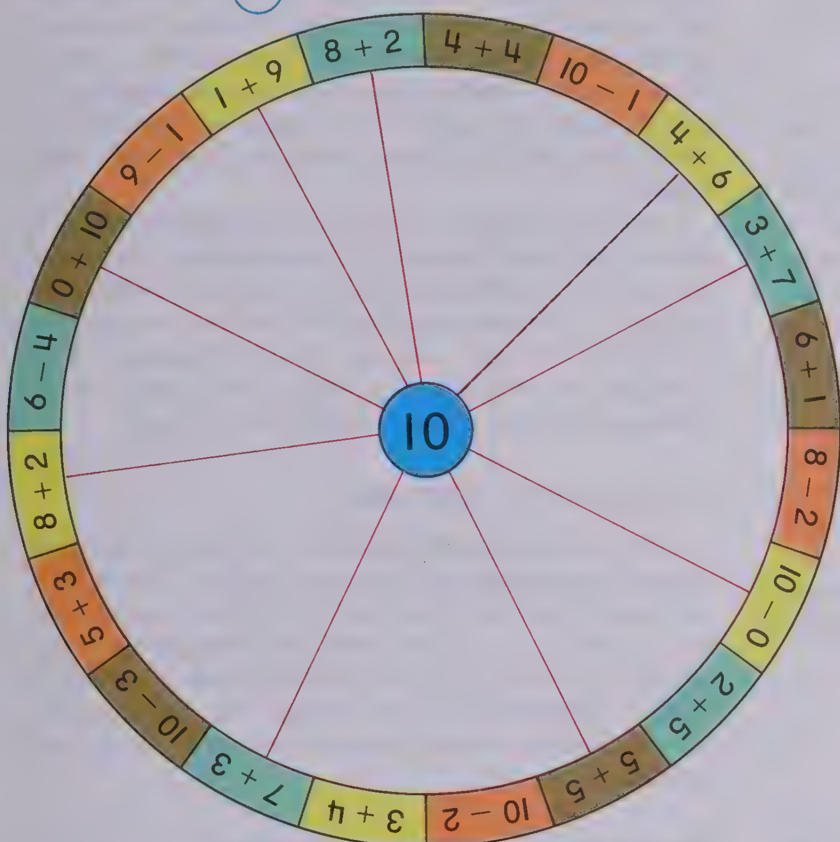
Activities similar to the one on page d-12 may be duplicated for other numbers such as 6, 7, 8, and 9. Or you might challenge the children themselves to choose a number and make a page similar to d-12 for a classmate to do.

Let's have fun



Which sums and differences equal 10?

Connect them to 10.



Addition, subtraction combinations

Teaching
Page d-12

Read the directions for the children and be sure they realize that for every sum or difference which equals 10 they should draw a line from the combination to the 10 in the centre. Stress that they should figure out each sum or difference so that they can be certain whether or not the combination equals 10. When they finish they might color the regions of the circle which contain sums of 10.

The combinations need not be written in the form of a circle; children might simply ring the correct phrases scattered around the given number.

4 + 3 10 − 2 5 + 3
2 + 7
10 − 3
7 + 1 8 2 + 5
10 − 4 8 + 0
4 + 4 2 + 6
7 − 1

Children might also enjoy a dice game. Number the faces of two small cubes from 0 to 5. Explain to the children that the object of the game is to roll a sum of 10, or near 10. If the first roll does not yield a sum of 10, the player may roll the dice again. However, he can add to his first sum only a number which will bring him nearer to or equal to 10; he may not go over 10. Thus, if his first roll is 3 and 4, his sum is 7. He should then roll again, but if his dice yield a sum greater than 3 he must keep 7 as his score. The player who, after 2 rolls, has a number nearest to 10 wins.

ORANGE MODULE, UNIT D

Sums and Differences 11 to 18 — Power Skills

Pages d-13 to d-24

General Objectives

To introduce the child to combinations of 11 through 18

To develop power skills for finding the sums and differences of these combinations

At the outset of this module, we wish to make one important point: The fact that combinations for the sums 11 through 18 are presented here should *not* in any way imply that the children are expected to master or memorize these combinations. Our intention is to expose the children to the ideas and techniques for finding these sums. We expect children to be able to look at an equation such as $8 + 7 = \square$ and by using one of several power skills to arrive at the sum.

It is also important to note that this is the first of two modules which deal with these combinations. The techniques which this module stresses for finding these sums and differences are basically counting techniques. Thus, the power skills developed in this module should be within the grasp of a first grader. The light green module, which should be considered optional for many classes, stresses a more advanced technique of regrouping, that is, of "making ten."

Mathematics

Since this is a power skill module, as opposed to a speed skill module, there are a number of important mathematical concepts used to develop sums 11 through 18. First, the idea of joining or forming the union of two sets is given as a method for finding sums. Secondly, concepts associated with lengths are used in the form of the centimetre strips and jumps on the number line. The strong emphasis upon the use of these mathematical ideas, union of sets and length, should not be interpreted as an indication that the speed skills, or memorization of the basic facts, is to be ignored. At this time the children are at the power skill stage, and this is the key time to develop the underlying mathematical concepts. The speed skills will be developed later.

Teaching Orange Module, Unit D

Approximate Time: 6 to 9 days

Materials

counters (at least 20 per child)
demonstration number line
number line for each child, if possible
strips, 1 set per child

There are three basic power methods developed in this module. These methods utilize three important kinds of materials, the counters, the strips, and the number line. It is essential to the development of the ideas treated here that these materials be available *for each child.* If erasable number lines are not available, the book provides a number line when appropriate. Most children will be able to use the number line without actually marking on it with pencil; for example, they might use the non-lead end of their pencil or simply make the jumps with their fingers.

Encourage children to develop an understanding of each of the three methods. It is expected that each child will be able to effectively use at least one method for finding the combinations, but help as many as possible to develop the ability to use all three methods. With such understanding the child will be able to find sums or differences which he cannot recall from memory.

Evaluation of Progress

Your daily observation will help you determine which children understand the processes studied. The evaluative page at the end of the module will help you see how well a child can use these methods independently. Remember that since it is not intended that the children master combinations of 11 through 18, automatic responses to these combinations should not be used as an evaluation criterion.

Resources for Active Learning

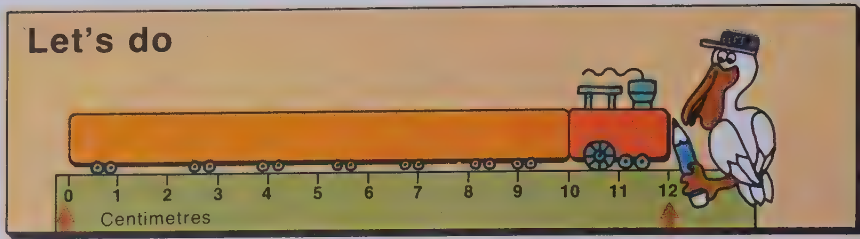
General Activities:
ENRICHMENT OF ARITHMETIC, Rhymes, 1/29–31, Webster, McGraw-Hill
Materials to study facts:
MATH ACTIVITIES, Materials 3/3–26, pp. 88–95, Allyn and Bacon
Manipulative Devices:
Cubical Counting Blocks, (Milton Bradley; school supplier)
Cuisenaire Rods (Cuisenaire Co.)
Unifix material (Educational Teaching Aids; Math Media; Responsive Environments Corp.)
Commercial Games:
Construct-a-cube Puzzle (Selective Educational Equipment)

Investigation
Page d-13

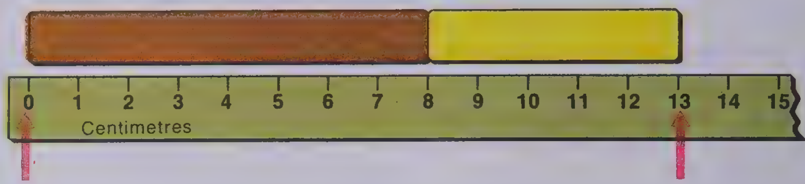
Read the directions with the children. Suggest that they first use their brown strip and yellow strip to place on top of the illustration. Elicit from them the meaning of the phrase $8 + 5$ and have them trace over the dashed numerals. You might then ask a child to suggest another strip, such as the dark green, and direct them to find the second strip they need to build a train that ends at 13. Remind them to record the strips they use to build each train in the space provided. If a child chooses $4 + 9$ and $9 + 4$ as two separate trains, he is correct, but point out that since he is using the same strips for both trains you would like him to find another combination of strips which could also be used. However, be sure he realizes that the trains $9 + 4$ and $4 + 9$ represent the same sum. As children work through this page help them see that the arrows under the ruler indicate the beginning and end of the trains.

If time permits, you might ask children to build trains for 11, 12, and 15. With centimetre rulers or duplicated copies of centimetre rulers, the children might also build trains for 16, 17, and 18. However, it is not intended that they work through all of these sums at this time.

Let's do



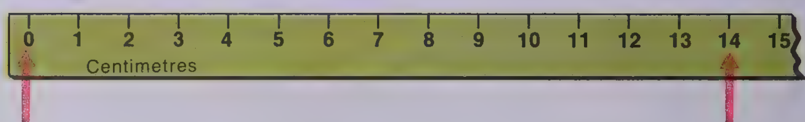
Find other two-strip trains that fit between the arrows.



8 + 5	3 + 10	4 + 9

Another is: 6+7.

How many two-strip trains can you find for these arrows?



7 + 7	8 + 6	9 + 5

Another is: 10+4.

Readiness for sums greater than 10

Purpose

To begin to develop power skills that might be used in finding sums of 11 through 18

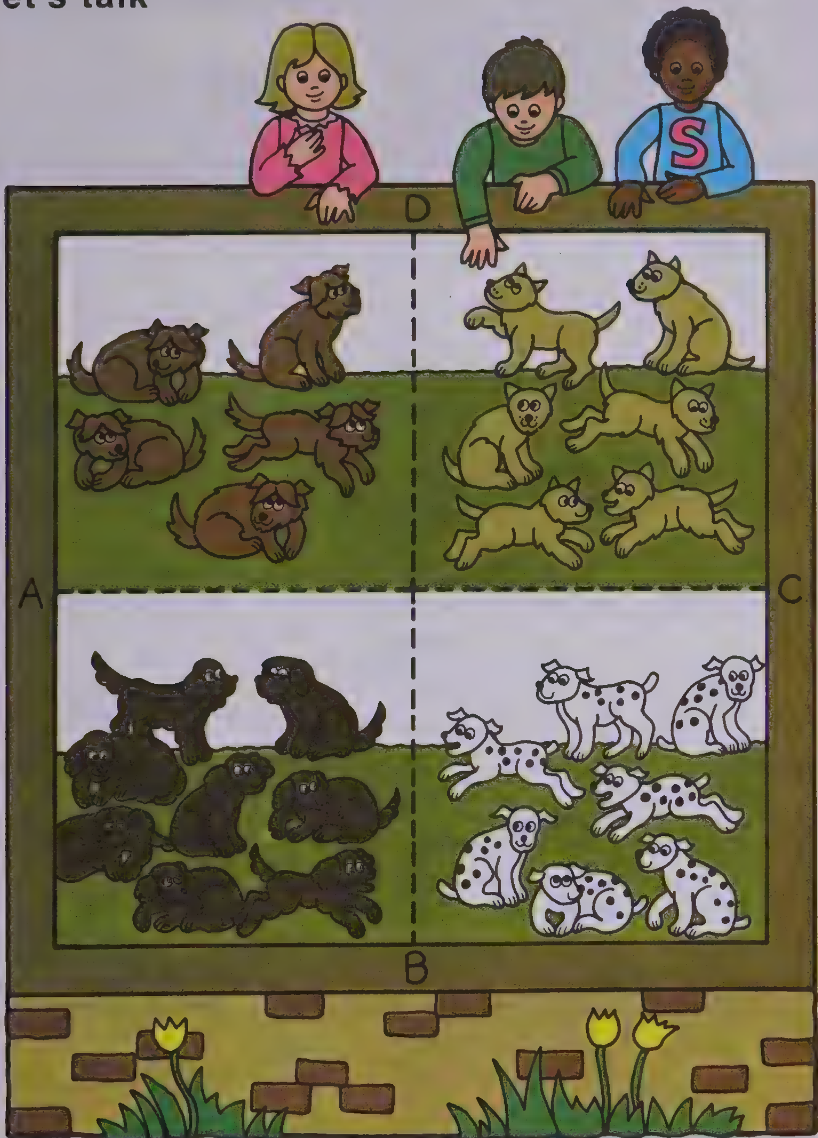
Preparation

Materials

centimetre rulers or a duplicated copy of six 18-centimetre rulers (optional)
set of strips for each child

To prepare for the investigation, allow the children a few minutes of free play with the strips. Then it would be helpful to review how to make trains with the strips by placing them beside a centimetre ruler. For example, ask the children to use the 2-strip and some other strip to match 7. Or ask the children to use a 2-strip and a 5-strip and find the mark on the ruler where the train ends. (If each child does not have a centimetre ruler, have him do this introductory activity on the ruler at the bottom of page d-13.) Have a child read the numeral where the train ends and point out that the sum of 2 and 5 may be found at this point.

Let's talk



Readiness for sums greater than 10

Discussion
Page d-14

Throughout this module various power skills for finding sums to 18 will be used. The method emphasized on page d-14 is that of counting. Direct the children to place 4 brown or black strips along the four dashed lines and imagine them as fences between the pens. Then ask the children to remove one of these fences and count the number of puppies in the large pen that has been formed. For example, direct them to remove the fence between the 5 puppies and the 8 puppies and ask, "How many puppies are in the larger pen?" In this case, they will find 13 puppies in the larger pen. You might observe with them that they can count the 5 in the top section and the 8 in the bottom or vice versa, beginning with 8 at the bottom and then counting 5 more in the top. Stress that in either case the total is 13.

Follow-up

The suggestion in the Investigation section to extend the activity to include sums of 16, 17, and 18 might be used as a follow-up activity.

Children would also benefit from counting and grouping themselves into a group of 15 or 16 or any number between 11 and 18. Then ask them to record the different combinations of two subsets they could make, and write these in equation form.

Resources for Active Learning

A CLOUDBURST, Vol. 1, No. 2221, Midwest Publications

Teaching
Page d-15

Explain to the children that the ringed area at the top of the page will be used to "hold" their counters. Point out the first equation and read the accompanying directions for them. Ask the children to put in 7 counters and then put in 5 more. Then ask them how they can find out how many they have in all. Most will simply count the counters in the ring. You might suggest that they put their counters in piles so that they will actually fit in the ring. When they have decided how many counters they have in all, direct them to complete the equation by writing 12 in the box. When the children understand what they are to do encourage them to continue independently. Move around the room to be sure children are counting their counters correctly and finding the correct sum. When the children finish, help them check their answers carefully.

Put in		Put in		How many?
7	+	5 more	=	$\boxed{12}$
6	+	7 more	=	$\boxed{13}$
5	+	9 more	=	$\boxed{14}$
8	+	7 more	=	$\boxed{15}$

Sums greater than 10—sets

Objective

Given an addition equation for a sum of 11 through 18, the child will be able to find the sum by counting sets of counters.

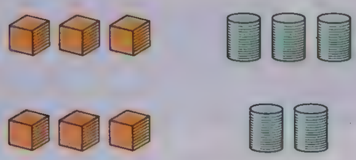
Pre-Book Activity

Materials

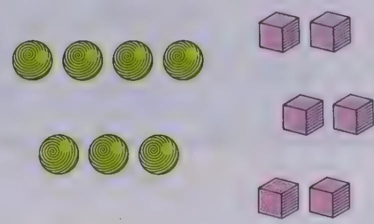
counters about 20 per child

Since the text material itself contains an activity, it would be appropriate to begin immediately with page d-15. However, if you prefer a warm-up activity, briefly review counting sets which contain 11 through 18 objects.

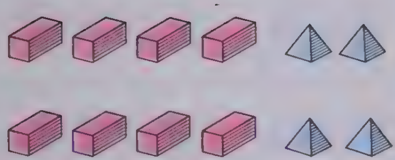
Find the sums.



$6 + 5 = \boxed{11}$



$7 + 6 = \boxed{13}$



$8 + 4 = \boxed{12}$

$9 + 5 = \boxed{14}$

$7 + 4 = \boxed{11}$	$3 + 9 = \boxed{12}$
$8 + 7 = \boxed{15}$	$8 + 9 = \boxed{17}$
$6 + 9 = \boxed{15}$	$6 + 8 = \boxed{14}$
$8 + 8 = \boxed{16}$	$9 + 9 = \boxed{18}$

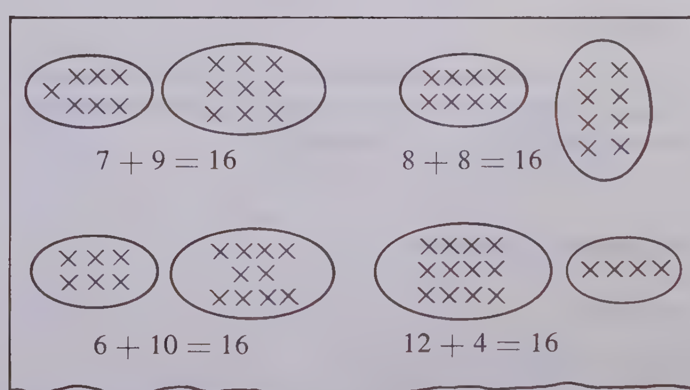
Sums greater than 10—sets

Teaching
Page d-16

Although the first four equations are accompanied by illustrated sets, many children will still benefit from using their counters for the equations on this page. Suggest that they continue to use the same method with the counters as they did on page d-15. Call their attention to the first frame. Ask them to explain how they can tell how many counters they should use. Stress that the numbers in the equation tell them how many counters to use so that even when there is no accompanying picture they should be able to solve the equation by using counters. When they finish, you might ask some children to show how they solved the equations.

Follow-up

Distribute a large sheet of newsprint to each child. Instruct the children to fold the paper in half one way and then in half the other way so that they have four sections. Suggest that they choose any number from 11 to 18 and draw four pictures showing set combinations which total their chosen number. They might also write an equation for each of their pictures.



Resources for Active Learning

For using the math balance refer to Book One, Unit D, Yellow Module, page d-10 under "Resources for Active Learning."

Teaching
Page d-17

Ask a child to explain what he thinks the large ringed area will be used for. Then call attention to the first equation. Read with the children the directions over each part of the equation. Stress that the sign of the equation indicates that they should subtract. Thus, after they have put 12 counters into the ring the subtraction sign tells them that they should take out 5 counters. They can then count the remaining counters to see how many are left in the ring. When children catch on to the procedure, encourage them to continue on their own. Move around the room to be sure they correctly count the remaining counters and write the correct numeral in the appropriate box.

Put in		Take out		How many?
12	−	5	=	7
14	−	9	=	5
13	−	7	=	6
15	−	7	=	8

Subtraction combinations for sums greater than 10—sets

Objective

Given a subtraction equation related to a sum between 11 and 18, the child will be able to find the difference by manipulating sets of counters.

Pre-Book Activity

Materials

counters: about 20 per child

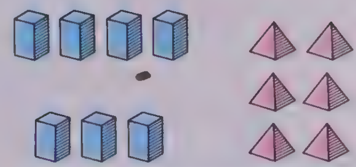
As with the previous lesson, you might appropriately begin with the text page immediately. However, if you prefer, review the order of numbers between 11 and 18. For example, ask questions such as: "What number is one less than 15?" or "What number comes just before 15?" (Recall that here we are referring to whole numbers.) Also show numeral cards from 11 to 18 and have the children put them in the correct order.

Follow-up

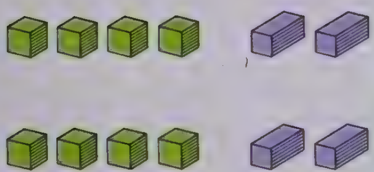
Prepare a pack of domino cards showing combinations on one end and addition or subtraction phrases on the other as described below. One child deals each of four players (himself one of the four) 6 cards. The dealer should then play one of his cards. The player to his left should try to match either end of the dealer's card with one end of one of his cards which has an equivalent value. The next player likewise tries to match equivalent values. The first child who successfully uses all of his cards wins. Certain rules are helpful. For example, if a child cannot

Find the differences.

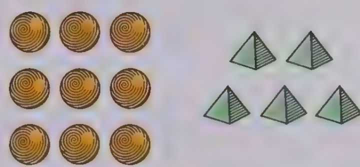
$11 - 5 = \boxed{6}$



$13 - 6 = \boxed{7}$



$12 - 4 = \boxed{8}$



$14 - 5 = \boxed{9}$

$11 - 4 = \boxed{7}$ $\quad$ $12 - 9 = \boxed{3}$

$15 - 7 = \boxed{8}$ $\quad$ $17 - 9 = \boxed{8}$

$15 - 9 = \boxed{6}$ $\quad$ $14 - 8 = \boxed{6}$

$16 - 8 = \boxed{8}$ $\quad$ $18 - 9 = \boxed{9}$

Subtraction combinations for sums greater than 10—sets

Teaching
Page d-18

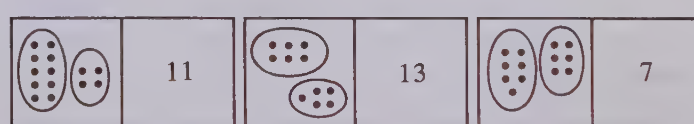
As on page d-16, illustrated sets are provided for the first four equations, but it is intended that the children solve most of the equations by using counters. Work through the first frame, relating the pictured sets to the equation $11 - 5 = \square$. Also have the children build a set of 11 counters, remove 5 and count those remaining. When you think the children are ready, encourage them to continue independently. As they work, move around the room watching in particular that children do not mix up the two subsets which are formed when they remove the number which was given to subtract. When they finish it would be helpful to have children use counters and explain how to solve several of the equations.

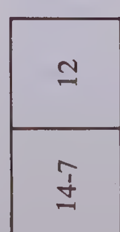
find a match, he should pass. If it happens that two children in a row must pass, the third player may be allowed to put down any of his cards, but not next to one already played. That is, he should start a new group.

Directions for cards: The following numbers, sets, and phrases would make a complete set. Use small cards. The phrases written in italics should be shown as sets.

7 + 4 / 5	*6 + 5* / 13	18 − 8 / 16
6 + 6 / 6	*9 + 4* / 14	*6 + 4* / 15
7 + 6 / 7	*8 + 4* / 15	13 / *9 + 8*
9 + 5 / 8	*9 + 2* / 16	12 / *9 + 9*
8 + 7 / 9	10 − 5 / 17	11 / *5 + 7*
8 + 8 / 10	12 − 6 / 18	13 / *7 + 7*
8 + 5 / 11	14 − 7 / 12	12 / *9 + 7*
9 + 3 / 12	16 − 8 / 14	11 / *9 + 6*

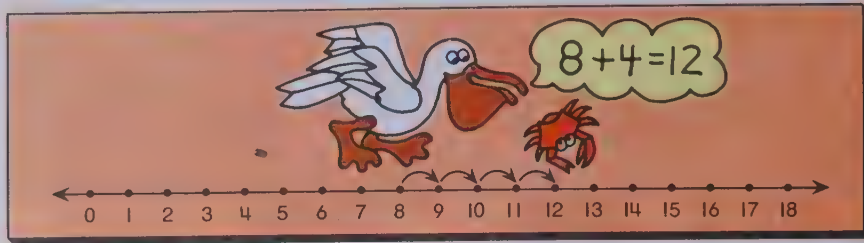
Sample plays:



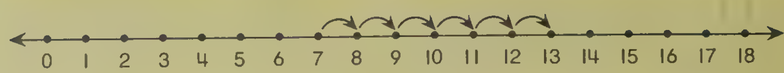


Teaching
Page d-19

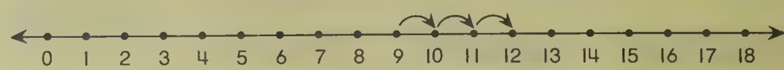
Again the illustration of the pelican and "friend" provide basis for introductory discussion. Then direct the children's attention to the first equation and the number line above it. Elicit from them what the arrows mean, why they start at 7, and how many jumps are pictured. Finally ask how this number line may be used as an aid in solving $7 + 6 = \square$, and have them write in the sum. Work through the other equations on this page similarly. Stress that the first number of the equation tells them how far from zero to start. The second number tells them how many jumps to make. And finally, the point on which they land after the last jump tells them the sum.



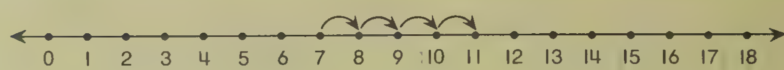
Solve the equations.



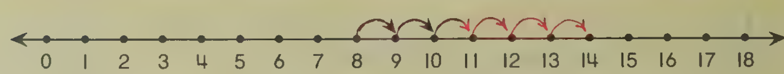
$7 + 6 = \boxed{13}$



$9 + 3 = \boxed{12}$



$7 + 4 = \boxed{11}$



$8 + 6 = \boxed{14}$

Objective

Given an addition equation for a sum of 11 through 18, the child will be able to find the sum by counting jumps on the number line.

Although this lesson stresses the method of using the number line as an aid in solving equations, children should be free to use counters or strips if they prefer. One of the main purposes of this module is simply to make available to the children various *power methods* suitable as aids in finding sums and differences for 11 through 18.

Pre-Book Activity

Begin with an oral review of the combinations of the numbers 10 or less. For example, say: "I am thinking of the sum of 6 and 3. What's my number?" After a few minutes of such a review, begin to present the more difficult combinations and to ask the children if they could use the demonstration number line as an aid in solving them. Work through three or four examples on the demonstration number line before proceeding into the text. Make up stories about the number-line pelican, and if children are mature enough, ask them to make up stories also.

Find the sums.

9 + 3 = $\boxed{12}$	8 + 4 = $\boxed{12}$
5 + 5 = $\boxed{10}$	6 + 8 = $\boxed{14}$
9 + 8 = $\boxed{17}$	5 + 4 = $\boxed{9}$
2 + 6 = $\boxed{8}$	9 + 6 = $\boxed{15}$
8 + 8 = $\boxed{16}$	6 + 7 = $\boxed{13}$
6 + 5 = $\boxed{11}$	0 + 5 = $\boxed{5}$

5 + 7 = 12	7 + 4 = 11	4 + 3 = 7	8 + 7 = 15	7 + 3 = 10	3 + 5 = 8
7 + 0 = 7	2 + 8 = 10	4 + 9 = 13	3 + 6 = 9	8 + 5 = 13	7 + 7 = 14

Teaching
Page d-20

Point out to the children the number line at the top. As a review you might have them count the points which are labelled. Explain to the children that the number line has been provided for their use as they try to solve the equations. They need not draw the arrows over the number line, but should simply use the line as a counting device.

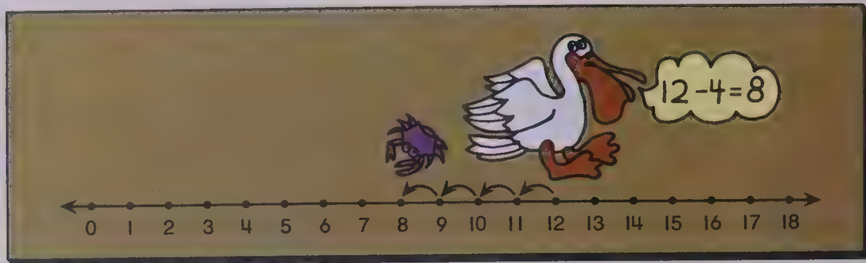
Emphasize again that they begin at the point indicated by the first number and take as many jumps as the second number indicates. Also point out that the exercises in vertical notation may be solved with the aid of the number line, since the children can think about these exercises in the same way they think about the equations above.

Follow-up

As a review of the combinations of 10 and less, allow the children to choose partners and to use their individual packs of flash cards. Let each child go through the pack once and record the combinations he missed. Then give the children ten minutes to study the cards they missed, and give them another chance to recheck these combinations with their partners. During this time, help individual children having difficulty with any of the power skills taught thus far.

Teaching
Page d-21

Again use the illustration to introduce the lesson. Then direct the children's attention to the first number line and equation. Elicit from them why the arrows begin at 13 and show 6 jumps to the left. Finally ask how this number line is being used as an aid in solving the equation $13 - 6 = \square$. When the children have written the numeral 7 in the box, continue to work through the remaining equations with the children. Notice that in the last frame children must complete the jumps on the number line themselves.



Find the differences.

$13 - 6 = \boxed{7}$

$12 - 3 = \boxed{9}$

$11 - 4 = \boxed{7}$

$14 - 6 = \boxed{8}$

Subtraction combinations for sums greater than 10-number line

Objective

Given a subtraction equation that is related to a sum of 11 through 18, the child will be able to find the difference by counting jumps on the number line.

Pre-Book Activity

Draw a number line on the chalkboard or overhead projector, and label it 0 through 18. Tell a subtraction story about the number-line pelican, and show the related jumps on the number line. Write the subtraction equation below the number line, and ask a child to tell how the number line can be used to help him find the difference. Continue with several other examples.

Find the differences.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

$11 - 5 = \boxed{6}$ $\quad$ $9 - 4 = \boxed{5}$

$10 - 6 = \boxed{4}$ $\quad$ $13 - 4 = \boxed{9}$

$16 - 7 = \boxed{9}$ $\quad$ $11 - 3 = \boxed{8}$

$8 - 2 = \boxed{6}$ $\quad$ $7 - 5 = \boxed{2}$

$13 - 5 = \boxed{8}$ $\quad$ $15 - 7 = \boxed{8}$

$9 - 6 = \boxed{3}$ $\quad$ $12 - 6 = \boxed{6}$

7	14	8	11	6	10
−3	−7	−3	−7	−2	−2
4	7	5	4	4	8

9	12	8	13	7	12
−3	−8	−4	−6	−7	−5
6	4	4	7	0	7

Subtraction combinations for sums greater than 10-number line

Teaching
Page d-22

As on page d-20, children are here provided with a number line to use as an aid in solving the equations. Remind the children that since these are subtraction equations, they must jump to the left. If some children prefer to use counters or strips as aids for solving these exercises, allow them to do so. However, most should be encouraged to use the number line.

Follow-up

Some children would benefit from a worksheet which has a number-line illustration for each equation they are to solve. An example of such a worksheet is found at the right.

Use the number line to find the difference.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

$15 - 7 = \square$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

$14 - 9 = \square$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

$12 - 6 = \square$

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18

$13 - 5 = \square$

TEACHING
Page d-23

Read the directions for the children. Point out the two sections of the page. Explain that for solving the equations in the top section they should use their counters or count those pictured on the page. For solving the equations in the bottom section they should use the number line. Also point out that some equations are addition, and some subtraction.

If a child strongly prefers the use of one aid over another, encourage him to use counters as suggested on the page, but allow him to use his preferred method if he chooses. Some children may check their answers by solving the same equation first with one method and then with another. You might also suggest that they use the strips to check some of their answers.

Show you know

Find the sums and differences. Use your counters.

$6 + 7 = \boxed{13}$	$11 - 4 = \boxed{7}$
$8 + 3 = \boxed{11}$	$14 - 7 = \boxed{7}$
$5 + 9 = \boxed{14}$	$12 - 5 = \boxed{7}$
$7 + 5 = \boxed{12}$	$13 - 4 = \boxed{9}$
$9 + 6 = \boxed{15}$	$15 - 6 = \boxed{9}$

Use the number line.

$8 + 4 = \boxed{12}$	$12 - 6 = \boxed{6}$
$8 + 7 = \boxed{15}$	$11 - 3 = \boxed{8}$
$2 + 9 = \boxed{11}$	$15 - 7 = \boxed{8}$
$8 + 5 = \boxed{13}$	$14 - 8 = \boxed{6}$
$7 + 7 = \boxed{14}$	$13 - 6 = \boxed{7}$

OBJECTIVE

The child will demonstrate his ability to work with the concepts presented in this module.

Since mastery of the combinations of 11 through 18 is not to be expected of first graders, page d-23 should be used to review or to evaluate a child's ability to use the *power* skills for finding sums and differences to 18.

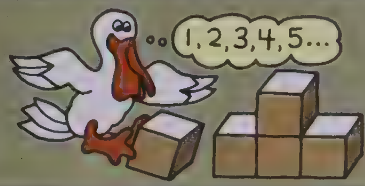
PRE-BOOK ACTIVITY

Materials

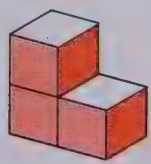
counters
number line
strips

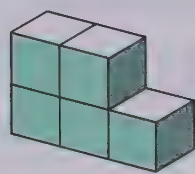
Display an equation from this module on the chalkboard, such as $7 + 6 = \square$. Remind children that they may solve this equation using any method they have just studied. Provide the materials and ask a child to show how he might use strips to solve this equation. Then ask a child to show the solution of the same equation with counters, and finally ask a third child to show it with the number line. Work through a subtraction equation such as $15 - 8 = \square$ in a similar manner.

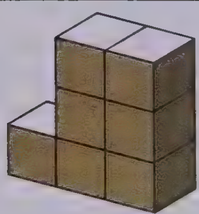
Let's have fun

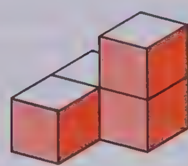


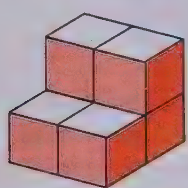
How many blocks?

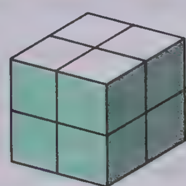

3

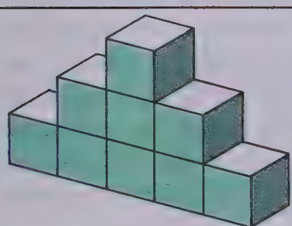

5

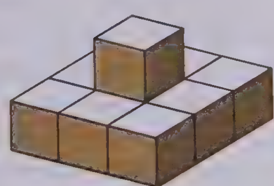

7


4


6


8


9


10

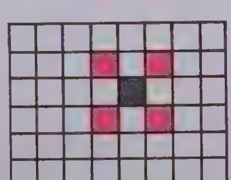
Readiness for volume concepts

Teaching
Page d-24

This change of pace page is simply an exercise in counting. However the three dimensional aspect of the illustrated cubes might cause difficulty. Suggest that children use actual cubes to build the pictures in each of the frames and then count how many cubes they used. Such an activity at this time should be treated with a light touch.

Follow-up

Allow children a time to use cubes or squares of paper to build shapes or create designs. Suggest for example that they build a ship out of 15 cubes, or a fort out of 9, or a tunnel out of 12, and so on. Encourage them to create their own shapes, and suggest that they might want to name each thing they build. The flat squares can be put into many designs, which may be recorded by coloring appropriate squares on graph paper.



RED MODULE, UNIT D

Fractions

Pages d-25 to d-34

General Objectives

To introduce number concepts other than whole number concepts
To introduce the idea of one half, one third, and one fourth of a given region
To introduce the idea of one half, one third, and one fourth of a given object

Much of the development of this module depends on the utilization of the geometric figures that can be made from the patterns provided on page 271. These figures allow the children to manipulate fractional parts of geometric shapes. This occurs when the children first cut out the figures and also when they choose and paste the appropriate piece onto the page. This sequence of handling a whole figure, breaking it apart into halves, or thirds, or fourths, and then repasting to reform the whole figure again provides definite physical advantages toward development of the fraction concepts.

Mathematics

The most important objective of this module is to expose children to number concepts other than whole numbers. At this stage, it is not necessary to introduce the symbols for fractions, since we are concerned only with exposing the children to the ideas, not with developing the symbols involved.

Do not give a detailed explanation of the meaning of fractions. Your discussions should be strictly intuitive. In describing one half, you should talk about sets of the same size (equivalent sets) or objects divided into parts of the same size.

Teaching Red Module, Unit D

Approximate Time: 5 to 7 days

Materials

construction paper for folding
crayons
geometric figures, marked in fractions
flannelboard
large geometric shapes cut or marked showing halves, thirds, and fourths
overhead projector
paste
scissors
sets of objects which can be divided evenly among 2, 3, or 4 children

Vocabulary

circle	one third
fraction	rectangle
one fourth	square
one half	triangle

As stated above, the most important physical aid in teaching this module is the set of figures that can be duplicated for each child from the patterns on page 271. However, demonstrations with objects or shapes on the flannelboard or with large geometric shapes are also useful. Also activities whereby children share sets among 2, 3, or 4 children provide important experiences for developing the concept of fractions.

Evaluation of Progress

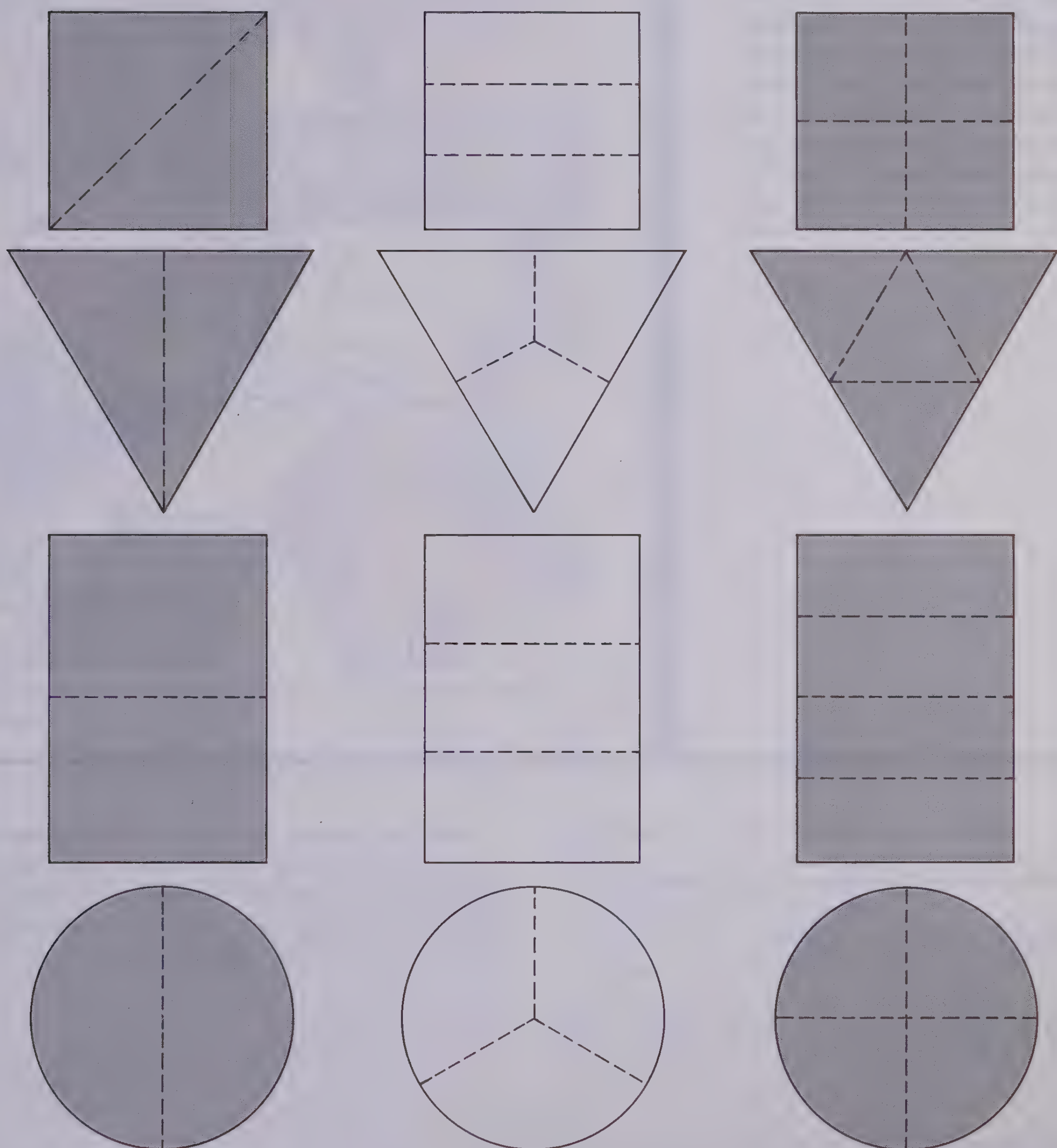
While it is important as an ongoing objective to have children understand that there *are* numbers other than whole numbers, this module initiates the germ of this idea basically by depending upon the idea of ratios such as 1 out of 2 or 3 out of 4 parts. Although it is hoped that children will be able to identify halves, thirds, and fourths, do not expect mastery at this level. Your daily observations of the children at work will be your most important aid in determining how much the children are grasping and therefore the kinds of activities most suitable for them.

Resources for Active Learning

General Activities:
MATHEX: Numeration No. 2, "Developing Fractional Number Concepts," pp. 48–50, Encyclopaedia Britannica Publications Ltd.
Manipulative Devices:
Discovery Blocks (Educational Teaching Aids)
Fraction Circles and Squares (Hammett, Lakeshore)
Geo-boards (Addison-Wesley)
Primary Shapes (Responsive Environments Corp.)
Commercial Games:
Competitive Fractions (Selective Educational Equipment)

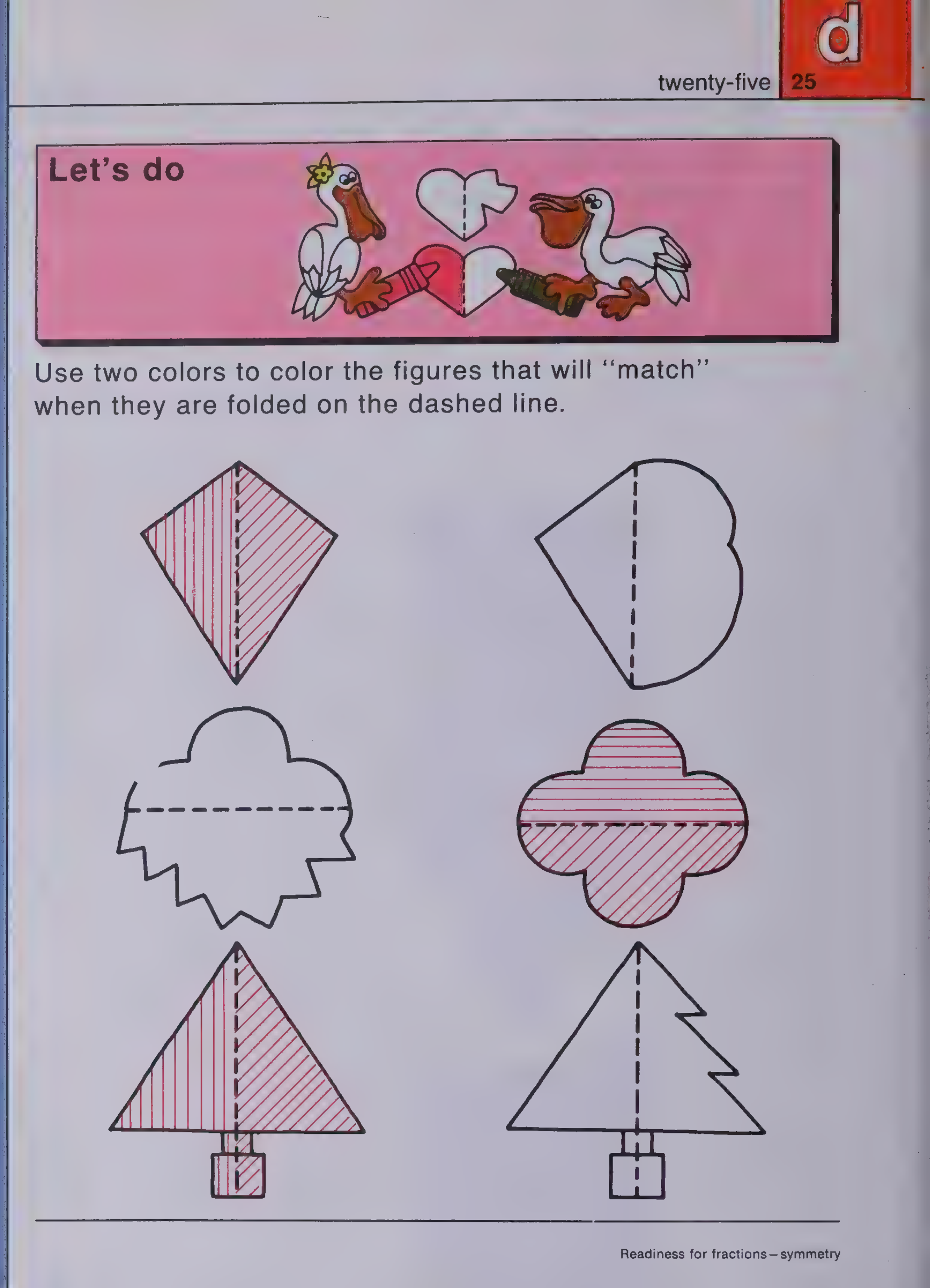
Patterns

The following patterns should be reproduced so that those figures sectioned in halves are orange, those sectioned in thirds are blue, and those sectioned in fourths are green. This may be done by reproducing them on paper of these colors or by having the children color them this way before they cut them out. If you have the children color the figures, be sure they have done so correctly so that all halves are orange, all thirds blue, and all quarters green.



Investigation
Page d-25

Explain to the children that the dashed line in each figure represents the folding line. Some of the figures would give them matching parts when folded along that line; other figures would not. They are to try to imagine whether or not the parts of the figure would match. If the parts of a figure would match when folded they should color each of the parts a different color. If the parts of a figure would *not* match when folded, you might suggest that they mark that figure with an X. When the children finish the coloring on this page and you have discussed their choices, ask them to try to draw and color a figure of their own which has two matching parts.


d

twenty-five 25

Let's do

Use two colors to color the figures that will "match" when they are folded on the dashed line.

Readiness for fractions—symmetry

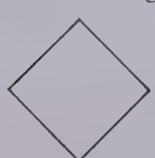
Purpose

To prepare for development of fraction concepts

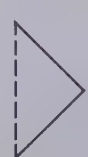
Preparation

To prepare for this lesson, display for the children a shape such as a diamond, which can be folded in half so that the halves "match" (that is, use a symmetrical figure).

Before folding



After folding



Help the children see that after folding the figure you have two parts whose shapes match when you place one upon the other. Use one or two other examples to be sure children realize what you mean when you speak of parts of a figure "matching" after they have been folded.

Let's talk

Readiness for fractions

Discussion
Page d-26

This discussion page shows four sets of objects cut into parts. In each set only one item has been cut into equivalent parts. For example, each of the three pies has ben cut, but only one has been cut into equivalent parts, namely, halves. Ask questions to stimulate discussion such as, "If two children wanted to share an apple equally which apple should they choose?" Help children see how one item in a set has been cut in halves and then discuss the differences in the other items of each set. Notice that each set deals with halves even though the stress should be on equal parts. For example, point out that only one kite has been painted so that equal parts are painted different colors, namely the yellow and tan kite.

Follow-up

Distribute to each child a piece of construction paper (which might be as small as 12 cm × 18 cm) and ask him to fold it in half. Then guide the children in cutting any shape you choose (it may be simply a free design or you might provide them with suitable forms to place along the fold). Be sure the children leave the fold intact as they cut. Finally ask them to unfold their cut figure and describe what they notice about the two halves.

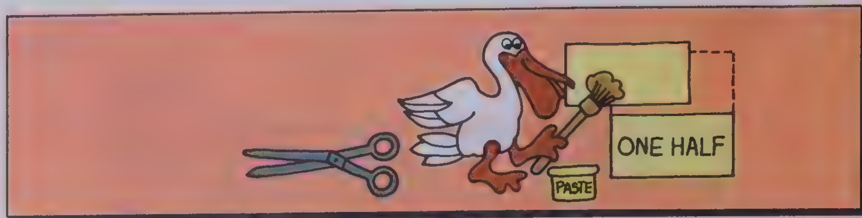
Resources for Active Learning

DEVELOPMENTAL MATH CARDS, "Paint a Splash," A^{3}2, Addison-Wesley

THINK AND COLOR, "Symmetry," pp. 88–92, Educational Science Consultants

Teaching
Page d-27

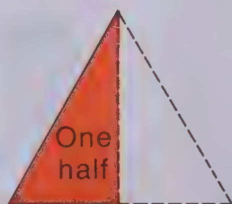
Call attention to the figure in the illustration and speak of the square as having been separated into two parts of the same size. Teach the words *one half*. Then have children look at the page and emphasize that one half of the rectangle shape is missing. Similarly one half of the circle shape, one half of the triangle shape, and one half of the square shape are missing. Explain that they should try to choose the missing halves from their figures, and then paste each in its correct place. Be sure that children save the pieces they do not use for the following lessons.



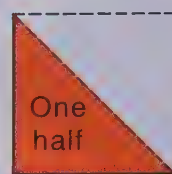
Paste in the missing half.

Rectangle shape

Circle shape



Triangle shape



Square shape

Objective

When given pictures of simple figures which have been divided into two parts, the child can identify whether or not that picture has been divided into halves.

Pre-Book Activity

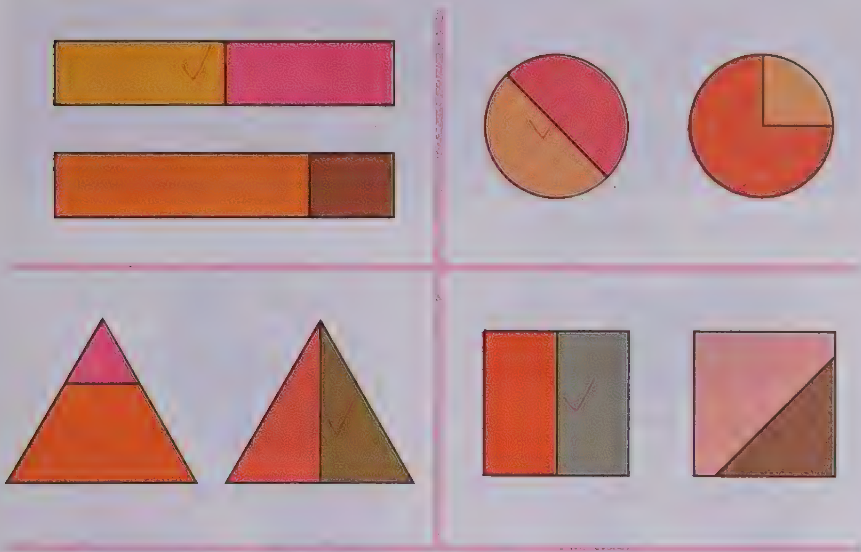
Materials

envelopes
fractional parts of geometric figures
paste

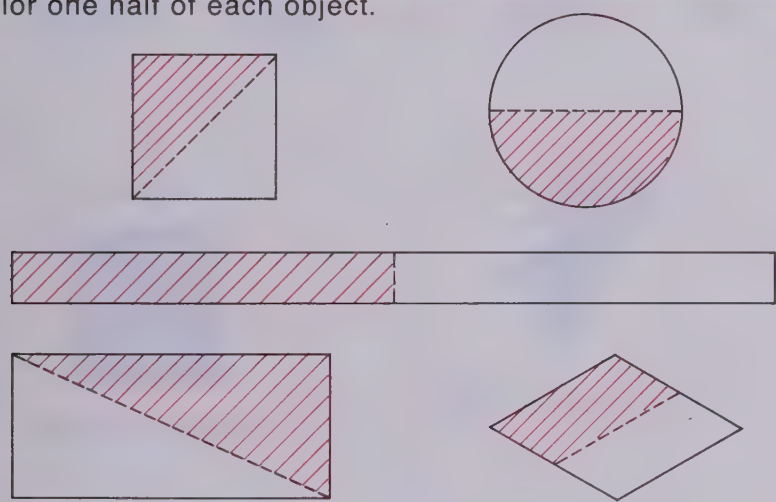
Since the lesson page itself provides an activity, prepare for it by having the children cut out the figures that have been prepared from the patterns on page 271.

Provide each child with an envelope to hold the figures. Review the names of the shapes: rectangle, circle, triangle and square.

Put a mark on the objects divided into halves.



Color one half of each object.



Either half can be colored.

One half

Teaching
Page d-28

Explain to the children that in the top section of the page there are pairs of figures. In each pair one figure shows two different colored halves and the other figure shows two different colored parts. They are to mark (X) on the figures which show halves.

In the second section they are simply to color one half of each figure.

Follow-up

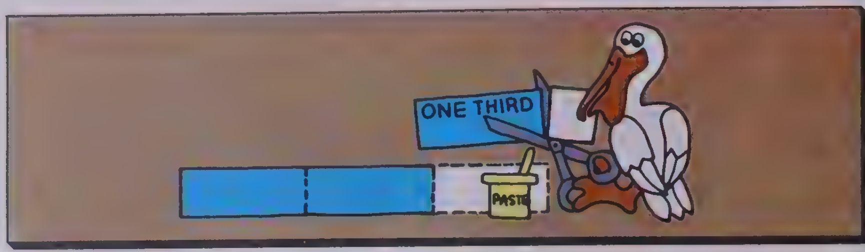
Follow-up activities might involve making a construction paper poster showing halves of larger figures or separating the geoboard into halves with rubber bands. Also helpful are activities which involve sharing. For example, give two children an even number of buttons and ask them to share them so each child has one half of the buttons.

Resources for Active Learning

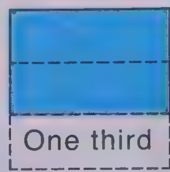
DEVELOPMENTAL MATH CARDS, "Looking for Halves," $B^1 6$, Addison-Wesley

Teaching
Page d-29

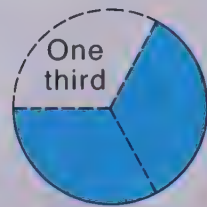
Call attention to the illustrated strip at the top of the page and teach as sight words the phrase *one third*. Explain to the children that they should find pieces in their envelopes which fit onto the incomplete regions. Stress that each figure has been divided into thirds – that is, we call each of the three "same-size" regions one third. Read the directions with the children to be sure they realize that they should paste in the missing third. When the children finish, it would be helpful to use the flannelboard and demonstration figures which show thirds.



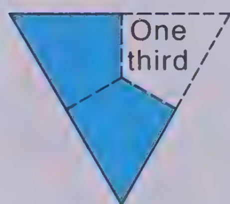
Paste in the missing third.



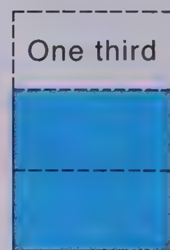
Square shape



Circle shape



Triangle shape



Rectangle shape

Objective

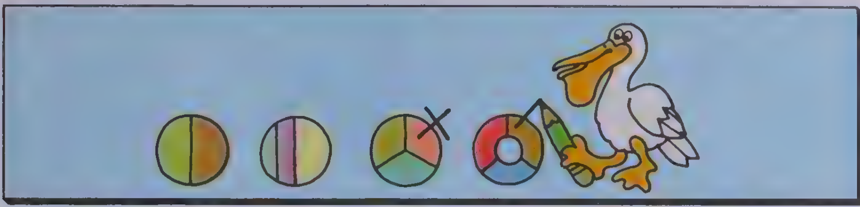
Given pictures of simple figures or actual objects which have been dividied into three parts, the child can identify one third of each.

Pre-Book Activity

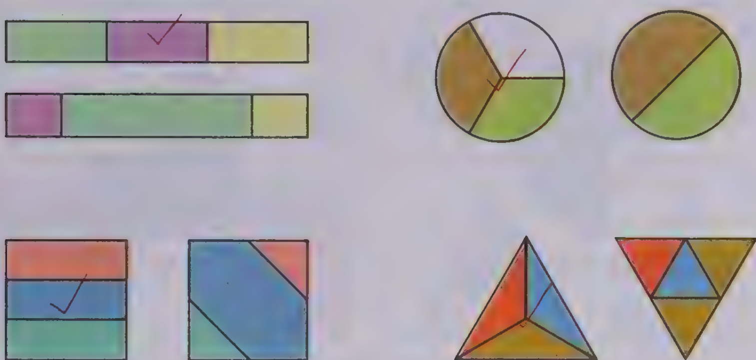
Materials

fraction pieces
paste

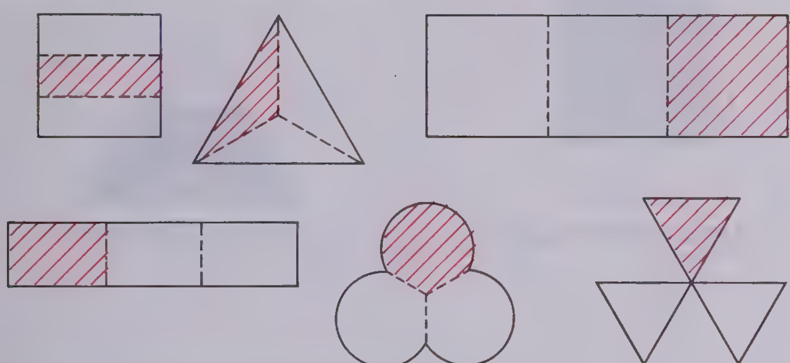
Again, this lesson page itself incorporates an activity, so you need no pre-book activity. The children should again have the cutout figures to use. However, if you prefer a specific pre-book activity you might adapt an activity suggested in the Follow-up.



Put a mark on the objects divided into thirds.



Color one third of each object.



Any one of three parts can be colored.

Teaching
Page d-30

Help the children understand that at the top part of the page they should mark each figure that shows thirds. Use the top left example to discuss how the regions that are thirds must be equal in size; in this frame both strips have been separated into three parts but only when parts are the same size can we say that the parts are thirds.

In the next section simply instruct the children to color one third of each object.

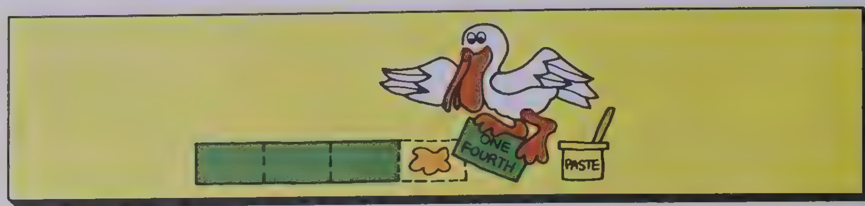
Follow-up

If children enjoy working with regions, challenge them to make a shape of their own and color one third of it. Some of the shapes children choose might be difficult to divide into three equal regions. This could be a basis for further discussion of how one third is one of three parts that are the same size.

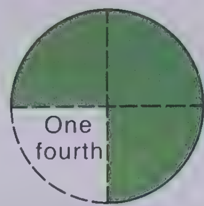
It would also be helpful to have available objects that can be divided into thirds. Show these objects being cut into thirds. Also have sets of 6, 9, and 12, and divide these sets into thirds by asking three children to divide the sets so that each child will get the same number. Each child would then have one third of the objects. Such activities with objects and sets may be adapted to demonstration materials, if you prefer.

Teaching
Page d-31

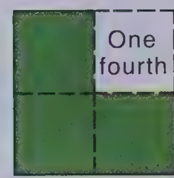
Call attention to the strip pictured at the top of the page. Teach the phrase *one fourth*. If children worked with strips before beginning the page, relate the illustration to these strips. Emphasize that each region is one fourth of the strip. Then direct children to look for the missing fourths among their cutout figures. When they have the correct pieces, they should paste them in place. Move around the room as children work and give guidance when necessary.



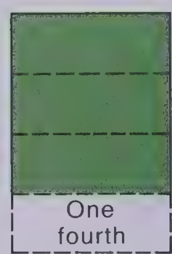
Paste in the missing fourth.



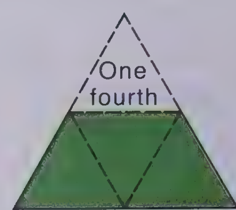
Circle shape



Square shape



Rectangle shape



Triangle shape

Objective

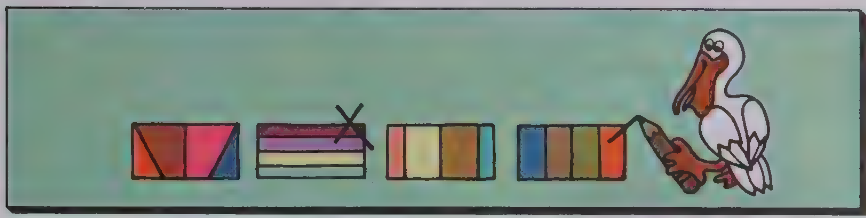
Given pictures of simple figures or actual objects which have been divided into four parts, the child will be able to identify one fourth.

Pre-Book Activity

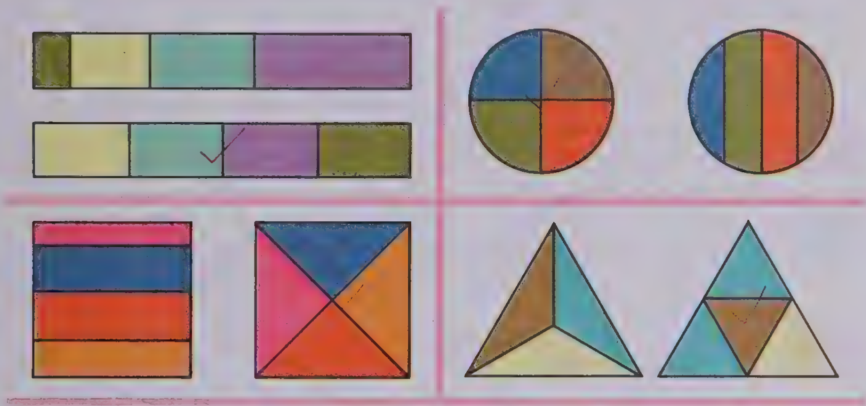
Material

cutout figures
paste

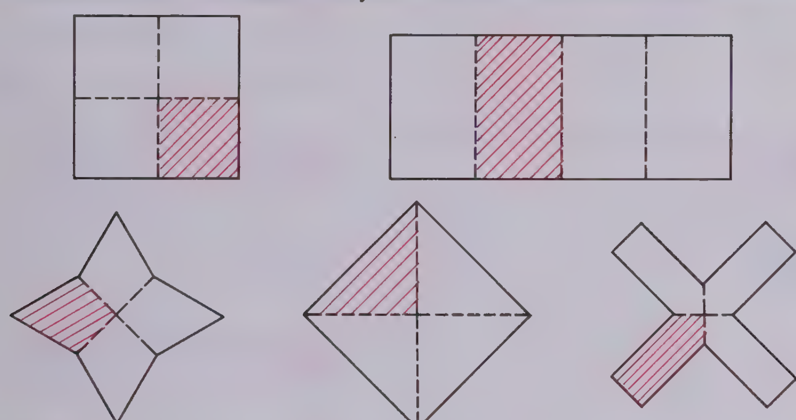
If you prefer an introductory demonstration, display sets of 8, 12, 16, and 20 objects and have children separate them equally into four parts. You might also distribute strips of paper and ask children to fold them in half and then in half again. Help children see that, since the strip has been divided into four same-size regions, each such region is one fourth.



Put a mark on the objects divided into fourths.



Color one fourth of each object.



Any one of the four parts can be colored.

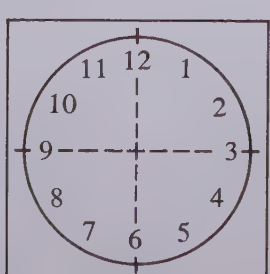
TEACHING
Page d-32

In the top sections of the page, ask the children to mark only those figures which show fourths. Again use examples to stress that the four parts of a figure must be the same size before they are considered fourths.

In the bottom section instruct the children to color one fourth of each figure.

FOLLOW-UP

Duplicate a clockface on square paper and distribute one to each child. Direct them to fold their paper in halves and in fourths as shown below. Help them mark the 12, 3, 6, and 9 in proper places where the fold makes a crease and ask them to write in all the clock numerals.



Suggest that they use two crayons, or two thin strips of cardboard to place on the clock as hands. Ask them to show various times on the clock. Relate the expressions you use to the fractions, half and fourth. Explain how the word *quarter* is used in telling time. Help them understand the meaning of the phrase "quarter past the hour." For example, if we say it is "quarter past two," we mean one fourth of an hour has passed since two o'clock. Some of your directions might be similar to the following:

1) Show half past three on your clock.
2) What part of an hour would have passed since six o'clock if the clock showed 6:15?
3) If the clock shows a quarter to four, how much of an hour has to pass before it is four o'clock?

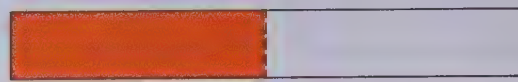
(The punchout clock face available separately would also be suitable for this activity.)

Teaching
Page d-33

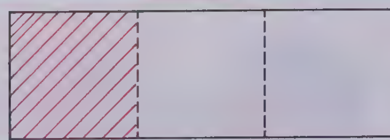
Discuss the first example at the top with the children. Point out that *one half* of the strip has been colored and so they should ring the phrase *one half.* Explain that for each figure they should first color one part. Then they should circle the correct phrase to show which part they colored. If some children have a reading difficulty, you might have them color one part of each figure and then explain to you orally what part they colored.

Show you know

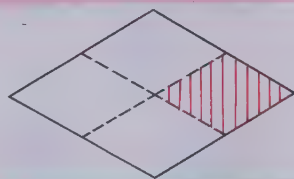
Color one part of each figure.
Circle the words that tell about the colored part.



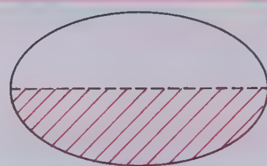
One half
One third
One fourth



One half
One third
One fourth

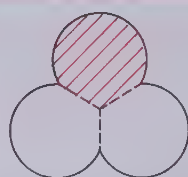


One half
One third
One fourth



One half
One third
One fourth

One half
One third
One fourth



One half
One third
One fourth

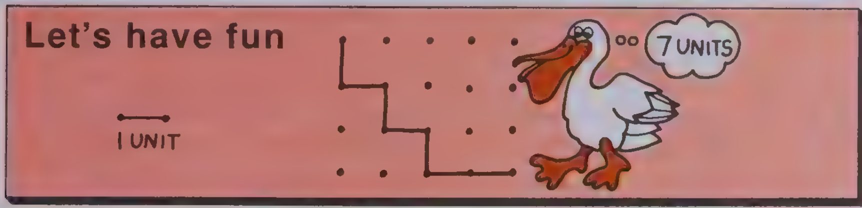
Objective

The child will demonstrate his ability to work with the concepts presented in this module.

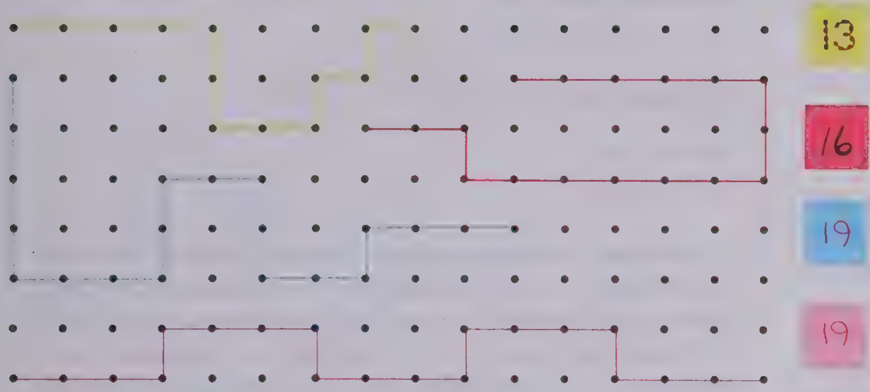
Pre-Book Activity

Use a variety of materials, particularly fractional parts of geometric shapes, to review halves, thirds, and fourths. As you review these fractional parts, always be sure children know what *whole* object, or figure, or set the fractional part relates to. Also, depending on the reading ability of the children, review the terms *one half, one third* and *one fourth.*

Let's have fun

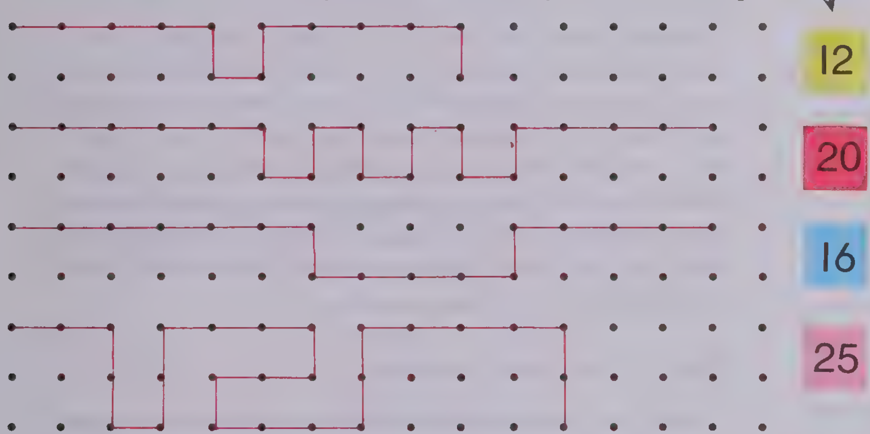


How long is each path? Unit



Path will vary. Examples are given.

Can you draw some paths this long?



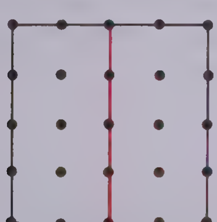
Lengths of paths

Teaching
Page d-34

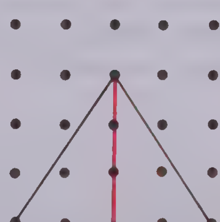
Page d-34 is a change of pace page and should be handled with a light touch. However children must realize that the length from one dot to the dot horizontally or vertically next to it is counted as one and we call this length a unit. This page might be challenging for some children so treat it like a game. Those who have difficulty should not feel discouraged.

Follow-up

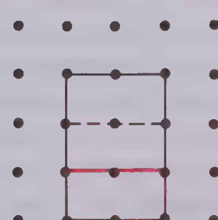
If geoboards are available, children might use one color rubber band to make a figure on the geoboards and then another color to show how to divide the figure into halves, or thirds, or fourths.



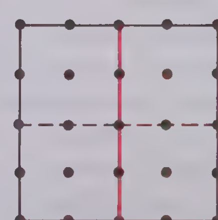
square; halves



triangle; halves



rectangle; thirds



square; fourths

LIGHT GREEN MODULE, UNIT D

Sums and Differences 11 to 18

Pages d-35 to d-44

General Objectives

To introduce regrouping in finding sums between 10 and 19

To provide informal experiences with the order and grouping principles

To strengthen mastery of combinations of ten or less

This module is intended as an optional study of a useful power skill for finding sums 11 through 18. Keep in mind that first graders are not expected to master these combinations. This module does not stress mastery. It simply provides a power skill more closely related to the base-ten number system. For example, because of the very meaning of place value a child knows that $10 + 7$ is 17 and he can simplify $9 + 8$ to this known sum by grouping $9 + 8$ as $10 + 7$.

The order principle and the "doubles" ($5 + 5$, $6 + 6$, $7 + 7$, and so on) are also introduced. The module concludes with the usual evaluative and change of pace pages.

Mathematics

Many of the ideas in this module are extensions of mathematical concepts already developed. This illustrates an important characteristic of mathematics: we are able to start with a few basic facts and build the desired structure from these facts.

In this module, the children learn to find sums greater than ten by utilizing the concept of place value (which is merely an agreement upon the use of symbols) and the basic principle of grouping. This learning process is one of reduction; that is, the children learn to solve a difficult problem by reducing it to several easy problems which they have solved before. For example, the addition of 7 and 5 is reduced to two problems that the children have already solved. First, the children break 5 into two parts, 3 and 2. Then, by regrouping, they add 7 and 3, a combination they have already learned. Finally, they add 10 and 2, utilizing their knowledge of place value.

Teaching Light Green Module, Unit D

Approximate Time: 5 to 7 days

Materials

counters (at least 20 per child)
floor number line
number line for demonstration (optional)
objects which may easily be grouped by tens, such as pencils, sticks, pipe cleaners
overhead projector
strips, 1 set per child

Vocabulary

forming ten
regrouping

Since the main point of this module is to find sums of 11 through 18 by "making ten" it is important to put special emphasis on recognizing the combinations of 10.

The use of strips is also given special emphasis since the strips lend themselves to representing the idea of regrouping to make ten. Children are accustomed to building trains and matching trains of the same length. In this module, they simply build the train which represents the sum, such as $5 + 7$, and then find the strip which, *when used with the ten strip,* matches that original train. For example, the train built of the 5 and 7 strips, matches the train consisting of the 10 and 2 strips. Children should be encouraged to do as many examples with the strips as they choose. However, it is hoped that most who try this module will be able to move from using the strips to making ten mentally.

Evaluation of Progress

Since this is an optional module, evaluation should not be stressed. Furthermore, since the skill being developed is a power skill, that is, mastery of the combinations is not expected, your evaluation of a child's understanding should be based more on your daily observation and discussion with the child than on his performance with particular exercises.

Resources for Active Learning

General Activities:

For games that provide mastery of facts, see Unit D, Yellow Module Introduction under "Resources for Active Learning."

MATHEX: Operations No. 3, "Operation Big Ten," pp. 16-23, Encyclopaedia Britannica Publications Ltd.

Multi-base arithmetic blocks:

THE DIENES M.A.B. Tasks and Manual, Herder and Herder NUMBER-BLOX, Creative Publications

Manipulative Devices:

Abacus or abacus board (Educational Teaching Aids; school supplier)

"Invicta" Math Balance (Math Media; Selective Educational Equipment)

Commerical Games:

Games of strategy:

Kalah (Creative Publications; Math Media)

Tac-Tickle (Creative Publications; Gamco; Wff 'N Proof)

(3-D) Tic-Tac-Toe (Childcraft; Creative Publications)

TRI-NIM (Childcraft; Gamco; Wff 'N Proof)

TUF (Creative Publications; Cuisenaire Co.; TUF)

Wff (Childcraft; Cuisenaire Co.; Wff 'N Proof)

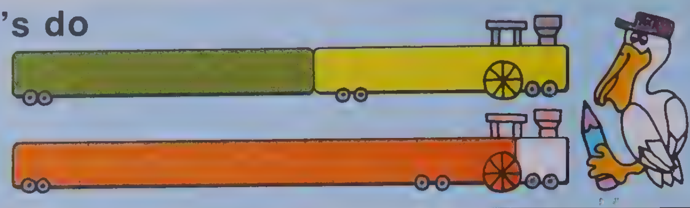
Investigation
Page d-35

Read the directions for the children. Explain that for each given train they are to try to build a matching train by using an orange strip and some white strips. Point out the answer box provided with each 10 for recording the number of white strips they use. As children complete the trains, discuss how each pair of trains consists of matching trains, by saying something such as: "8 and 5 matches 10 and 3."

It would be helpful to extend this activity by suggesting other trains for the children to match. For example, write a list such as the following on the chalkboard and ask children to build the trains for each set.

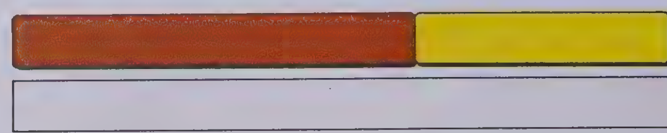
8 + 7	matches	10 + ☐
9 + 7	matches	10 + ☐
8 + 9	matches	10 + ☐
9 + 9	matches	10 + ☐

Let's do



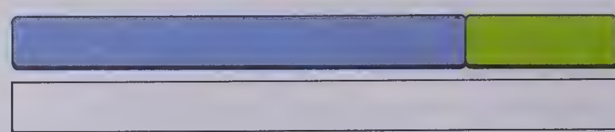
Make a matching train with orange and white strips.
Record the number of white strips in the box.

8 + 5



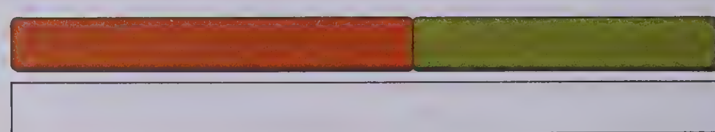
10 + $\boxed{3}$

9 + 3



10 + $\boxed{2}$

8 + 6



10 + $\boxed{4}$

Readiness for sums greater than 10—grouping by 10's

Purpose

To introduce informally the idea of regrouping for finding sums 10 through 18

Preparation

Materials

strips, 1 set per child

After a time of free play with the strips, review with the children some of the trains which can be built to show numbers 18 or less. For example, write the numbers 12, 15, 16, 17, or 18 on the chalkboard and ask children to build trains that show some of these numbers. Then ask them to describe the train they built for one of the numbers such as 15. As children suggest various trains for 15 such as 7 and 8, or 6 and 9, point out that these trains all show the same number, 15. If your children have grown quite familiar with the strips, you might prefer to begin immediately with the investigation.

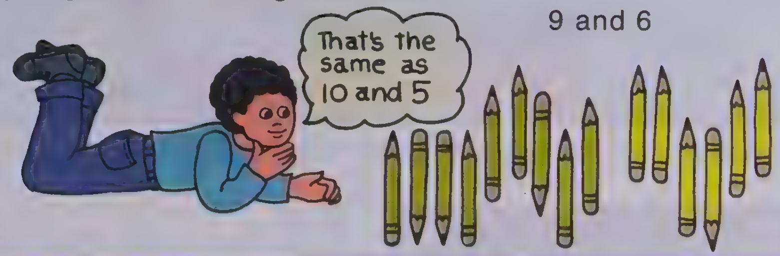
Follow-up

A math balance of the type shown the next column may be used as a tool for helping children learn to relate the combinations for 11 through 18 with combinations of 10 and another number less than 10. You should not try to use the balance to explain the operation of addition, but only as a skill-building device. A homemade balance may be constructed if done carefully. A piece of pegboard in which holes have already been punched may be cut and placed on a stand to balance. Weights may be made from paper clips and same-size washers. However, it would be

Let's talk

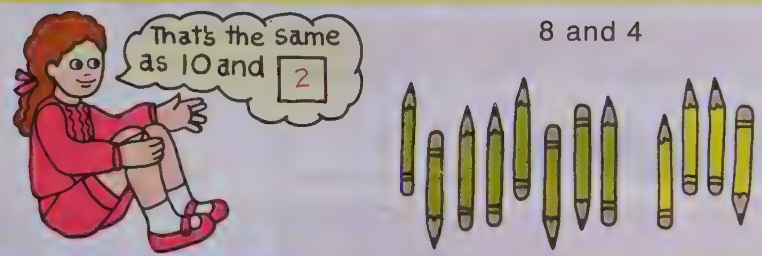
Can you give the missing numbers?

9 and 6



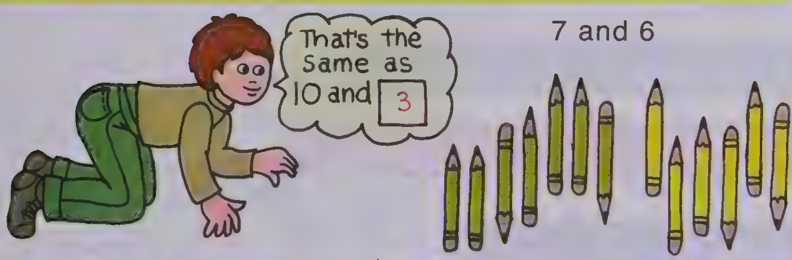
Tom

8 and 4



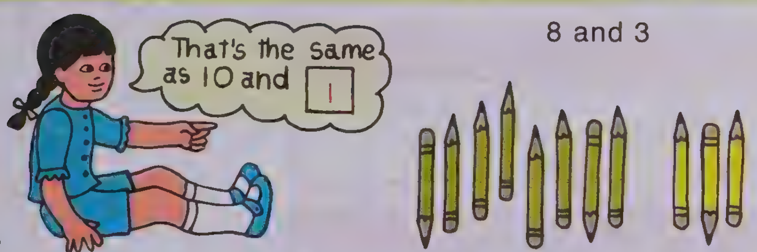
Sue

7 and 6



Bill

8 and 3



Jane

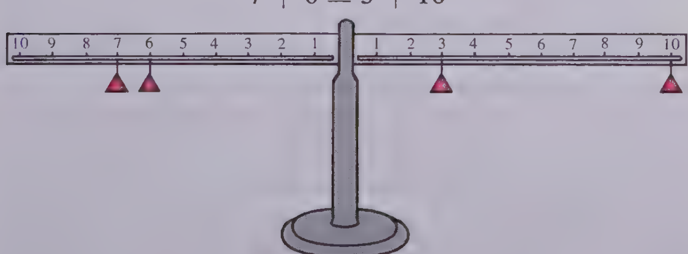
Readiness for sums greater than 10—grouping by 10's

Discussion
Page d-36

Have available at least 20 pencils or sticks which children can use during the discussion of the exercises on page d-36. One of the main purposes of this page is to help children verbalize the thinking involved in "making 10" to find sums. Ask someone to explain what Tom is thinking and to show with the pencils or sticks how 9 and 6 is the same as 10 and 5. Elicit from the children that if one of the 6 pencils is put with the 9 pencils to make 10, the grouping is changed from 9 and 6 to 10 and 5. Suggest that they ring 10 of the pencils and count the 5 not ringed. Work through each of the illustrations similarly. It would also be helpful to use demonstration strips to relate this discussion to the activity the children did on page d-35. Stress that the sum does not change, only the grouping changes.

important to test the weights of the washers to be sure that they are the same. Children may then place two weights on one side of the balance and two other weights on the other side to show combinations for the same sum.

$$7 + 6 = 3 + 10$$



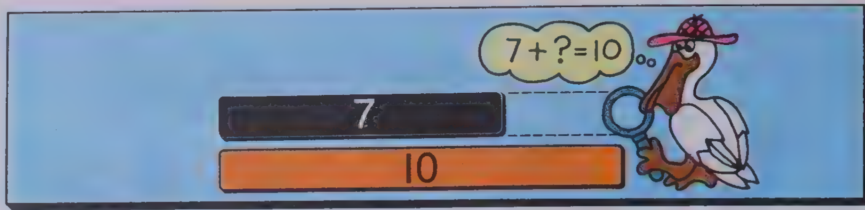
Resources for Active Learning

MATHEX: Numeration No. 2, ". . . Cardinal Number Concept," pp. 8–9, Encyclopaedia Britannica Publications Ltd.

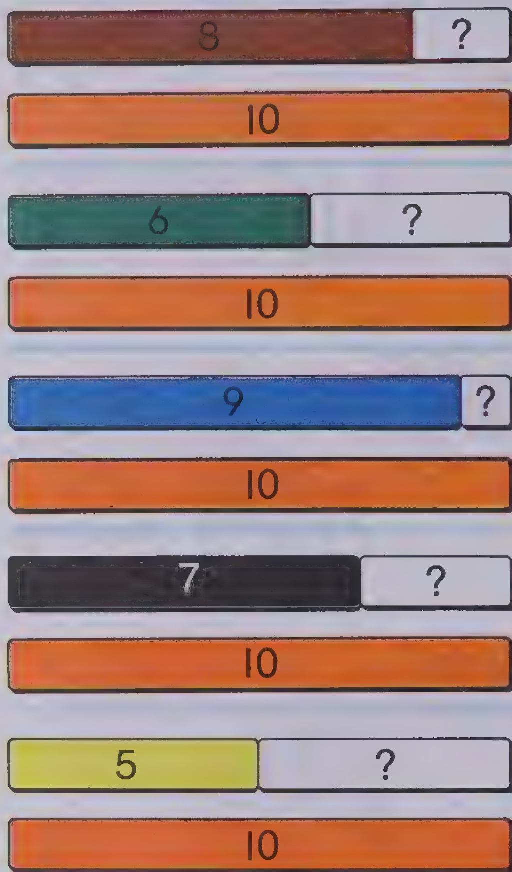
MATH WORKSHOP: Games and Enrichment Activities, "Naming Numbers . . ." and "Fewest Rods Game," pp. 44–49, Encyclopaedia Britannica Educational Corp.

TEACHING
Page d-37

Each child will need a set of strips for this page. Explain to the children that in each frame they are to find the strip which will complete the train to match the ten-strip. They should then record this missing strip, or missing addend, in the box provided to complete the equation. Work through the first frame with the children, but encourage them to do most of the exercises independently. You may also suggest that they color the space for the missing strip.



Find the missing strip. Then solve the equation.



$8 + \boxed{2} = 10$

$6 + \boxed{4} = 10$

$9 + \boxed{1} = 10$

$7 + \boxed{3} = 10$

$5 + \boxed{5} = 10$

OBJECTIVE

Given an equation with a missing addend for a combination of ten, the child will be able to find the missing addend.

This lesson stresses the combinations for ten. Unless children have a mastery of these combinations, the power skill of making 10 to find sums for 11 through 18 will become a difficult task of little value to the child.

PRE-BOOK ACTIVITY

Materials

strips for each child

Use an oral review game to practice combinations of 10. For example, play "I am Thinking of a Number." You might say: "I am thinking of a number; the sum of my number and 4 is 10. What is my number?" or you might say "I am thinking of two numbers whose sum is 10. One of the numbers is 3; what is the other number?" Occasionally, include numbers from a sum less than 10 to keep the children alert.

You might also include an activity that leads the children from known to unknown combinations. Since most of the children are familiar with the "doubles" (1 + 1, 2 + 2, 3 + 3 and so on), ask them "What goes with five to make ten?" (This is an abuse of mathematical terminology, but children may understand this terminology more readily than if you ask what number added to 5 gives 10.) When the children respond "five," immediately ask: "What goes with six to make ten?" Many of the children will be able to make the transfer and respond "four." Ask again, "What goes with nine to make ten?"

Make 10 in all. Then solve the equations.

$$6 + \boxed{4} = 10$$

$$7 + \boxed{3} = 10$$

$$8 + \boxed{2} = 10$$

$9 + \boxed{1} = 10$	$6 + \boxed{4} = 10$
$7 + \boxed{3} = 10$	$8 + \boxed{2} = 10$
$6 + \boxed{4} = 10$	$9 + \boxed{1} = 10$
$5 + \boxed{5} = 10$	$7 + \boxed{3} = 10$

Teaching Page d-38

Call attention to the first frame. Ask children to count the number of circles. Relate this number to the numeral 6 of the equation $6 + \square = 10$. Explain to the children that they should draw as many other circles as are needed to make 10 in all. They should record in the box how many more they needed to make 10, that is, how many circles they drew. Some children will figure out how many more of each set are needed and then draw that many. This represents a greater mastery of the ten combinations.

The equations at the bottom may be solved using any method the children prefer. However, encourage children who are able to do so to complete them without depending on concrete materials. Those who have mastery of the ten combinations and understand the concept of missing addend should be able to solve them mentally.

When the children respond "one," ask: "What goes with one to make ten?" The children should make the transfer again and respond "nine." Ask, "What goes with two to make ten?" Some children will see at once that 8 goes with 2 to make 10.

Follow-up

The following activity is an enjoyable way for children to practice finding a missing addend for a combination of 10. It may be adapted to a variety of materials.

Place on the table two musical instruments such as, two xylophones, or two drums, or two triangles. Ask a child to come to one of the instruments and play some number of sounds less than 10. Ask another child (or two) to record on the chalkboard the number of sounds played. Then ask a second child to play on the second instrument the number of sounds needed to have 10 played in all. The recorders should then add this number to the first to complete an equation whose sum is 10.

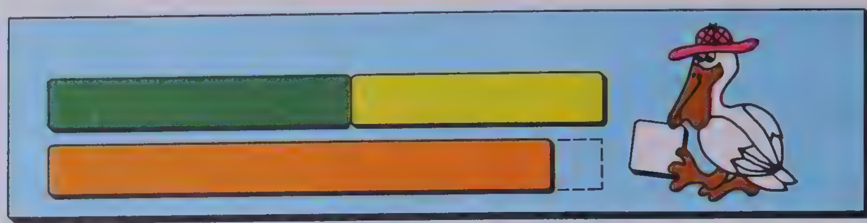
For example, child A and child B each stand by an instrument. Child A plays 7 sounds. Recorders write $7 + \square = 10$. Child B should then play 3 sounds. Recorders erase the $\square$ and complete the equation $7 + 3 = 10$. Since this activity involves sound, all the children in the classroom can listen to and count the sounds to see that the performers do their tasks correctly.

Teaching
Page d-39

Use the illustration as the introductory example. Ask children to make the trains shown. Then ask them how they can express their trains with numbers. Elicit from them that 6 and 5 is the same as 10 and 1.

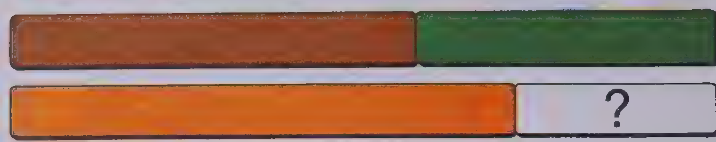
Then ask the children to build the trains shown in the first frame. Be sure they realize that they must find the strip to complete the train with the ten-strip. Draw from them that the $8 + 6$ train matches the $10 + 4$ train, so they must both be equal to the same number. Elicit from them the kind of thinking developed in the pre-book activity: "8 and 6, that is the same as 10 and 4, and that is 14. So 8 and 6 are 14." Stress that they have found the sum $8 + 6 = 14$ by making 10 and 4.

Work through the next frame in a similar manner. Also work through other examples related to other sums of 18 or less until children have a grasp of how to use the method of "making ten" to find sums of 11 through 18.



Find the missing strip. Give the missing numbers.

8 + 6



10 + $\boxed{4}$

8 + 6 = 14

9 + 4

10 + $\boxed{3}$

9 + 4 = 13

Objective

Given an addition equation of a sum of 11 through 18, the child will be able to solve the equation by thinking of the given sum as a combination of 10 and another number.

Pre-Book Activity

Materials

strips, 1 set per child

After a free play period with the strips, ask the children to build a train for a number between 11 and 18, such as 13. Call on various children to describe the strips they used. Lead them into discussion of the sums by using language such as $7 + 6$ is the same as $9 + 4$; $8 + 5$ is the same as $10 + 3$. Show on the chalkboard or flannelboard a combination which shows how the 10-strip can be used in a train to express 13. For example, the 8- and 5-strips matching the 10- and 3-strips. Stress that both of these trains match the number 13. Elicit "8 and 5, that matches 10 and 3, and that is 13. So 8 and 5 are 13." Work through other examples similarly and help children verbalize this way of thinking. Since this activity closely follows the text material on page d-39, you might prefer to begin immediately with the text page and extend it to include this discussion.

Find the missing numbers.

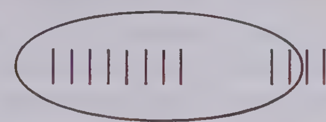
9 and 5	10 and 4
7 and 4	10 and 1
6 and 6	10 and 2
9 and 6	10 and 5
8 and 7	10 and 5
7 and 6	10 and 3
5 and 7	10 and 2

Solve the equations.

$8 + 4 = \boxed{12}$	$9 + 6 = \boxed{15}$
$9 + 5 = \boxed{14}$	$8 + 7 = \boxed{15}$
$7 + 4 = \boxed{11}$	$7 + 6 = \boxed{13}$
$6 + 6 = \boxed{12}$	$5 + 7 = \boxed{12}$

Teaching
Page d-40

Call attention to the first phrase 9 and 5 and explain that 9 and 5 may be expressed as a combination of 10 and 4. Suggest that children find the combinations by using their strips in the method developed on page d-39. However, the child should feel free to find the combinations of ten according to the method of his choice. Some children may simply draw X's grouped according to the first expression and then circle them to make ten. You might suggest this method to any child who has difficulty with the strips or with groups of sticks.



8 + 4
is the same as
10 + 2

Notice that most of the equations on the bottom of the page are related to the expressions in the top. It is not necessary to point this out to the children, but if a child has completed the top section successfully, he should better understand how he might solve the equations on the bottom by making ten.

Follow-up

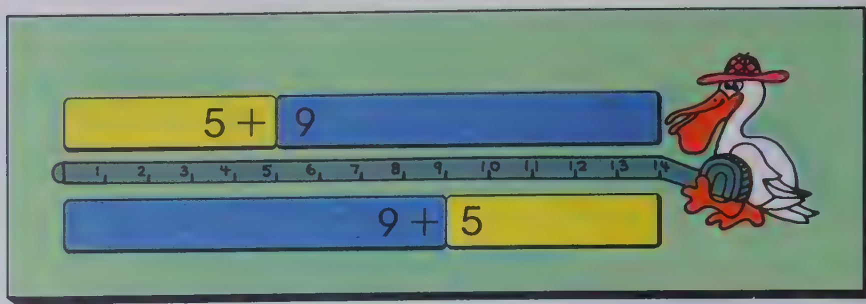
To help children think of the patterns of 10 and use them in solving higher equations, duplicate an exercise similar to the one in the next column, or write it on the chalkboard.

Solve the equations.	
$8 + \square = 10$ $8 + 7 = 10 + \square = \_\_$	$5 + \square = 10$ $5 + 8 = 10 + \square = \_\_$
$9 + \square = 10$ $9 + 4 = 10 + \square = \_\_$	$9 + \square = 10$ $9 + 6 = 10 + \square = \_\_$
$6 + \square = 10$ $6 + 8 = 10 + \square = \_\_$	$7 + \square = 10$ $7 + 9 = 10 + \square = \_\_$
$7 + \square = 10$ $7 + 7 = 10 + \square = \_\_$	$6 + \square = 10$ $6 + 6 = 10 + \square = \_\_$

Teaching
Page d-41

Be sure children realize that the illustration at top does not show actual size strips. Then have children look at the first frame on page d-41. Ask them if they notice anything the same about the two equations. Elicit from them that the same numbers, 9 and 3, are being added. Then ask what is different about the two equations. Draw from them that aside from the second equation having no sum yet, the numbers 9 and 3 are in different order. Discuss whether or not a change in order changes the sum of two numbers by relating this discussion to the pre-book activity and to the demonstration art. Help children realize that, if they know, or find, the sum of two numbers given in one order than they also know the sum of those same numbers given in reverse order.

Encourage children to work through several of these exercises independently. Observe the children and help any child who tends to solve each equation separately to see the relationship between each pair. Some children may need to work through some pairs with concrete materials before they become comfortable in relying on the order principle.



Solve the equations.

$9 + 3 = 12$	$8 + 5 = 13$
$3 + 9 = \boxed{12}$	$5 + 8 = \boxed{13}$
$7 + 6 = 13$	$9 + 8 = 17$
$6 + 7 = \boxed{13}$	$8 + 9 = \boxed{17}$
$9 + 4 = \boxed{13}$	$8 + 6 = \boxed{14}$
$4 + 9 = \boxed{13}$	$6 + 8 = \boxed{14}$
$7 + 5 = \boxed{12}$	$9 + 7 = \boxed{16}$
$5 + 7 = \boxed{12}$	$7 + 9 = \boxed{16}$

Order principle

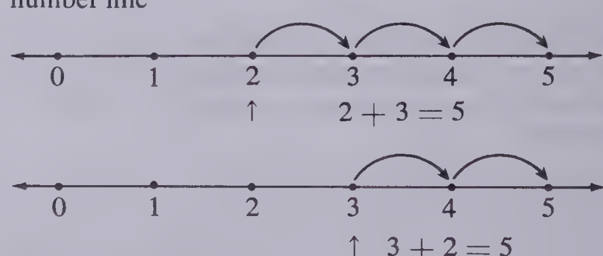
Objective

Page d-41. Given a pair of addition equations which demonstrate the commutative (order) principle, the child will be able to solve both equations by finding the sum of one of them.

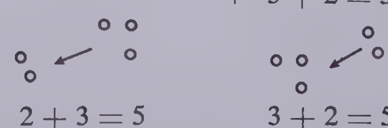
Pre-Book Activity

Before introducing children to the use of the commutative principle as an aid in finding sums of 11 through 18, it would be helpful to develop an understanding of the commutative (order) principle with sums of 10 or less. This may be done by using the number line, sets, or strips as shown in the next column.

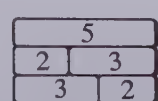
a) number line



b) sets



c) strips



Since $6 + 6 = 12$
I know $6 + 7 = \boxed{13}$.

Solve the equations.

Since $5 + 5 = 10$, I know $5 + 6 = \boxed{11}$.

Since $6 + 6 = 12$, I know $7 + 6 = \boxed{13}$.

Since $7 + 7 = 14$, I know $7 + 8 = \boxed{15}$.

Since $8 + 8 = 16$, I know $8 + 9 = \boxed{17}$.

Since $6 + 6 = 12$, I know $6 + 5 = \boxed{11}$.

Since $8 + 8 = 16$, I know $8 + 7 = \boxed{15}$.

Since $5 + 5 = 10$, I know $5 + 4 = \boxed{9}$.

Since $7 + 7 = 14$, I know $7 + 6 = \boxed{13}$.

Teaching
Page d-42

Since this page is not related to the order principle studied on page d-41, it may be used as a separate lesson, or given a specific introduction. For example, it would be helpful to work through the sum $6 + 7$ at the top of the page with set materials. Place two groups of six objects side by side or have children do this with counters and ask children to find the total. Then add one object to one of the sets and ask them to guess how many there are all together now. Have them check their guess by counting. Point out that since $6 + 6 = 12$, $6 + 7 = 13$ or one more than twelve.

It would be helpful to discuss each of the sentences on page d-42. Explain that phrases such as $5 + 5$, $6 + 6$, $7 + 7$, and so on are called doubles. Stress that knowing the doubles by memory will often help them find sums of combinations which seem more difficult.

You might ask a child to hop out an equation such as $4 + 3 = \square$ which you write on the chalkboard, on the floor number line. Then ask a second child to hop out $3 + 4 = \square$. Ask all the children what they notice about the two equations. Suggest other pairs of equations and elicit from the children that the same numbers are being added, only in different order, and that the sums are the same for each pair of equations.

Follow-up

Duplicate an addition table such as the one that follows and challenge the children to work together and use the power skill methods they have been learning to complete the table by filling in the sums. Give guidance at the beginning, but encourage them to do most of the table with a partner. Then use the overhead projector or a large demonstration table to help them check their tables. With some more capable children, you might want to point out certain patterns. For example, the sums of the doubles fall on a line from the top left corner to the bottom right corner (the diagonal).

+	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5										
6										
7										
8										
9										

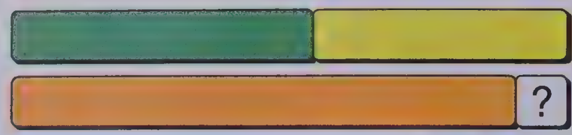
Teaching
Page d-43

Children should have no difficulty understanding what is expected of them on this page. However, be sure they realize that they are free to use whichever method they choose to find the sums. Encourage them to think of combinations of ten and the grouping techniques that they have just studied. Observe the children carefully, particularly those not using the strips, to determine, if possible, whether some children still prefer the counting methods studied in the Orange Module.

Show you know

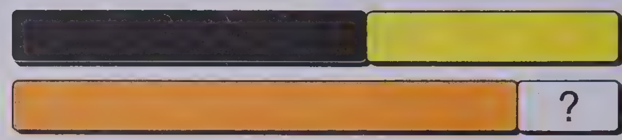
Find the missing numbers.

$6 + 5$



$10 + \boxed{1}$

$7 + 5$



$10 + \boxed{2}$

Find the sums.

$6 + 5 = \boxed{11}$	$7 + 5 = \boxed{12}$
$9 + 3 = \boxed{12}$	$8 + 4 = \boxed{12}$
$7 + 7 = \boxed{14}$	$4 + 8 = \boxed{12}$
$7 + 6 = \boxed{13}$	$5 + 7 = \boxed{12}$
$6 + 6 = \boxed{12}$	$5 + 8 = \boxed{13}$

Objective

The child will demonstrate his ability to work with the concepts presented in this module.

Since this module is intended as optional material, mastery should not be a major emphasis. However, page d-43 should help you evaluate the understanding achieved by those children who completed the module.

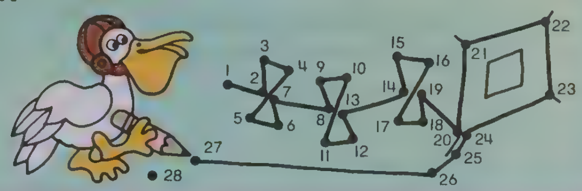
Pre-Book Activity

Materials

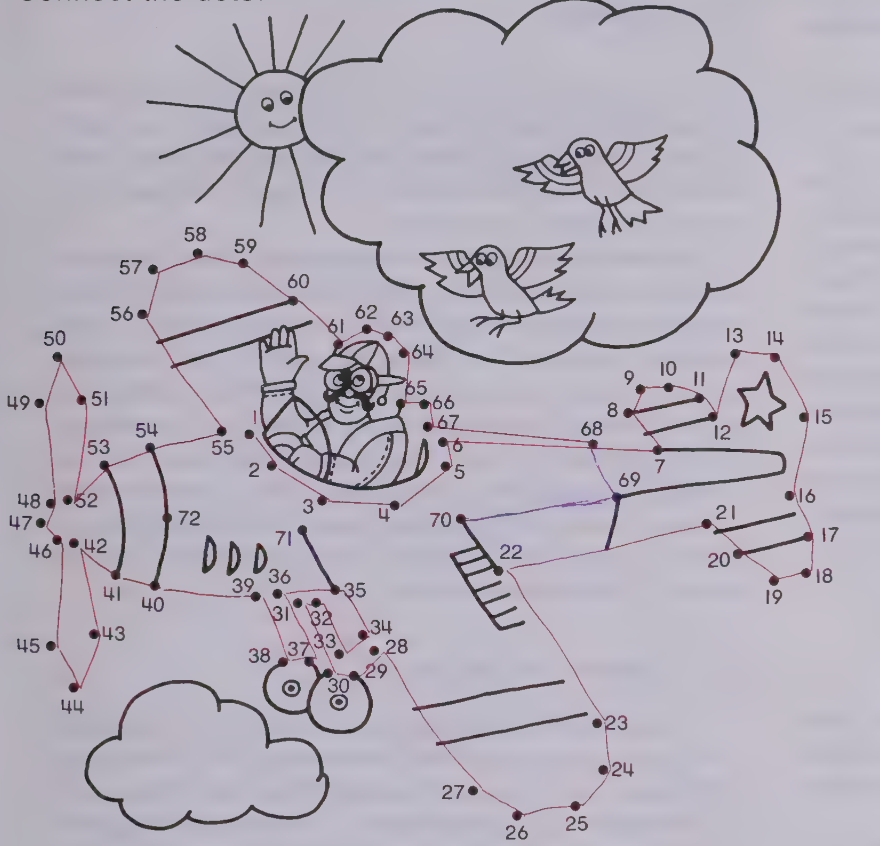
strips, 1 set per child

It would be helpful to review with the children how the strips may be used as an aid in finding the sums 11 through 18. Ask the children to build matching trains with the 7- and 8-strips and the 10-strip and another strip to find the sum of $7 + 8$. Have a child describe how a train of the 7- and the 8-strips matches a train of the 10- and 5-strips, so we know that $7 + 8 = 15$. Work through other examples according to the need of the children. If you feel that such a review with the strips is not necessary, use an oral game to review combinations of 10.

Let's have fun



Connect the dots.



Teaching
Page d-44

The dot picture is provided here as an enjoyable change of pace. Note that the starting point is indicated by a red number one. You might use sections of the picture to have the children practice counting different sequences. For example, pick out a point (such as the nose of the plane) and count from 47 to 60.

Follow-up

Invite the children to play a game of ringtoss. Let each child try to toss two rings over large bottles weighted with sand or dirt. (Large bleach or detergent bottles make good targets, and embroidery hoops or heavy rope rings bound together with cloth or electricians' tape will make satisfactory rings.) Use felt markers to label 9 bottles from 1 through 9. Let each child keep his own score, by adding the numbers on the bottles he rings. The child with the highest score wins.

DARK GREEN MODULE, UNIT D

Inverses and Missing Addends

Pages d-45 to d-54

General Objectives

To extend work with missing addends to include combinations of 18 or less

To continue developing the relation between addition and subtraction

To develop the ability to find differences for combinations of 18 or less by thinking of missing addends

This module extends the ideas developed in the Blue Module, Unit C to include the combinations of 10 through 18. Many of the same methods are employed. Children are introduced to the concepts by an investigation similar to that of the Blue Module, Unit C. Since most of the difficulty with missing addends pertains to the written notation, the child is first given a light touch introduction to missing addend equations. Following this, the ideas are developed by the use of sets, and the number line. Related addition and subtraction equations are then studied and "families" of four related equations are developed. The module concludes with the customary evaluative and change of pace pages.

Mathematics

Related addition and subtraction equations are considered so that finding the difference of an equation such as $12 - 5 = \square$ may be thought of as finding the missing addend of the equation $5 + \square = 12$.

The missing addend concept is based on a formal definition of subtraction:

> If a, b, and c are whole numbers and $a + b = c$, then $b = c - a$.

Thus b, the difference of two whole numbers, is an addend in the addition statement.

Although children can consider subtraction solely from the interpretation of take away, this approach is very limiting. The missing addend approach lays a foundation for development of more difficult topics at a later level, particularly for mastering the combinations of 11 through 18 in Book 2.

Teaching Dark Green Module, Unit D

Approximate Time: 5 to 7 days

Materials

cards showing sets for combinations of 18
counters, about 20 per child
demonstration number line
objects for set demonstrations
2.5 cm squares, 4 or 5 per child

Vocabulary

difference
missing addend

Set illustrations, counters, and the number line should be considered the basic aids for this module. However, if you think some children may benefit from work with the strips, the suggestion in the Follow-Up on page d-50 may be used for this purpose.

Children should be encouraged to use these materials only as long as necessary. The materials should be considered as power tools for finding missing addends in an addition equation. The children hopefully will be able to solve a subtraction equation by using their solution of a related missing addend equation. That is, do not always encourage the children to use concrete materials for the subtraction equations. Lead them, rather, to the understanding that once they have found the missing addend of an equation, they have found the difference for the related subtraction equation.

Evaluation of Progress

Throughout this module, keep in mind that the emphasis is not on mastery of the facts through 18. *First grade children are not expected to master the combinations of 11 through 18.* The emphasis is on understanding how to find a missing addend in an addition equation and on the fact that a difference in a subtraction equation may be thought of as a missing addend. Thus, you will want to evaluate the children's power skills for finding missing addends and their understanding of the relationship between a missing addend and a difference. Daily observations and individual discussion with the children will be your best basis for evaluation, but page d-53 is also provided as an evaluative instrument.

Resources for Active Learning

General Activites:

MATHEX: Operations No. 3, "Operation Big Ten," pp. 16–23; "Placeholders," pp. 26–27, Encyclopaedia Britannica Publications Ltd.

INVESTIGATION
Page d-45

Each child should have at least four counters. Present this activity as a game, perhaps introducing it as a "remembering" game. Read the directions for the children. Help them understand that they are to cover only blue numerals. They are not looking for sums; rather, each blue numeral they cover becomes a missing addend. Explain that when a numeral is covered they should try to remember which numeral is hidden. In order to help them remember, they might think of ways to figure out the missing addend. Have the children cover one blue numeral on each equation at the beginning so they will be challenged to figure out the missing addend. It might be helpful for children to work in pairs.

Let's do

5 + ▢ = 13

In each equation, cover one of the blue numerals with a square.
Can you remember what numeral is under each square?

$$5 + 4 = 9$$

$$9 + 5 = 14$$

$$7 + 3 = 10$$

$$8 + 4 = 12$$

Readiness for missing addends

PURPOSES

To introduce missing addends for combinations of 10 to 18 in preparation for finding related differences

To provide experience in finding differences by thinking about missing addends

PREPARATION

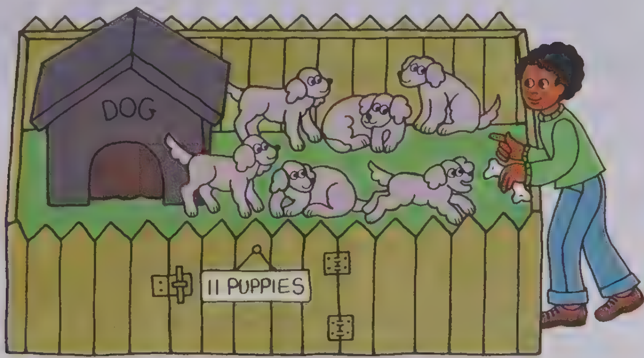
Materials

2.5 cm squares, 4 or 5 per child

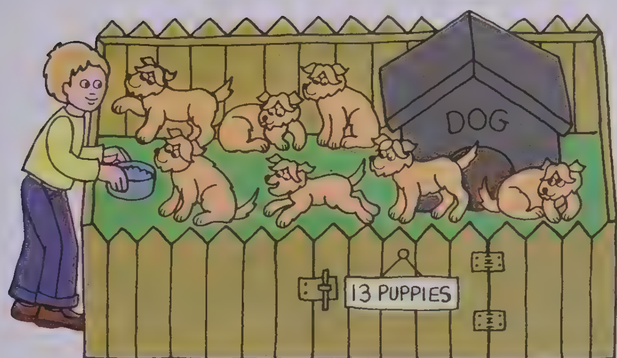
Due to the nature of the investigation, you might begin immediately with the printed page. However, if you prefer, you might use an oral warm-up activity to review combinations of 10.

Let's talk

How many puppies are in the house?
Solve the "doghouse" equation.



$$6 + \boxed{5} = 11$$



$$7 + \boxed{6} = 13$$

Discussion
Page d-46

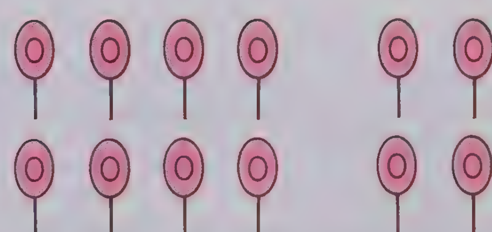
After reading the directions for the children, point out the sign on the gate showing how many puppies are in the first pen. Then have them relate the six illustrated pups to the 6 in the equation, and the total eleven to the 11 in the equation. Help the children realize that some of the pups are in the doghouse. Encourage them to share their ideas on how to figure out how many pups are not shown.

Follow-up

Give each child a large sheet of newsprint. Direct the children to fold it into four sections. Write on the chalkboard several missing addend equations such as the following:

$8 + \square = 12$ $\quad 6 + \square = 13$
$7 + \square = 15$ $\quad 9 + \square = 14$
$5 + \square = 11$ $\quad 8 + \square = 13$

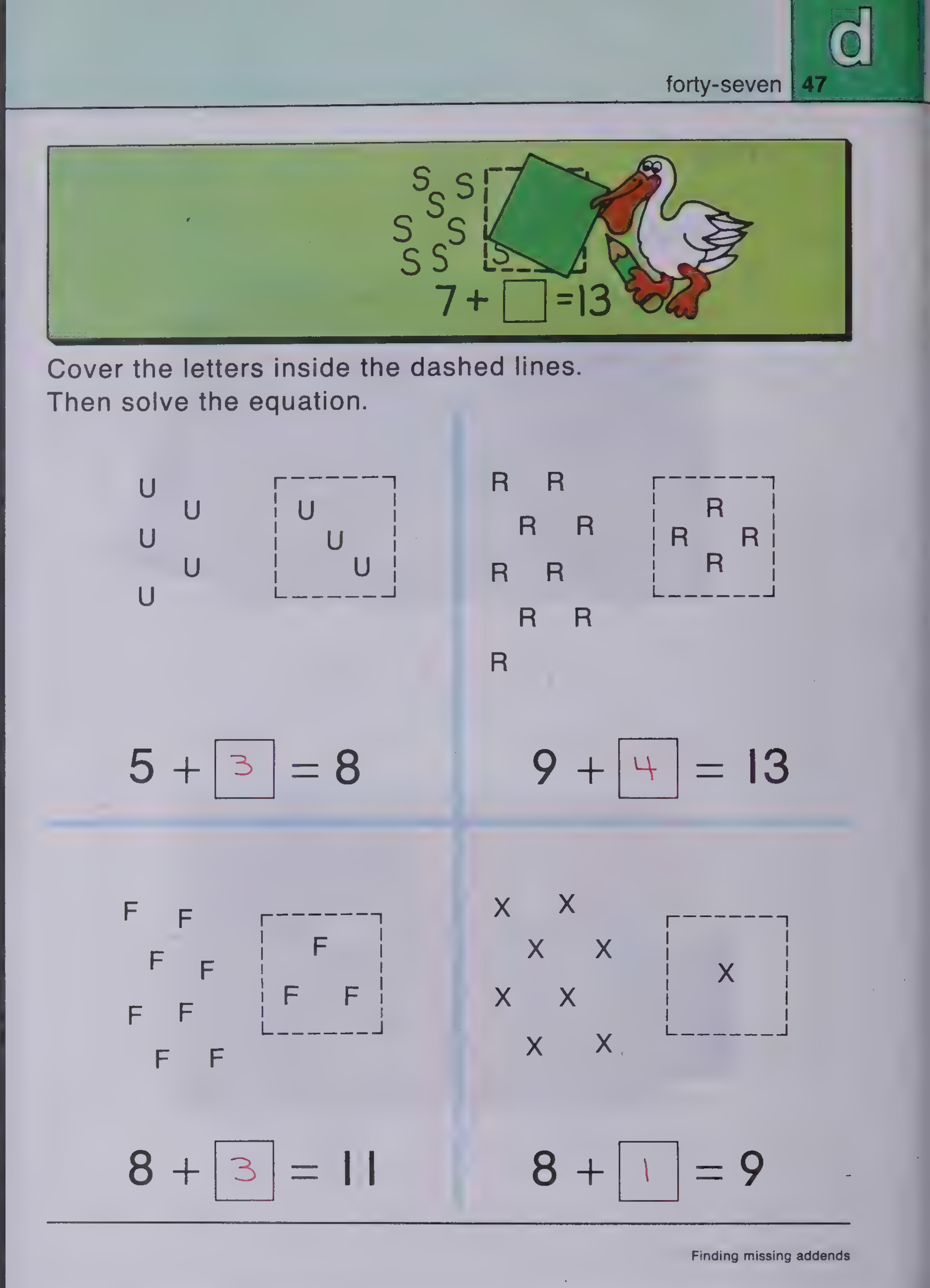
Suggest that they choose 4 equations to solve and then draw pictures to use as aids in solving these equations.



$$8 + \boxed{4} = 12$$

Teaching
Page d-47

Each child will need four squares for this page. Call attention to the first frame. Point out the dashed line square and ask the children to try to cover it with one of their squares. When they have covered it, hiding the letters within, ask them to try to solve the equation. After they have written the numeral which they think completes the equation, they should lift the square and check their answer. As soon as children understand the procedure, encourage them to work independently. However, some children may need your directions step by step for each frame.


forty-seven 47 d

$7 + \square = 13$

Cover the letters inside the dashed lines.
Then solve the equation.

$5 + \boxed{3} = 8$ $\qquad$ $9 + \boxed{4} = 13$

$8 + \boxed{3} = 11$ $\qquad$ $8 + \boxed{1} = 9$

Finding missing addends

Objective

Given an addition equation with a missing addend, the child will be able to find the missing addend by studying a related set illustration.

Pre-Book Activity

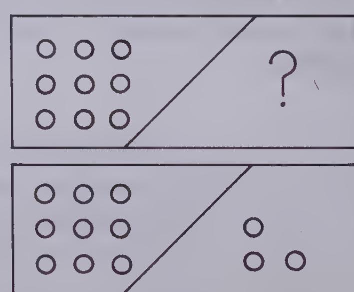
Materials

counters, at least 18 per child
2.5 cm squares

Give each child 18 counters. Direct children in using these counters in missing addend situations. For example, you might give groups of three or four children cards on which you have printed equations with missing addends and ask them to show with sets of counters the situation the equation suggests. Children might draw dots to show the sets they have used.

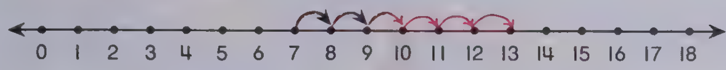
$9 + \square = 12$

$9 + 3 = 12$

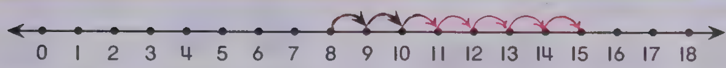


If you prefer to direct the children more closely, ask them to build with their counters a set, such as a set of seven. Then ask them to find the set they need to combine with it in order to have a total of say, 16 counters.

Solve the equations.



$$7 + \boxed{6} = 13$$



$$8 + \boxed{7} = 15$$

$4 + \boxed{3} = 7$

$6 + \boxed{4} = 10$

$9 + \boxed{0} = 9$

$7 + \boxed{5} = 12$

$6 + \boxed{3} = 9$

$5 + \boxed{3} = 8$

$6 + \boxed{5} = 11$

$4 + \boxed{4} = 8$

$9 + \boxed{3} = 12$

$5 + \boxed{4} = 9$

$3 + \boxed{5} = 8$

$7 + \boxed{3} = 10$

Missing addends – number line

Teaching
Page d-48

It would be helpful to demonstrate a few equations with missing addends on the demonstration number line before assigning page d-48. Notice that the number line is being used here basically as a counting device. Preparatory use of the floor number line, numbered to 20, would be appropriate. For example, have a child stand on 6. Explain that his goal is 14. Ask children for guesses as to how many jumps are needed to reach 14, writing $6 + \square = 14$. Then ask a child to jump to 14, while the other children count his jumps to complete the equation $6 + 8 = 14$.

With such a preparation children should readily see how the number line on the page may be used to solve $7 + \square = 13$. Have them complete both the dashed line jumps and the equation for each example. The remaining equations should be completed independently by the children. Be sure they observe the blank number line provided for their use as they work these equations.

Write the missing addend equation on the chalkboard, $7 + \square = 16$. Ask a child to draw his sets on the chalkboard and another child to complete the equations you have written, $7 + 9 = 16$.

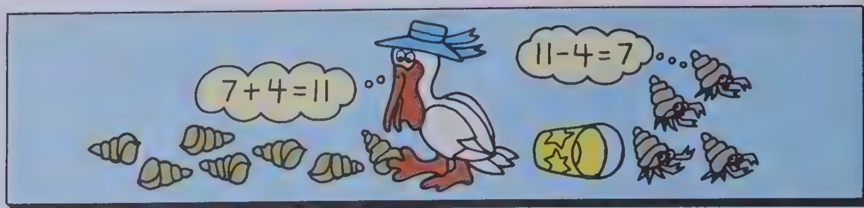
Work through other examples. Give children many opportunities to relate their sets to the equations you write on the chalkboard.

Follow-up

The floor number line may be used for various games. For example, give a group of four or six children two piles of cards, one with numerals 1–9, the other with numerals 10–18. Have the group split itself into two teams. A child from one team should draw a card from the first pile. This numeral designates his first number in a missing addend equation and he should stand on that space. A fellow team member should then hold the second pile (10–18) for him to draw from in order for him to pick the sum of the equation, or his "goal." The third team member should write the missing addend equation on the chalkboard such as $8 + \square = 12$. The child standing on the number line should then work through the equation by jumping out the solution. Each time a team correctly solves their equation, they should get a point. When children pick numerals which form equations that are not among the basic facts, such as 17 and 2 for $2 + \square = 17$, you can have them pick another card or, if they are capable, jump out the spaces to find the answer. Notice that the purpose of this activity does not emphasize speed in finding the missing addend, but rather it emphasizes how the number line may be used as a power tool for finding missing addends.

Teaching
Page d-49

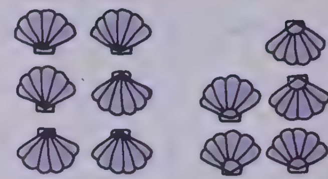
Call attention to the first frame. Help the children relate the illustrated sets to the equations. They should first think of the addition equation $5 + \square = 9$ and complete it by filling in the missing addend. Point out that counters or the number line may be used if they choose, but they might also simply use the pictured sets. When they have completed the addition equation, they should use it as an aid in completing the subtraction equation. Note with the children that the last frames are not accompanied by illustrations. Suggest that they use materials of their choice or draw their own pictures of sets of dots to solve the equations.



Solve the equations.

$5 + \boxed{4} = 9$

$9 - 5 = \boxed{4}$



$6 + \boxed{5} = 11$

$11 - 6 = \boxed{5}$

$7 + \boxed{3} = 10$

$10 - 7 = \boxed{3}$

$8 + \boxed{4} = 12$

$12 - 8 = \boxed{4}$

$5 + \boxed{8} = 13$

$13 - 5 = \boxed{8}$

$6 + \boxed{6} = 12$

$12 - 6 = \boxed{6}$

Objective

Given a pair of related addition and subtraction equations with a missing addend, the child will be able to find the difference by thinking of it as a missing addend.

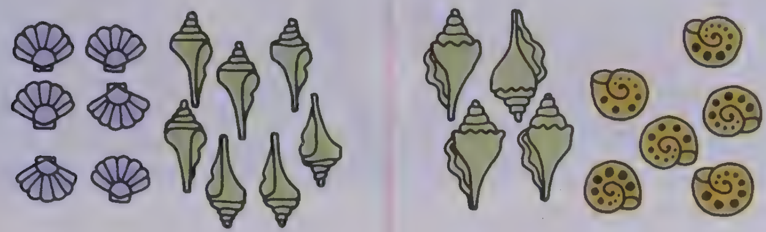
Pre-Book Activity

Materials

counters, at least 18 per child

Ask the children to build with their counters two sets whose sum is between 11 and 18 (such as a set of 6 and a set of 8). Observe with the children that $8 + 6 = 14$ by asking them to count the counters in these two sets. Write the addition equation $8 + 6 = 14$. Stress that they have 14 counters in the two sets combined and write the numeral 14 under the addition equation. Then ask the children to remove the 8 counters in one set. As they do this, continue the second equation so that you have $14 - 8 = \square$. Remind the children that they should think about starting with 14, removing 8, and finding how many are left. Since they have 6 left, they know that $14 - 8 = 6$. Point out that 6 was one of their original two sets. Work through several examples in this manner. Stress that knowing the addition equation can aid them in finding the difference. Finally introduce addition equations with missing addends. In these equations children must find the missing addends by counting out the counters needed to make the sum. Then they can use that completed equation to solve its related subtraction equation.

Solve the equations.



$6 + \boxed{7} = 13$ $\quad$ $4 + \boxed{6} = 10$

$13 - 6 = \boxed{7}$ $\quad$ $10 - 4 = \boxed{6}$

$2 + \boxed{9} = 11$ $\quad$ $8 + \boxed{2} = 10$

$11 - 2 = \boxed{9}$ $\quad$ $10 - 8 = \boxed{2}$

$3 + \boxed{5} = 8$ $\quad$ $9 + \boxed{3} = 12$

$8 - 3 = \boxed{5}$ $\quad$ $12 - 9 = \boxed{3}$

$7 + \boxed{7} = 14$ $\quad$ $8 + \boxed{6} = 14$

$14 - 7 = \boxed{7}$ $\quad$ $14 - 8 = \boxed{6}$

Relation between addition and subtraction

TEACHING
Page d-50

Since this page follows the same format as page d-49, direct the children to continue with the same procedure. Stress that they may use any materials they prefer: counters, strips, number line, or their own pictured sets.

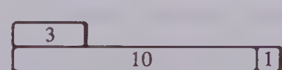
FOLLOW-UP

To help the children review subtraction combinations, place miniature subtraction tables, such as the following ones, on the chalkboard or overhead projecter. These tables can be used as a written or an oral activity, depending on the needs of the children.

Subtract 6		Subtract 5		Subtract 4	
10		12		10	
12		9		8	
8		6		6	
14		10		4	
9		8		11	
6		5		9	

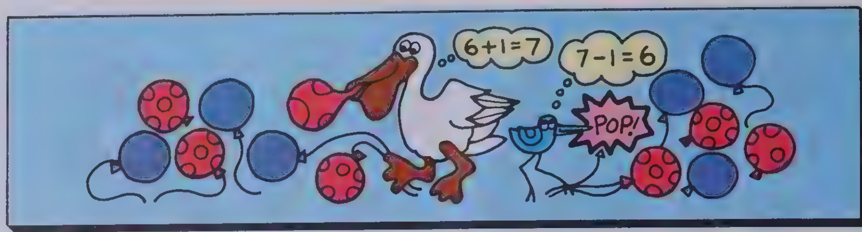
Alternatively, if children have grown familiar with the strips, you might direct them in using the strips to find missing addends. Write a missing addend equation on the chalkboard. Ask children first to build a train which represents the sum by using the orange strip plus one other strip for any sum greater than 10. Then ask them to take the strip for the first addend and try to figure out what strip will fit with it to make a matching train.

$3 + \square = 11$ $\quad$ $6 + \square = 15$

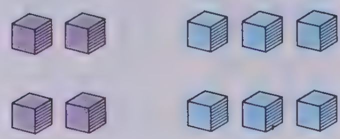


Teaching
Page d-51

Direct the children's attention to the first frame. Have them give the number in each subset and relate these objects to the two addition equations. Then explain how the objects may be thought of as a set of 10. Relate the subtraction equation to the idea of removing a set, in one instance 6, and in the other instance 4, from this total of 10. Again point out that knowing the addition equations should help them solve the subtraction equations. Work through the second frame similarly. If children need illustrations for the last two frames, suggest that they draw sets themselves.



Solve the equations.



$4 + 6 = \boxed{10}$	$5 + 3 = \boxed{8}$
$6 + 4 = \boxed{10}$	$3 + 5 = \boxed{8}$
$10 - 6 = \boxed{4}$	$8 - 5 = \boxed{3}$
$10 - 4 = \boxed{6}$	$8 - 3 = \boxed{5}$
$8 + 5 = \boxed{13}$	$5 + 6 = \boxed{11}$
$5 + 8 = \boxed{13}$	$6 + 5 = \boxed{11}$
$13 - 8 = \boxed{5}$	$11 - 5 = \boxed{6}$
$13 - 5 = \boxed{8}$	$11 - 6 = \boxed{5}$

Relation between addition and subtraction

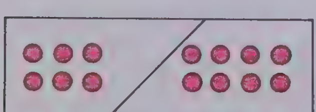
Objective

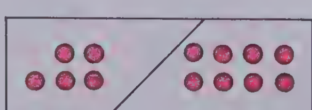
Given a partitioning of a set of 18 or less, the child will be able to solve two addition and two subtraction equations related to the partitioning of the set.

Pre-Book Activity

Materials

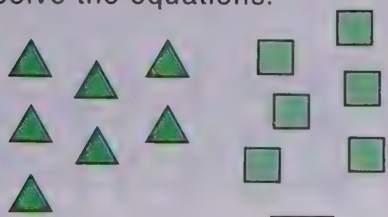
cards showing sets of combinations for 18 or less, at least one card for every 2 children. Sample cards:





Assign partners or have the children work in groups of three. (This activity may be done individually by each child if you prefer.) Give each group one of the cards you have prepared. Explain that each card represents a combination for 18 or a number less than 18. Tell them that you would like them to try to write four equations suggested by their card. You might explain that two of their equations should be addition and two should be subtraction. Then encourage them to work with their partners to find the equations. As they work move around the room asking questions to direct their efforts. If some groups finish very quickly, you might give them a second card or ask them to draw the sets and write the four equations they found on the chalkboard. When the children have written the equations, ask several children to explain their cards and equations.

Solve the equations.

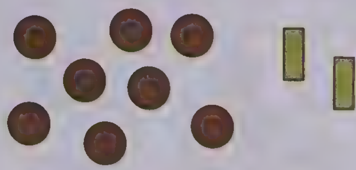


$7 + 6 = \boxed{13}$

$6 + 7 = \boxed{13}$

$13 - 7 = \boxed{6}$

$13 - 6 = \boxed{7}$



$8 + 2 = \boxed{10}$

$2 + 8 = \boxed{10}$

$10 - 8 = \boxed{2}$

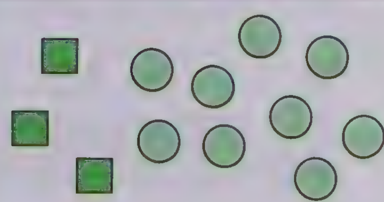
$10 - 2 = \boxed{8}$

$5 + 2 = \boxed{7}$

$2 + 5 = \boxed{7}$

$7 - 5 = \boxed{2}$

$7 - 2 = \boxed{5}$



$3 + 9 = \boxed{12}$

$9 + 3 = \boxed{12}$

$12 - 3 = \boxed{9}$

$12 - 9 = \boxed{3}$

Relation between addition and subtraction

Teaching
Page d-52

The format of page d-52 is very similar to that of page d-51. However, it would be helpful to observe with the children that here we are considering two sets of different elements, but thinking of combining them into a total so that we can still answer the question of how many items there are altogether. Encourage the children to complete the equations independently. When they are finished, have children explain their ways of finding the answers.

Follow-up

A worksheet like the one below would be helpful in developing understanding of the relationship between missing addends and differences. However, children should be encouraged to use whatever materials they need: counters, number line, or strips, to help them find the missing numerals.

Since $3 + 8 = 11$, then $11 - 3 = \square$
Since $7 + 9 = 16$, then $16 - 7 = \square$
Since $8 + 5 = 13$, then $13 - 8 = \square$

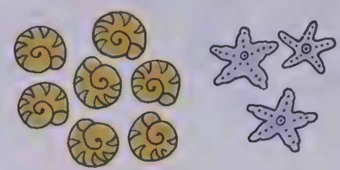
$2 + \square = 9$	$9 + \square = 18$
$9 - 2 = \square$	$18 - 9 = \square$
$5 + \square = 12$	$7 + \square = 14$
$12 - 5 = \square$	$14 - 7 = \square$

Teaching
Page d-53

Children should know what is expected of them on this page, so ask a child to explain what he thinks they are to do. Then encourage the children to complete the equations independently. Be sure, however, that they feel free to use aids such as the counters, number line, or strips if they choose.

When the children finish, ask volunteers to give their answers and explain how they found them.

Show you know
Solve the equations.



$7 + \boxed{3} = 10$

$10 - 7 = \boxed{3}$

$6 + \boxed{7} = 13$

$13 - 6 = \boxed{7}$

$5 + \boxed{4} = 9$

$9 - 5 = \boxed{4}$

$8 + \boxed{3} = 11$

$11 - 8 = \boxed{3}$

$5 + \boxed{5} = 10$

$10 - 5 = \boxed{5}$

$5 + \boxed{9} = 14$

$14 - 5 = \boxed{9}$

$5 + 7 = \boxed{12}$

$7 + 5 = \boxed{12}$

$12 - 5 = \boxed{7}$

$12 - 7 = \boxed{5}$

Module review

Objective

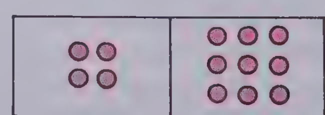
The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

Materials

objects for set demonstrations

Exhibit a set of 5 objects. Hold one hand hidden or have a box which children cannot look into and tell the children that you have enough objects hidden to make a total of 13 all together. Ask a child to write an equation for this situation and have someone complete $5 + \square = 13$. Show the complete set and review ways by which they could have figured out that 8 were hidden. Remind them that they might have used counters, the number line, or the strips. After working through several examples of this kind, show two sets, such as a set of 4 and a set of 9. Elicit from the children the various ways these sets might be thought of: combining the two, or considering the whole and removing one set. Lead them to a development of the four equations which may be written for these sets.



$9 + 4 = 13 \qquad 13 - 4 = 9$

$4 + 9 = 13 \qquad 13 - 9 = 4$

Develop other examples of this kind according to the children's needs.

Let's have fun

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

0	1	2	3	4	5	6	7	8	9

Finish each row.

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

0	2	4	6	8	10	12	14	16	18

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

1	3	5	7	9	11	13	15	17	19

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

0	3	6	9	12	15	18	21	24	27

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

0	4	8	12	16	20	24	28	32	36

0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22

0	5	10	15	20	25	30	35	40	45

Skip counting sequences

Teaching
Page d-54

This change of pace page develops a use of the number line which is new to the children. Until now this program has developed the use of single unit jumps on the number line, basically as an aid in counting. On this page, jumps range from jumps of 2 to jumps of 5. However, since this is an extension module, it is recommended that children be encouraged to try to figure out the patterns with as little guidance as possible. Explain that the row of numerals in the boxes below each number line matches the jumps shown on the number line. By studying both the jumps and the numerals, they should try to trace over the dashed lines and complete the jumps along each number line. Then they should write the correct numerals in the boxes. The use of a demonstration number line would be helpful *after* children have completed the page. Encourage children to describe on the display number line their completed patterns.

Follow-up

An "In the Bag" activity would serve as a review of combinations of 10. Choose a set of 10, for example, 10 paper cups. Make sure the children know there are 10. Tell them to put their heads down and close their eyes. Put some of the cups in a large brown paper bag. Then say: "Heads up!" "How many cups do you see?" "How many are in the bag?" When the children have answered these questions correctly, continue by allowing one child to put some cups in the bag and asking the others to try to tell how many they see and how many are in the bag.

BLUE MODULE, UNIT D

Inequalities

Pages d-55 to d-62

General Objectives

To introduce the child to the inequality symbols: >, greater than; and <, less than

To develop ideas of inequality and order

To introduce work with inequalities involving numbers from 0 to 99

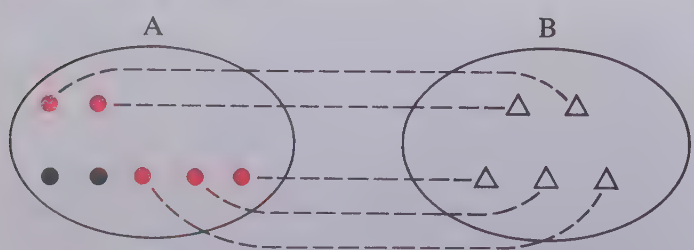
After the greater than and less than concepts are reviewed, children are introduced to the written notation. The inequality symbols are presented first with numbers of 10 or less to help the child gain a feel for the symbol while working with numbers that are not difficult. Children next compare multiples of ten, (such as $90 > 70$, $30 < 80$). Then, through carefully programmed exercises, they are led to a comparison of numbers within a given decade (such as $95 > 92$, $23 < 28$), and of numbers having the same unit digit (such as $51 < 91$, $62 > 42$). Finally, they compare any two numbers less than 100. Due to the nature of the work with inequalities, this module includes a review of place value. Comparison of numbers helps maintain and strengthen the understanding of place value.

Mathematics

The key concept of this module involves inequalities. The idea of one number being greater than another or that the numbers have a specific order is intuitively obvious to the child from the very beginning; however, it is in this module that the basic idea underlying inequalities is formally introduced and dealt with. The following is a precise mathematical definition of this idea based on sets.

> If a set from cardinal number A has a proper subset that is equivalent to a set from cardinal number B, then cardinal number A is greater than cardinal number B ($A > B$), and cardinal number B is less than cardinal number A ($B < A$).

This definition is illustrated in the following diagram by comparing cardinal numbers 7 and 5.



Notice that the set of colored circles is equivalent to set B. Furthermore, the set of colored circles is a proper subset of set A since it is a subset of A that does not contain all of A. From the definition, 7 is greater than 5 and 5 is less than 7. An intuitive explanation of why 7 is greater than 5 is merely that a set of 7 has more objects than a set of 5.

Teaching Blue Module, Unit D

Approximate Time: 4 to 6 days

Materials

counters: 30 per child

objects that can be used for demonstration and easily bundled into tens (pencils, straws, sticks, pipe cleaners)

Vocabulary

greater than
inequality
less than

Children should recognize that 37 is greater than 29 by comparing sets of objects as well as by thinking of 37 as being to the right of 29 on the number line. Once the children have caught on to the idea of comparing two-digit numbers, you will want them to begin thinking about comparing the number of tens and then, if these are the same, comparing the number of ones.

Recall, however, that even though sets are used to build ideas of comparison, *greater than* and *less than are relations defined for numbers* and not for sets.

Evaluation of Progress

Although one of the purposes of this module is to familiarize children with the inequality symbols, evaluation of the child's understanding of the concepts of greater than and less than should be of prime consideration. A child's incorrect use of or confusion of the two symbols, > and <, does not necessarily mean he has a faulty understanding of the greater than and less than concepts. Therefore, treat a child's performance with the written symbol as an indication of his understanding, but not as the sole criterion. Also, be patient with development of the children's ability to read phrases such as $27 < 29$, or $31 > 25$. Often a child will have a satisfactory understanding of the concepts and be able to place the written symbol correctly, but not be able to read the phrases easily.

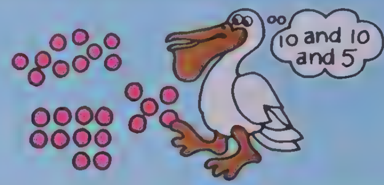
Resources for Active Learning

General Activites:

EARLY NUMBER MULTI-GROUP LAB ACTIVITY CARDS, "Number Relations," Nos. 24–27, Responsive Environments Corp.

ENRICHMENT OF ARITHMETIC, "What's My Number Name: 1/19–20," Webster, McGraw-Hill

Let's do



Can you build some sets that have **more than** 20 and **less than** 30?

Put your counters inside the ring.

Give the numbers of your sets.

20 21 22 23 24 25 26 27 28 29 30

Investigation
Page d-55

Be sure each child has at least 30 counters or sticks. Explain to the children that they are to build as many sets as they can that are more than 20 and less than 30. Point out the circled space on their page where they may place their counters and the section provided where they are to write the numbers of the sets they build. Notice that this recording space is one solid box, rather than individual boxes, to avoid suggesting that all numbers between 20 and 30 must be found. The children should not feel they have to build all the sets 21, 22, . . . 29. As the children work and you move around the room, encourage them to group their counters by tens so they may more easily do the counting and write the correct numeral. When they have finished, discuss several sets which children chose. Notice that none of their sets should contain twenty or thirty.

Purpose

To introduce the concepts of more than and less than for two-digit numbers

Preparation

Materials

a minimum of 30 counters per child

To prepare for this lesson, it would be helpful to review the meaning of place value. For example, hold up two bundles of ten sticks and five and ask a child to write the numeral for this number of objects. When he has written 25, point out that the 2 means 2 tens because of where it's positioned in the numeral. Then hold up 3 tens and 2 and repeat this procedure. Stress that the 2 in 32 simply means 2. Continue with other demonstration bundles. You might also write a two-digit numeral on the chalkboard and ask the children to explain its meaning by showing the correct number of bundles and single sticks.

Let's talk

Readiness for inequalities

Discussion
Page d-56

This illustration is intended to provide a basis for discussion of the *more than* and *less than* concepts. Notice that the groups of fruit may be compared without actually counting every piece. You might use phrases such as: "Suppose there are 40 apples, how many plums do you think there might be?" As you mention specific numbers, appropriate to the items illustrated, encourage the children to make guesses about more than and less than even if some of their guesses seem "wild." As they mention specific numbers, write appropriate phrases on the board such as:

"3 is greater than 1"
"40 is more than 30"
"3 is less than 7"
"50 is less than 60"

"75 is more than 40" and so on. If time permits, have some children make sets using bundles of ten to show these inequalities.

Follow-up

Oral activities may be used to reinforce the children's understanding of the vocabulary of this lesson. Ask children to name and compare sets around the room. For example, there are 8 windows and 1 door, 8 is more than 1; there are 28 children and 2 adults, 28 is more than 2; there is 1 flag and 12 erasers, 1 is less than 12.

Another activity would be to play a "more than" or "less than" game. Start by saying: "My number is 58. Can anyone name a number greater than my number?" This phraseology can be reduced to one child giving a number (less than 100) and a second child giving a number greater than the given number. For example, first child: "26," second child "36," or "44." This same procedure may be used for "less than." After the first child gives a number you might say "greater than" or "less than" as a direction for the next child to respond.

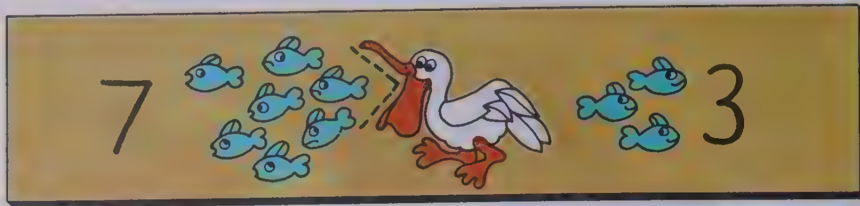
Resources for Active Learning

DEVELOPMENTAL MATH CARDS, A#15, Addison-Wesley

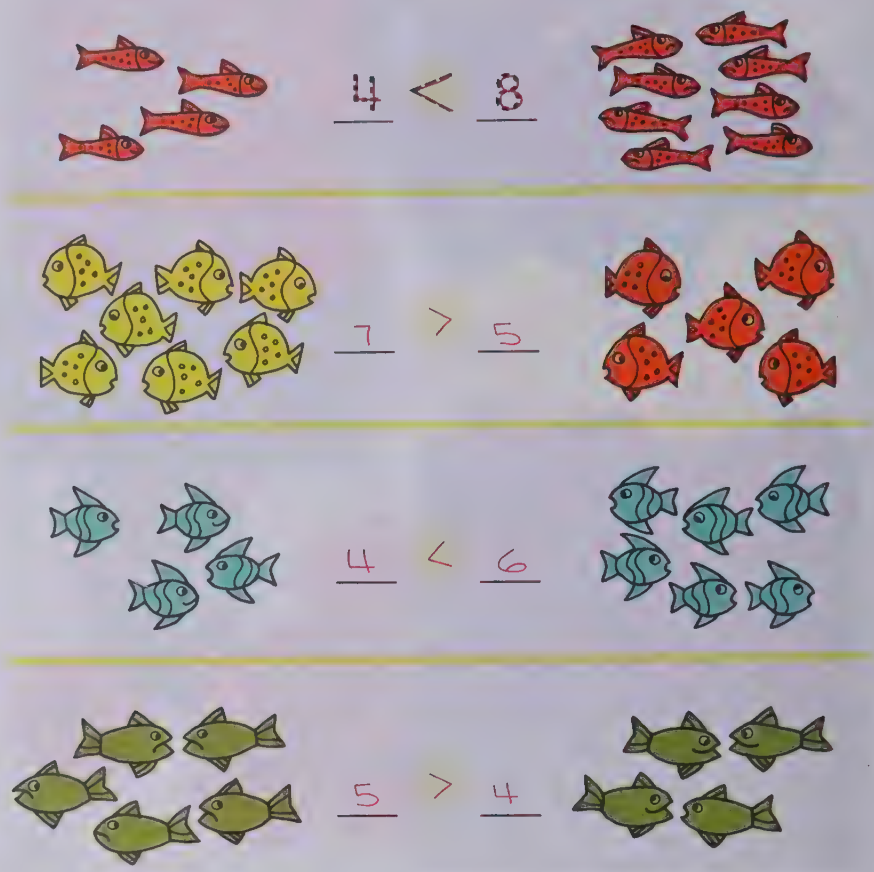
Teaching
Page d-57

Call attention to the pelican at the top and relate it to the story you have just told. Have children identify the numbers of the two sets of fish, and decide which is greater. Help them realize that the greater number, 7, is next to the wider space of the "pelican's mouth," that is, of the inequality symbol. Write $7 > 3$ on the chalkboard and read with the children "seven is greater than three." Have them trace over the dashed line symbol. Then read the directions for the exercises. Continue development with the first frame, writing $4 < 8$ on the chalkboard and have the children read "four is less than eight." Stress again that the greater number always stands near the wider end of the symbol.

Before children begin their work, be sure they understand that they should first find out how many there are in each set and write the numeral in the space provided and then place the inequality symbol between the numerals.



Tell how many fish. Then put the "mouth" in the .



Introduction to the inequality symbol

Objectives

Given the symbol >, the child will identify it as meaning greater than.

Given the symbol <, the child will identify it as meaning less than.

Pre-Book Activity

Since this lesson is concerned with the introduction of the written symbols used to express inequalities, you might prepare for it by telling the story of the hungry pelican who is pictured on page d-57. Give the pelican a name if you have not already done so and tell the children why he might be hungry. For example, he may have been in a storm so the fish stayed too deep in the water for him to catch them.

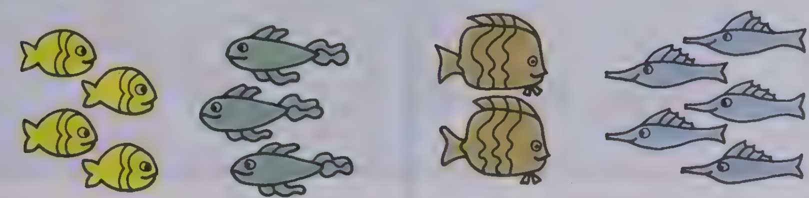
Follow-up

A written follow-up may take the following form. The second section would have to be carefully explained to the children.

Use <, >, or = to make each statement true.

5	4	6	0
8	8	4	5
8	9	5	7
2	1	4	4
3	3	3	2
$2 + 7$	9	$3 + 4$	6
$6 - 6$	0	$6 - 4$	4
$4 - 2$	6	$1 + 8$	9
$7 + 2$	8	$9 - 5$	3
$8 - 3$	7	$2 + 5$	7

How many in each set?



4 is greater than 3

$4 > 3$

2 is less than 5

$2 < 5$

Put $>$ or $<$ in each .

$6 > 2$	$3 < 5$	$7 < 8$
$1 < 7$	$6 > 2$	$8 > 3$
$6 < 9$	$4 > 2$	$4 < 9$
$9 < 10$	$10 > 9$	$10 > 2$
$8 < 9$	$9 > 2$	$0 < 1$
$4 < 10$	$3 > 0$	$6 < 8$

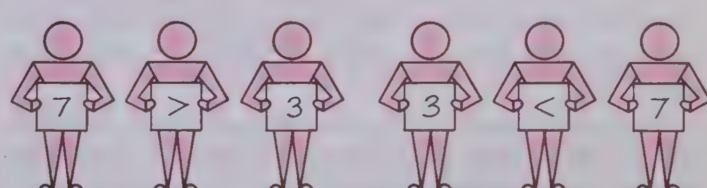
Teaching
Page d-58

Call attention to the sets at the top of page d-58. Observe with the children how many there are in each of the first two sets and which of the two sets is greater. Stress that 4 is greater than 3. Then remind the children that the symbol, $>$, may be used to express that 4 is greater than 3. Work through the second frame. Ask the children to explain how the phrases should be completed. Again emphasize that the wider part of the symbol is always closer to the greater number. The stress at this level should be on the concepts of greater than and less than and on the correct choice of the written symbols; exactness in reading the symbols will come gradually.

Encourage the children to complete the phrases independently. Suggest that for each phrase they decide which number is the greater and place the correct inequality symbol accordingly. To check their answers, do not read through the columns. Ask children to *write* the phrases on the chalkboard so they can see which symbol has been used. Use phrases such as: "Since 1 is less than 7 the wider end of the symbol is next to the 7."

Alternatively, an activity of the following type might be used. Using a large card for each numeral, make two sets of cards with the numbers 0 to 10, and two with the symbol $>$. (This symbol card may be turned over to show "less than") Divide the class into two teams. Distribute one set of cards to each team. (If necessary, increase the sets to include 11 and 12.) Have the teams stand on opposite sides of the classroom. Explain to the children that you will call out two numbers. Three children from each team, the two holding the two numbers called and one holding the symbol, should come to the front and arrange themselves to form a correct statement. The first team whose 3 members have positioned themselves correctly should be given a point. Since the children with the symbol cards will be involved for each statement, have children take turns holding them.

Sample play: Teacher calls 7, 3
Children may respond with either arrangement.



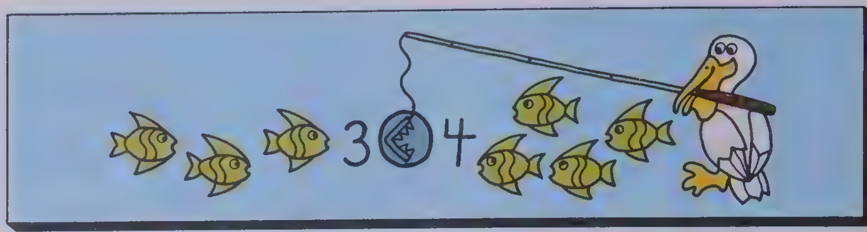
The other team members should act as judges to determine which group was the first to arrange a correct statement.

Resources for Active Learning

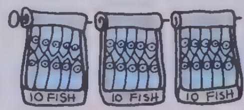
DEVELOPMENTAL MATH CARDS, "Drop On." B[1] 14, Addison-Wesley

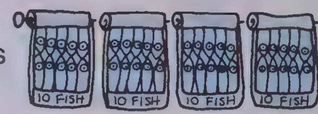
Teaching
Page d-59

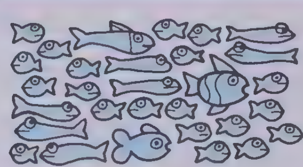
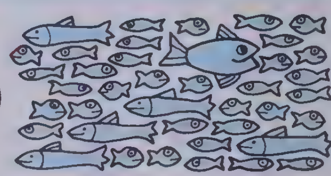
Work through the top three illustrations with the children. Elicit from them that since 3 is less than 4, 3 tens or 30 is less than 4 tens, or 40. In each frame, review with them the correct choice of symbols. You might use a similar example such as 30 > 20 in order to further review both the greater than and less than symbols. Remind the children that the wider end of the symbol is always written closer to the greater number. Then encourage the children to complete the exercises, and to continue on to page d-60.



Put > or < in each



3 tens < 4 tens



30 < 40


40 < 80	90 > 80	50 > 40
30 > 20	50 < 60	90 > 50
70 > 50	30 < 90	30 < 60
40 < 60	80 > 30	50 > 20
10 < 20	40 > 10	20 < 60

Objectives

Given two multiples of ten which are less than 100, the child will be able to identify the greater number by correctly placing the inequality symbol > or < between them.

Given two numbers which are less than 100, the child will be able to identify the greater number by correctly placing the inequality symbol, > or <, between them.

Pre-Book Activity

Materials

objects for set demonstrations which can be easily bundled in tens

Since the text page is carefully programmed, do not over prepare. It would be helpful simply to review the place value of two-digit numerals and compare sets, without emphasizing the written symbol. For example, write the numeral 56 on the chalkboard and ask someone to use the objects available to show a set that has 56 in it. When the child has shown 5 bundles and 6, ask another child to come and show a set which is larger than 56. When this has been done, have the numeral for the second set written on the chalkboard. Then ask someone to build a set that is less than the number you just wrote on the board and continue similarly. It should not be necessary to build many such sets. Keep this activity as a brief review of the meaning of two-digit numerals and of the concepts of greater than and less than.

Put > or < in each

20 < 30	60 > 40	50 < 90
24 < 34	62 > 42	51 < 91
27 < 37	69 > 49	54 < 94
7 > 3	2 < 7	6 > 4
47 > 43	12 < 17	56 > 54
67 > 63	32 < 37	96 > 94
29 < 30	29 > 19	12 > 9
30 > 29	15 < 22	64 > 59
40 > 39	42 > 38	27 < 35
39 < 40	27 < 32	44 > 38

Teaching
Page d-60

This page should be considered a direct continuation of page d-59. Encourage the children to proceed in the same manner by deciding which of the numbers in each pair is greater and placing the correct symbol accordingly.

Note that the exercises in the two top frames follow a careful development. When children have completed the page, be sure to point this out.

The children's understanding of inequalities will be indicated by their answers in the last frame. If any children show a lack of understanding, give them simpler inequalities before continuing.

Follow-up

The following exercise emphasizes that thinking about simple inequalities will help more capable children solve more difficult inequalities. It may be presented on the chalkboard or on the overhead projector.

Write > or < in the ○.

4 < 5	4 tens < 5 tens	40 < 50
7 > 6	7 tens > 6 tens	70 > 60
9 ○ 8	9 tens ○ 8 tens	90 ○ 80
0 ○ 1	0 tens ○ 1 ten	0 ○ 10
5 > 4	50 > 40	53 > 43
6 ○ 7	60 ○ 70	68 ○ 78
4 ○ 3	40 ○ 30	47 ○ 37
4 ○ 5	40 ○ 50	45 ○ 55

Resources for Active Learning

DEVELOPMENT MATH CARDS, "Four Sets," B-1 19, Addison-Wesley

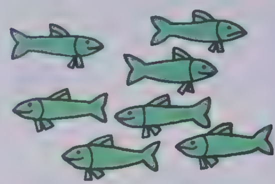
MATH ACTIVITIES, Activity 2/2, p. 25, Allyn and Bacon

Teaching
Page d-61

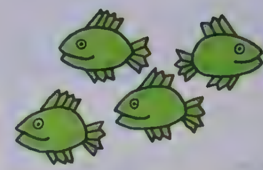
Have children observe that the top frames are accompanied by sets. Suggest that they study the sets and numerals carefully in order to place the correct inequality symbol. For the remaining pairs of numbers, ask the children to decide which number of the two is greater and to write the correct inequality symbol in the space provided.

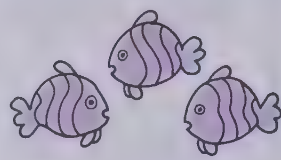
Show you know

Put > or < in each .

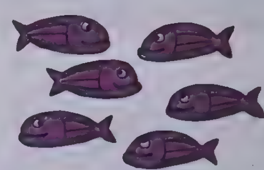


7 > 4





3 < 6



7 > 3	30 < 40	27 < 28
3 < 6	20 < 60	34 > 33
4 < 9	70 > 30	26 < 32
0 < 1	72 > 32	43 > 38
5 > 2	76 > 36	29 < 30

Objective

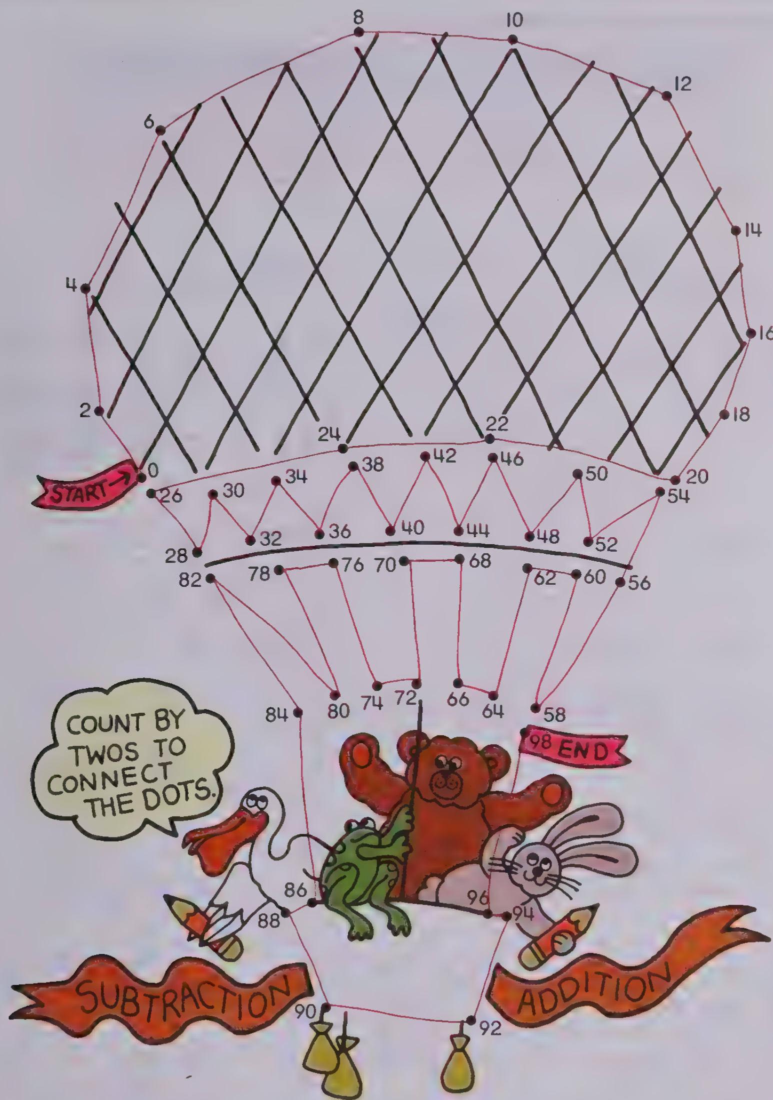
The child will demonstrate his ability to work with the concepts presented in this module.

Pre-Book Activity

A simple review of the symbols would be appropriate for this lesson. For example, write on the chalkboard statements such as those below. Notice that the children may give one of many numbers to make each of the statements true.

37 < ____	83 < ____	____ > 19
6 > ____	38 > ____	____ > 29
93 < ____	____ < 38	____ > 39
____ < 19	65 > ____	____ < 49

Let's have fun



Teaching
Page d-62

This change of pace page is simply an activity in counting by twos. You might challenge some capable children to figure it out on their own, but for most you will want to discuss with them that 0 is the starting point. Since there is no 1, have them guess where they should draw their first connection. Continue working with them only until you are sure they understand what to do.

Follow-up

Children who have worked this module might enjoy a more challenging activity. For example, you could give them an exercise with directions like: "Write *T* if the sign makes the sentence true, and write *F* if the sign makes the sentence false."

Write T or F.	=	>	<
2 + 3 □ 3 + 2	T	F	F
8 − 8 □ 4 − 4	T	F	F
9 − 2 □ 3 + 5			
0 □ 1 + 2			
4 − 1 □ 1 + 1			
7 − 1 □ 6 + 1			

Teaching
Page d-63

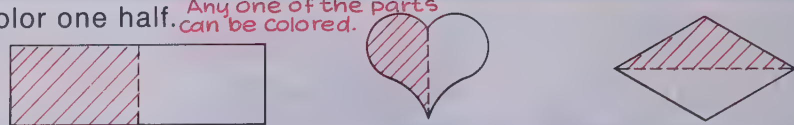
Explain to the children that they may use any materials they choose to help them solve the equations in the top section of the page. In the bottom section of the page they should color the fractional part of the figure according to the directions for each frame. If children need help in reading these phrases help them as needed.

Looking back

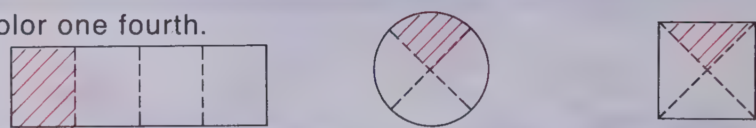
Find the sums and differences.

$7 + 2 = \boxed{9}$	$12 - 3 = \boxed{9}$
$8 + 4 = \boxed{12}$	$16 - 9 = \boxed{7}$
$6 + 5 = \boxed{11}$	$13 - 4 = \boxed{9}$
$9 + 6 = \boxed{15}$	$15 - 8 = \boxed{7}$
$7 + 7 = \boxed{14}$	$11 - 7 = \boxed{4}$
$9 + 7 = \boxed{16}$	$17 - 9 = \boxed{8}$
$8 + 5 = \boxed{13}$	$14 - 6 = \boxed{8}$

Color one half. Any one of the parts can be colored.



Color one fourth.



Color one third.

Cumulative review

Objective

The child will demonstrate his ability to work with the concepts presented in Unit D.

Pre-Book Activity

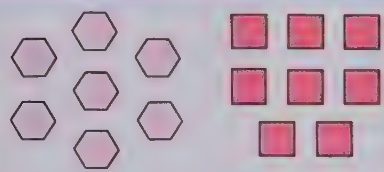
Review any concepts which seemed particularly troublesome for the children. For example, you might use the "I'm Thinking of a Number" game to review the basic addition and subtraction facts. Notice that the material treated on page d-64 is related to the concepts developed in the extension (optional) modules so should not be used with those children who did not study these modules.

Find the missing number.

7 + 6

10 + 3

Solve the equations.



9 + 5 = 14	7 + 8 = 15
5 + 9 = 14	8 + 7 = 15
14 − 5 = 9	15 − 8 = 7
14 − 9 = 5	15 − 7 = 8

Put > or < in each

8 > 6	40 < 60	56 > 37
15 > 12	80 > 30	39 < 41

Cumulative review

Teaching
Page d-64

Page d-64 is related to the concepts treated in the extension modules and should be used only by those children who studied these modules. You might choose to work through these exercises with those children.

Follow-up

Children might enjoy working out the following code.

8	9	10	11	12	13	14	15	16	17	18
A	F	H	V	N	O	E	I	T	U	C

1) $3 + 7 = \square$
2) $16 - 8 = \square$
3) $5 + 6 = \square$
4) $7 + 7 = \square$
5) $18 - 9 = \square$
6) $8 + 9 = \square$
7) $7 + 5 = \square$
8) $6 + 7 = \square$
9) $6 + 6 = \square$
10) $7 + 4 = \square$
11) $17 - 9 = \square$
12) $9 + 9 = \square$
13) $15 - 7 = \square$
14) $8 + 8 = \square$
15) $6 + 9 = \square$
16) $7 + 6 = \square$
17) $8 + 4 = \square$

Solution: HAVE FUN ON VACATION

Index

A Text for Teachers

Investigating Mathematics Learning

Phares G. O'Daffer

Robert E. Eicholz

Charles R. Fleenor

Introducing the Metric System

James Sherrill

J. Norman C. Sharp

ADDISON-WESLEY (CANADA) LTD.

DON MILLS, ONTARIO • READING, MASSACHUSETTS • MENLO PARK, CALIFORNIA • LONDON

INVESTIGATING MATHEMATICS LEARNING

I. Some Thoughts About Learning

Almost everyone has some observations on teaching and learning. A recent quote that has been making the rounds is: "If we tried to teach children to speak, they would never learn." However, in *The Process of Education* (Harvard University Press, 1960), Jerome Bruner observes, "Any subject can be taught effectively in some intellectually honest form to any child at any stage of development." But Linus (of *Peanuts* cartoon fame), considerably less optimistic, laments: "How can I learn 'New Math' with an 'Old Math' mind?"

In a more critical vein, John Holt in *How Children Fail* (Pitman Publishing Co., 1964) asserts: "In our classes, we begin with words, carry on with words, and often fail to get beyond words." He also says, "All too often the mathematics classroom becomes a temple of worship for the right answers, and the way to get ahead is to lay plenty of them on the altar." We know, of course, that many teachers for many years have been doing an excellent job helping elementary school children learn mathematics. Yet, it is worthwhile for us to reevaluate our approaches and, if possible, find even better ways to create situations where children learn more effectively.

The implications of the research of Piaget and others in how children learn mathematics and the observations of countless classroom teachers concerning the directions we should take are well summarized by a familiar Chinese proverb:

I hear and I forget.
I see and I remember.
I do and I understand.

The message of this proverb is that hearing and seeing are not enough: to learn with understanding, the child should experience *active involvement* with mathematical ideas. In order for the child to become actively involved, it has been found that the use of *physical materials* which contain the seeds of the mathematical ideas are valuable and often necessary. Coupled with the idea of active involvement with physical materials is the idea that teachers should encourage *student responsibility* and create conditions in which the student is not always encouraged to rely solely on the teacher but rather to take initiative for figuring out some things for himself.

Z. P. Dienes summarized a multitude of suggestions from researchers and teachers when he said: "It is suggested that we shift the emphasis from teaching to learning, from our experience to the children's, in fact, from our world to their world."

Teachers vary considerably in their views of how best to help children become actively involved with mathematics. While one teacher desires to convert his classroom immediately into a mathematics laboratory, another teacher may prefer a very modest beginning with a limited amount of active student involvement with physical materials inserted into his usual classroom approach. In this text we suggest a number of approaches for modest beginnings and indicate ways in which these approaches might be expanded to provide for a total laboratory approach and a more extensive individualized program.

To introduce one possible approach, let us simulate a teaching strategy by outlining one way to organize a specific lesson. Thus, suppose a teacher wanted to devise a lesson which would help children understand the idea of congruent segments in geometry. First the teacher provides each child with a geoboard and a sheet containing several 5-by-5 arrays of dots. Then he reviews, very briefly, the idea of a segment and the endpoints of a segment. Next, after helping the children see that they can use a rubber-band around two nails to represent a segment on the geoboard, he passes out the investigation suggested in Figure 1.

INVESTIGATION

How many different segments with endpoints just as far apart as these can you find on your geoboard? Record your results on dot paper.

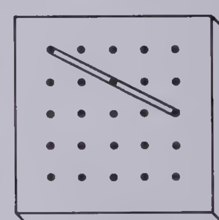


Figure 1

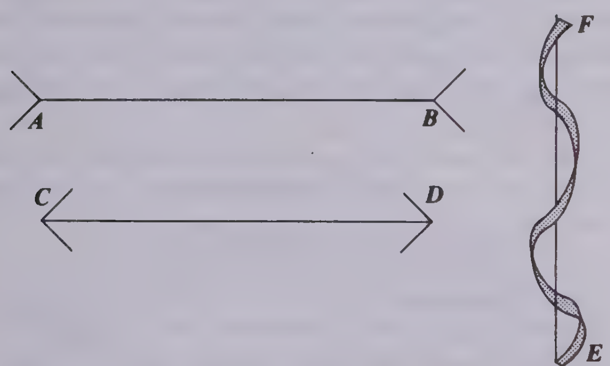
The teacher may choose to have chairs or desks rearranged so that children can communicate with each other as they become involved in the investigation. The teacher will check to be sure that everyone understands the investigation question; then he should encourage the children to find their own way to answer the question and record their findings. (To gain a fuller appreciation of an investigation situation, play the role of the child and complete the investigation yourself.)

Brief discussions among children or between teacher and children may occur during investigations, but the main discussion is most effective after the investigation has been completed. At this time, the teacher might ask such questions as: "How many different

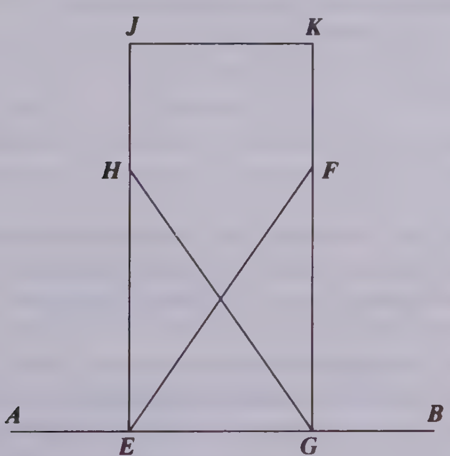
segments did you find?" "How can you be sure that you have found all such segments?" "How could you convince someone that each of your segments has endpoints just as far apart as all the others?" Such questions could then be followed up with a definition of congruent segments: When the endpoints of one segment are just as far apart as the endpoints of another segment, we say the segments are *congruent*. Then ask, "Can you think of some other ways to tell when two segments are congruent?" This question might lead into a discussion of how tracing paper, compasses, or marks on the edge of a piece of paper can be used to determine whether or not two segments are congruent.

After the children have discussed the ideas, the teacher may provide them with some problems which *utilize* these ideas. The child would probably be encouraged to use the ideas for testing congruence of segments that were developed in the discussion. The following are examples of possible exercises.

1. Find 2 segments below that are congruent to each other.



2. Name each pair of congruent segments in this picture.



One way to individualize a lesson is through an *extension* of the exercises. Extending the exercises can provide for remediation, reinforcement, or enrichment. As an extension to individualize this lesson, the teacher might give certain students the follow-up investigation below. (For a fuller appreciation of this lesson, complete the exercises and the investigation yourself.)

INVESTIGATION

Segment AB is not congruent to segment CD.
How many different segments (no two congruent) can you find on your geoboard? Record your results on dot paper.

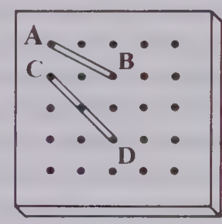


Figure 2

This abbreviated "lesson" provides a preview of one possible technique for encouraging children to become actively involved with physical materials in situations where they take more responsibility in the learning of mathematical ideas.

In the next section of this text, the parts of lessons such as the one described above will be analyzed and discussed. An outline for planning such lessons will be given, and various suggestions for carrying out each part of such a lesson will be proposed.

Since the investigation phase of the lesson provides the encouragement for active involvement by the child and since the kind of investigation used depends upon the type of learning involved, Section III in this text will focus on specific types of learning in elementary school mathematics. For example, the "lesson" described above helped children learn the *concept* of congruent segments; other lessons might be concerned with developing a *skill,* forming a *generalization,* learning a *fact,* or developing an *attitude*. Each type of learning will be analyzed and related to activity-oriented lessons that provide modest beginnings toward an active approach to mathematics learning.

Edith Biggs and James MacLean, in their book *Freedom to Learn* (Addison-Wesley, 1969), state: "A few schools scattered throughout the world are responding with some speed to a message which has been repeated with increasing urgency for some three hundred years. It is a simple message: Schools should be organized, not for teachers to teach, but for children to learn." In the same book, there appears an extensive list of "homemade" materials and devices that can be easily acquired for use in the mathematics classroom. Many materials, from newsprint and drinking straws, to string, popsicle sticks, beans, and homemade geoboards, can be made available to children at minimal cost. Rather than dismiss the possibility of actively involving children with materials in the classroom because no funds are available, a teacher should study this list carefully; he may be amazed by how much can be done with minimum expense.

Teachers sometimes feel that to involve children with physical materials and allow them to communicate with other children in the classroom is to invite

chaos. On the contrary, it has been found that, when children really become involved in using materials to investigate a situation, there may be a bit more low-keyed noise about the room but the usual discipline problems are almost nonexistent. It is helpful if there are tables available in the classroom so that children can work in small groups. If tables are not available, desks could be moved to assist in small-group work. On occasion, an investigation might call for children to leave their desks and to engage in other activity in the room. A simple set of "ground rules" should suffice to make the situation quite manageable.

It is interesting to consider the number of elementary school teachers who prefer to say that they are "helping children learn mathematics" rather than that they are "teaching mathematics." What one says, of course, does not always describe accurately what one does. It does seem important, however, in the light of recent studies and observations about how children learn mathematics, to focus on the child and try to create an environment in which the child has a greater opportunity to make decisions and to become really interested in his study. It is hoped that the following sections of this teachers' text will provide some ideas which may help you improve your ability to "help children learn mathematics."

EXERCISE SET 1

1. What was your reaction to the investigations in this section? **A** Did you become involved in the activity? **B** Were you interested? **C** Did you watch the clock? **D** Did you talk to anyone else while completing the investigation? (If so, was it helpful?) **E** Did the investigation situation help you better understand the idea involved? **F** What other feelings did you have?
2. Which quotation in this section seemed most significant to you? Why?
3. **A** Do you think most teachers teach the same way they were taught as elementary school children? **B** What do they do differently? **C** What are some ways you think our teaching of elementary school mathematics might be improved?
4. Look through the *Investigating School Mathematics* text at your grade level. How do the comments in this first section of the text relate to the approach taken in the child's text?

II. A Plan for a Learning Experience

First consider the practical matter of how the teacher proceeds in the daily task of helping children learn mathematics. A structured outline (inherently flexible) around which daily learning experiences may be planned can be a valuable organizational aid for the teacher and can give him a fresh insight into the role of new approaches to instruction.

Here is the outline that was used in planning the "lesson" in Section I. It has proven to be quite useful, especially for those teachers who have desired to make a beginning toward providing children more opportunities for active involvement with mathematical materials and ideas.

Preparation and Investigation
Discussion
Utilization and Extension

Since this outline offers a variety of possibilities for a teacher to reevaluate his approach to classroom instruction, the following sections provide an examination of its individual elements.

Preparation and Investigation

The investigation phase (often called simply "the investigation") is central to the learning experience. In this phase, the children are encouraged to become actively involved, individually or in groups, in the investigation of a situation that contains the seed for the central idea of the lesson. The investigation should be the "main event" in terms of pupil activity and involvement. The teacher should think of the investigation as a child-centred activity. Completion of the investigation in Figure 3 will help clarify the ideas of investigation.

INVESTIGATION

Can you find an investigation in a text from the *Investigating School Mathematics* series that

(a) uses centimetre strips?
(b) utilizes paper folding?
(c) has a question like "How many can you find?"
(d) involves the geoboard?
(e) encourages children to use graph paper?
(f) asks the children to record their findings?
(g) directs the children to use reference material?

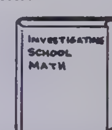


Figure 3

Homemade or commercially produced manipulative materials often provide the stimulus for the situation to be investigated. At other times, even more simple teacher-devised activities provide this stimulus. For example, the suggested investigation in Figure 4 might have been made by a teacher to initiate an investigation in a lesson designed to help children form the generalization, "You can rearrange three addends any way you please, and the sum will always be the same."

Sometimes by asking appropriate questions about a situation of interest to the children the teacher may involve them in an exploration of a central idea to be developed.

Regardless of how an investigation is initiated, a teacher should remember that the investigation situation is specifically designed to encourage children to

INVESTIGATION

Make three slips of paper like these.

2 3 4

Then turn them over and mix them up. Pick any two slips and add the numbers on them. Then add the number on the other slip.

If you do this five times, will you get the same number each time?

Figure 4

take responsibility for the thinking and exploring. Too much "teacher help" can hinder the achievement of these aims.

In an investigation, it is not uncommon to see children deeply involved and assuming full responsibility for completing the task at hand. The teacher, who plays a key role in initiating the investigation, may appear not to be needed as he moves about the room. Occasionally, a brief discussion between teacher and child occurs, but most of the larger-group discussion occurs after the investigation. The investigation itself should embody an attitude toward learning that could be easily stifled by too many words from the teacher. Perhaps, in an investigation, a new adage should replace the old: the teacher, rather than the children, should be "seen but not heard."

The investigation is predicated on the assumption that the best way to minimize the need for words is to substitute an appropriate question for a wordy explanation at a time when a child's interest in a mathematical situation is beginning to ripen.

For example, suppose a certain group of children understand the concept of a triangle and are ready to consider characteristics that distinguish one type of triangle from another. An appropriate question to initiate an investigation might be the one shown in Figure 5. (Try this investigation yourself.)

INVESTIGATION

How many triangles with different shapes can you make that have no nails inside?

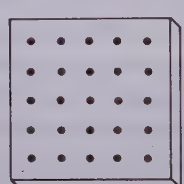


Figure 5

This question is both activity-stimulating and activity-sustaining. It helps involve the child in a search which he will continue with little further motivation. Notice also that the answer is not as important as the experiences the child will have as he responds to the question. Further, the question is sufficiently clear that the child immediately becomes involved with the challenge of the investigation rather than dissipating energy in efforts to understand the question. Another characteristic of this type of question is that it provides for individual differences: when the child is asked "How many can you find?" he can feel successful even if he finds only one. Of course, not all investigations can or should be introduced by this type of question, but it is important for the teacher to recognize that as the children respond to these questions, they will achieve in widely differing ways. In an investigation, the teacher should give recognition for all levels of achievement.

It should be noted that the amounts of time used for the investigations may vary considerably. One investigation may involve a very brief "happening" which sparks a simple idea within the child. Another investigation may utilize a large part of the period of time available for the mathematics lesson and might involve the child in a sustained exploration of a game or a set of manipulative materials.

To set the stage for an investigation of any duration, a preliminary *preparation* phase is sometimes needed. This phase provides for a brief review of key ideas needed for the investigation and for any motivational activity helpful in introducing it. This phase should be kept fairly short and care should be taken to see that this preliminary work does not preempt the central idea or activities involved in the investigation or the work that follows it.

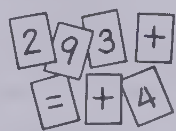
In summary, the investigation phase is the child-involvement phase. It often requires materials, and is usually motivated by a carefully selected question which focusses the student's attention on the central idea of the lesson. Proper consideration of this phase in your lesson planning can be highly rewarding.

EXERCISE SET 2

1. Find some investigations in the *Investigating School Mathematics* text that contain features not mentioned in Figure 4.
2. Choose a lesson from an *Investigating School Mathematics* text and write a description of the role you think the teacher would play in using the investigation phase of the lesson.
3. Choose an idea to be taught and prepare an investigation situation which has the potential of involving the child in working with this idea.
4. Two investigations follow. Give the central idea of a possible lesson based on the use of each one.

A

INVESTIGATION



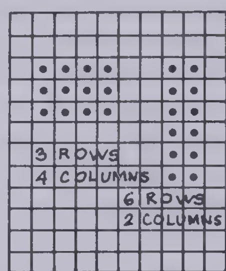
Cut out 7 slips of paper. Put one of these numerals or one of these signs on each one.

How many different equations with 3 addends can you write with your slips of paper? Record each equation you find.

B

INVESTIGATION

The graph paper shows two different ways to arrange 12 counters in a rectangular array.



How many different ways can you arrange 24 counters in a rectangular array? Record your findings by drawing pictures on graph paper.

5. Here is an interesting investigation you may like to try. Through it, you will be introduced to a basic idea of mathematics. Be sure to record your findings and be ready to discuss them further in the next section.

Copy and continue this array of numbers until you reach 52.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16
17	18	19	20
	. . .		

Then circle all the prime numbers in the array.

Notice that the numbers in the right-hand column can be written as $4 \times$ (a whole number). For example: $8 = 4 \times 2$, $12 = 4 \times 3$, and $20 = 4 \times 5$.

Can you make a statement about prime numbers that is suggested by this activity?

Discussion

Following the investigation, a *discussion* phase allows teacher and children to further share ideas in a discussion of what they found in the investigation. The teacher has an excellent opportunity in this phase to ask questions and to supply examples to help children further develop their understanding of the ideas germinated by the investigation.

Another valuable aspect of the discussion phase is that it provides additional opportunities for children to communicate with other children as a means of shaping their ideas. In a good discussion, it is not unusual for children, having reached an impasse in *their* thinking and communication about an idea, to ask the teacher if he can clarify the point. This is when the teacher as a resource person emerges. At other times, when ideas new to the teacher arise, the teacher participates in the discussion, not as a resource person, but as a fellow-learner. Both of these situations can contribute to a comfortable, meaningful discussion, but its potential benefits may never be realized if the teacher monopolizes the discussion to the extent that the children are denied the opportunity to draw their own inferences and make their own decisions. Since it is the child who is involved in the investigation, the child's ideas about the findings should be of primary importance, and the child should supply as many details leading to the understanding of the idea as possible.

By listening to the child and asking appropriate questions, the teacher can build on the child's initial ideas and help him develop a deeper understanding in preparation for further work. This understanding cannot be developed, however, by always asking questions which require simply that a child remember a fact correctly or perform a practical skill. Nor is it sufficient to ask questions to which a child can respond with a guess of "Yes" or "No." Rather, the questions that should be asked often are those that require a deeper thinking on the part of the child.

For examples of the more effective type of question, consider again the investigation described in Figure 7. This investigation, designed to set the stage for the development of the concepts of isosceles triangle, right triangle, scalene triangle, and equilateral triangle, might be followed by a discussion in which the teacher would ask questions such as the following:

1. Can you choose a pair of triangles you found and describe ways in which one is different from another?
2. In what ways are some triangles you found alike? (Note: Children may respond, "Some have a square corner," "Some have two sides the same," "Some have no sides the same," "Some are large," and so on.)
3. How would you describe a triangle that is different from any of the triangles you formed on the geoboard?

As the teacher asks thought-provoking questions and listens to the children's responses, he will be able to find ways to clarify the basic idea of the lesson and to prepare the children for the independent work which is to follow. It is in the latter stages of the discussion that the teacher may want to explain more carefully, show additional examples, and, in general, lead the child to a deeper mastery of the ideas involved.

EXERCISE SET 3

1. Can you find a question in the "Discussing the Ideas" section of an *Investigating School Mathematics* text which **A** asks the children to recall something previously learned? **B** asks the children to restate or explain an idea in their own words? **C** asks the children to interpret a diagram, picture, or explanation? **D** asks the children to analyze a given situation? **E** asks the children to evaluate a given situation?
2. What do you think about the effectiveness of the investigation described in Figure 5 as a means of meeting the goals indicated?
3. Write five questions you might ask while conducting a discussion in a mathematics lesson of your choice.
4. The following discussion exercises refer to the investigation presented in exercise 5 of Exercise Set 2. **A** What statement did you make about prime numbers? **B** Can you find a prime number that does not appear in the first or the third column? Can you find more than one? **C** $4 \times n$ is an algebraic expression. What algebraic expression can you devise to describe the prime numbers in the third column? in the first column? **D** Of the prime numbers less than 100, which type of prime occurs more often? **E** 113 is a prime number. Which type of prime is it?
5. Investigation questions may be open ("In how many ways can you measure a ball?") or closed ("Can you find the circumference and diameter of this ball?"). Discuss the merits of open and closed questions.

Utilization and Extension

The *utilization* phase allows each child to work on his own and to use the ideas developed in the investigation and discussion phases.

Often children need to practice recalling facts that have been developed or introduced in the lesson. Appropriate exercises requiring written answers are often valuable in providing this practice.

In another lesson, a child may have learned an algorithm or a skill. In order to refine this skill, he may need considerable practice using it. Appropriately designed written exercises which children complete independently can be quite helpful in polishing these skills.

In another lesson, a new idea may have been presented. In order to become more familiar with this idea and to understand how it relates to other ideas, the child may need thought-provoking problems which involve the idea. The *utilization* phase presents an opportunity for the child to solve problems which involve ideas that have been presented previously or to look at an idea that is different but closely related to one he has already encountered.

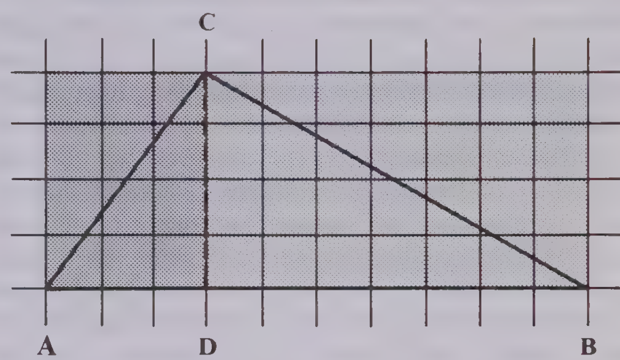
Creative activities for independent work can do much to extend the learnings developed in the investigation and discussion phases. The utilization exercises in examples A and B below are sequenced in such a way that the child has an opportunity to discover a new procedure or new ideas as a result of his work.

EXAMPLE A

Find the differences.

75	75	75	75	75	75	75
−32	−33	−34	−35	−36	−37	−38
43	42	41																								

EXAMPLE B

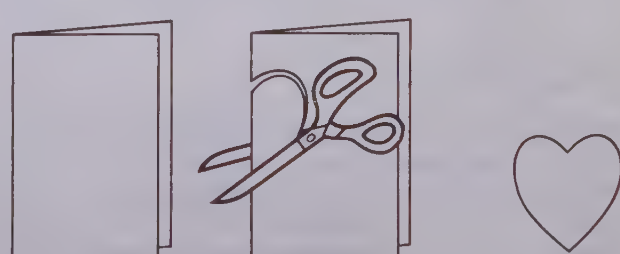


What is the area of the region shaded dark gray?
What is the area of the region shaded light gray?
What is the area of the two regions together?
What is the area of triangle *ADC*?
What is the area of triangle *BDC*?
What is the area of triangle *ABC*?
The area of triangle *ABC* is what part of the entire shaded region?

The teacher should appreciate the great potential value of discovery-sequenced exercises such as these, and should look for opportunities to make his own exercise sets using such sequences. Another set of utilization exercises might encourage the child to independently delve more deeply into the idea initiated in the investigation. Further activities with mathematical materials often provide opportunities for the child to use and extend the idea of the investigation. Example C provides an opportunity for the child to reinforce his concept of symmetry.

EXAMPLE C

Do this to make symmetrical figures.



Fold a piece of paper.

Make a cut that starts and ends on the fold.

Unfold the piece you cut out. It will be symmetrical.

Make cuts so that the unfolded shape will be:

A a rectangle	**D** a square	**G** a rocket
B a leaf	**E** a house	**H** a hexagon
C a triangle	**F** a pumpkin	**I** a butterfly

Regarding the utilization phase, it should be noted that on occasion it may be more valuable to have pairs or small groups of children work the exercises together.

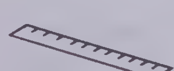
Finally, the *extension* phase provides for use of remedial, maintenance, or enrichment activities to further individualize the learning opportunities. This individualization offers numerous advantages. The slower children can avoid the frustration of having to proceed to new ideas before the previously presented ideas are understood, and the more capable children are spared the tedium of completing long lists of drill problems involving ideas they already understand.

The teacher might look for creative ways to meet individual differences in the ability to learn mathematics. For example, the slower child might profit from additional drill on certain facts and skills. Drill tapes or audio cassettes made by the teacher might provide a novel way to present the necessary practice. Duplicator masters and commercial workbooks are also available to provide extra work for those who need it. For other situations, an appropriate programmed instructional unit might serve the needs of the slower child. Single-concept film loops, which the child can play again and again, often are useful in helping him grasp an important concept. Appropriately conceived tutorial situations, in which classmates who understand the ideas work with children who do not, can be quite effective. Simple investigations utilizing physical objects which clarify more abstract ideas can also provide remedial work for certain children.

The teacher must also be concerned with those children who understand the basic ideas of the lesson and who can quickly work all the utilization exercises provided. These children can often become quite interested in activity cards which contain "open-ended" questions, such as the card shown below. (You are encouraged to try the suggested activity yourself.)

ACTIVITY CARD 10

In how many different ways can you measure yourself?


Make as many different measurements of **you** as you can and make a chart to show the information. Here are just a few suggestions:

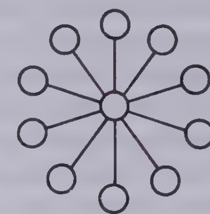
Pulse	Length of step
Height	Number of calories used
Weight	Area of bottom of foot
Arm span	Distance you can jump

Activities such as these give the child an opportunity to make his own decisions about which ideas he uses from the lesson and how he uses them.

Puzzles or riddles can also provide a useful extension of ideas for your children. Consider, for example, those shown in Figure 6.

Think

Draw a figure like this one on your paper. Place the numbers 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 in the circles so that the sum along any line is 21.



Think

I can be found halfway between Twenty-seven and seventeen.

WHO AM I?

27 ? 17

Figure 6

Conceptually fertile games can also provide valuable experiences to supplement the basic lesson. For example, the game "Sleuth" (3M Company) is fun for children and gives them valuable experience in classification and drawing logical inferences.

The methods suggested for extending the ideas for slower children are often suitable for use in certain situations with more capable children. Similarly, the more exciting modes of extension suggested for faster children can often be quite stimulating and valuable if used appropriately for the slower children.

It is to be hoped that the teacher will share a sense of excitement in providing extra stimulation to broaden the mathematical perspective of the children. Perhaps, he will also see that much of the extension activity can truly be fun for children while at the same time inspiring new interest and involvement in mathematical ideas. In using this suggested lesson outline, if the teacher chooses to maximize the investigation phase while deemphasizing the others, it might justly be said he is using the laboratory approach. On the other hand, should he maximize the discussion phase, he may find increased options for a guided discovery approach to mathematics learning. Also, it is possible that maximization of the utilization phase accompanied by appropriate student materials would allow the teacher to embark on a course of individually prescribed instruction.

EXERCISE SET 4

1. Find an example of an exercise set in which a learning sequence occurs in an *Investigating School Mathematics* text.

2. Choose a mathematics topic and write a set of exercises which might lead the student to discovery of a central idea.
3. Can you find a lesson in an *Investigating School Mathematics* text in which the "Using the Ideas" section provides for varying degrees of student ability.
4. Choose a learning experience appropriate for your children and list some possible specific activities for use in the extension phase of this learning experience.
5. Describe your views concerning the role of drill for slow, average, and bright children.
6. Select and play a game that could be used to extend a lesson with children.
7. In Exercise Set 2, you investigated an idea of mathematics. In Exercise Set 3, you had an opportunity to discuss this idea. The exercises below enable you to use the idea you learned, and suggest an extension of the idea.

Complete each exercise.

A List five prime numbers of the "$4n + 1$" type that are greater than 50.

B List five prime numbers of the "$4n - 1$" type that are greater than 50.

C 997 is the largest prime number less than 1000. Is it a "$4n + 1$" or a "$4n - 1$" prime?

D Suppose you used a continuation of the array of numbers shown below and circled all the prime numbers. What does this suggest about another way to classify the primes?

1	2	3	4	5	6
7	8	9	10	11	12
13	14	15	16	17	18
19	20	21	22	23	24
. . .					

III. A Focus on Specific Types of Learning

In considering the more specific aspects of mathematics learning it is helpful to categorize the general types of things children learn. A simplified categorization is given below.

Concepts
Skills
Generalizations
Facts
Attitudes

It is important to recognize that each of these types of learnings has unique characteristics. Because of this, the approaches and children's activities chosen to promote these learnings may often be quite different. In the sections that follow, we will consider each of these types of learning and suggest possible approaches and activities.

Concepts

Suppose that a child is having difficulty and comes to the teacher for assistance. When the teacher asks what the difficulty is, the child points to the multiplication 9×6 and says, "I can't do this because we haven't had it yet." This reflects a common attitude among children who have been in school for a few years. Somehow they learn to feel that they are incapable of figuring out anything new in mathematics. Literally, they can do nothing that they "haven't had yet."

If this child had confidence in his ability to "figure something out" and had a clear understanding of the *concept* of multiplication, he could have found the product by perhaps adding sixes, using sets, or making jumps on the number line. Another child who knew no division "facts" but who had a clear concept of division (as illustrated below) could use his knowledge of multiplication to find any of the basic quotients desired.

$$\overset{\mathbf{P}}{72} \div \overset{\mathbf{F}}{8} = \overset{\mathbf{F}}{n} \;\leftarrow\; \text{You find this quotient,}$$

$$\text{when you find this factor.} \;\rightarrow\; \overset{\mathbf{F}}{n} \times \overset{\mathbf{F}}{8} = \overset{\mathbf{P}}{72}$$

A *concept,* then, may be thought of as an idea which, when properly understood, will help the child to solve problems he "hasn't had yet," to figure something out for himself. As another example, consider the concept of prime number. Once a child understands that a prime number is a whole number with exactly two factors, he has the power, providing he understands how to find the factors of a number, to seek out and list those numbers that are prime. Of course, the task of deciding whether or not a given number is prime may be quite laborious, but understanding the concept does give the child the power to succeed.

To look more carefully at what concepts are and how they are taught, consider a model in which concept learning is relatively easy, namely, that of a set of attribute pieces. Suppose there are pieces of four different shapes (triangles, squares, circles, and rectangles), of three different colors (red, blue, and yellow), and of two different sizes (large and small), as pictured in Figure 7. (In the figures the colors red, blue, and yellow are denoted by the initials R, B, and Y.)

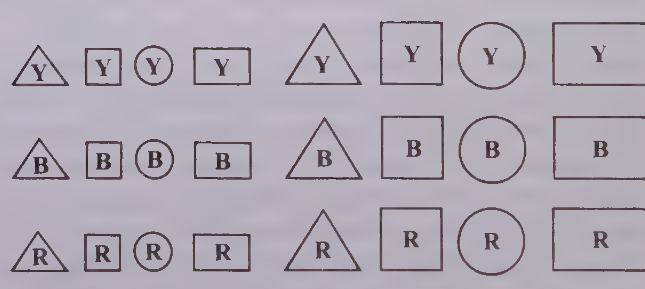


Figure 7

Now consider the Concept Card in Figure 8.

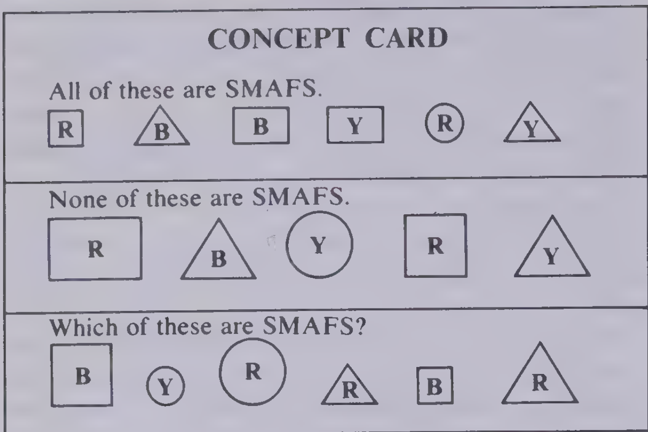


Figure 8

If you study the preceding Concept Card carefully, you will develop the simple concept of a SMAF. Notice that the key means used to teach this concept is by examples, along with *non-examples*. Both examples and non-examples play important roles in teaching many concepts in mathematics. The concept of a triangle may be taught to young children by using the Concept Card in Figure 9.

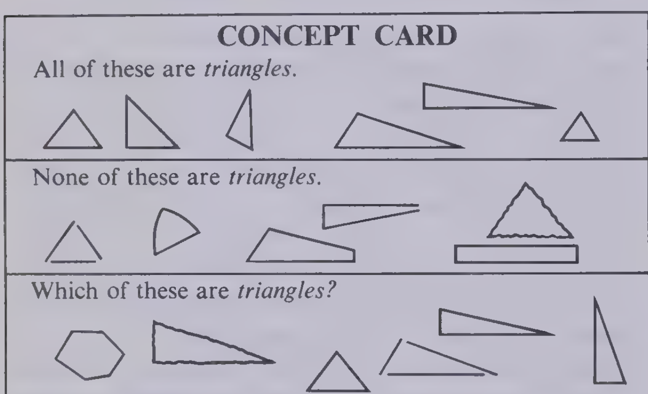


Figure 9

Clearly, the child would need further experiences in order to develop fully the concept of triangle, but the beginnings are embodied in the Concept Card shown in Figure 9.

One of the important ideas to remember when considering concepts is that concepts, unlike some other things that children learn, are developed over a period of time. Simple concepts may be developed very quickly, but other, more complicated concepts must be germinated when the child is very young and broadened through a spiralling return to the concept at various stages throughout the child's development. Many concepts are not fully developed until the child becomes an adult and encounters the idea in a variety of situations. For example, the concept of a fraction or fractional number may be introduced in grade 1 or grade 2, but a full understanding of this concept may not come until many years later. The child may acquire only an embryonic idea of a concept the first time it is presented, so it is important for the teacher to recognize the true nature of concepts and be willing to return often to the idea and carefully nurture its growth within the child. If he does not expect complete mastery after the initial presentation, he will spare himself considerable frustration when he recognizes later that the child needs further development of the basic idea.

Another key feature of concept learning suggested by the experiments of Piaget and supported and extended by the theories developed by Z. P. Dienes concerns the role of physical manipulative materials in young children's concept learning. In general, the implication of these authors' works is that it is through child involvement with physical environment that a firm basis for the development of more abstract concepts is laid. In fact, it is suggested that concept learning is facilitated by exposing children to as many different physical situations which embody the concept as possible.

It should be recognized that there are different levels of concept development and different types of concepts within these levels. For example, in the very earliest stages of mathematical learning, most concept learning involves the *concept of physical objects* such as balls, blocks, and circular or triangular objects. Very soon, the concept of certain *relations between objects* is developed: above, below, taller, shorter, larger, wider, longer, behind, and so on. A subsequent stage involves the concept of a *set of objects* such as a set of golf clubs, a set of dishes, a box of crayons, a set of blocks, a collection of stamps, or the children in a classroom. A slightly higher level of concept learning involves *relations between sets of objects:* equivalent, equal, has more than, has less than, and so on. It is at this stage that the important concept of *number* arises. For, in a sense, the concept of number involves a consideration of a set of equivalent sets. At a higher level of abstraction, the concept of certain *relations between numbers* (is less than, is greater than, is equal to, and so on) is developed. Ascending the ladder of abstraction, another level of development might involve the concept of *sets of numbers,* such as odds, evens, primes, composites, and perfects.

Clearly, the realm of concepts is vast, and the elementary teacher need not concern himself directly with many of the types of higher-level concepts. He must recognize, however, that the beginning stages in the development of many important concepts occur in the elementary school and that, through utilization of a variety of manipulative materials and appropriate strategies, he can do much to help the children learn concepts appropriate for their level.

EXERCISE SET 5

1. Use the attribute pieces shown in Figure 13. Invent a concept, name it, and make an appropriate concept card for it.

2. Choose at least two *Investigating School Mathematics* concepts from the list given below and develop concept cards which illustrate the use of examples and non-examples to teach the concepts you have chosen.
   A quadrilateral
   B simple closed curve
   C odd number
   D greater than (the relation)
   E right triangle
   F is congruent to (the relation)
   G lowest-terms fraction
   H parallelogram
   I diagonal of a polygon
   J parallel lines
   K one half
   L isosceles triangle
   M equivalent sets
   N symmetrical figure
3. Answer the questions on the sets of Creature cards from the set of attribute materials published by the Webster Division of McGraw-Hill Book Company (if available).
4. Choose an unusual concept of your own invention and make a concept card from which a person might discover your concept.
5. The investigation in Figure 1 was used to teach the concept of congruent segments. Make a card to teach this concept using examples and non-examples.
6. Complete "Learning a Concept" on pages I-18 and I-19; then answer the following questions.
   A What are some examples of the concept you learned?
   B Give some characteristics of the concept you learned.
   C What were your feelings about the lesson? How could the lesson be improved to make the learning of the concept easier?

Skills

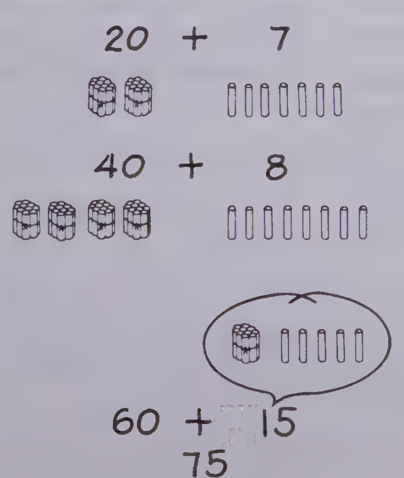
Broadly speaking, there are several types of skills that children develop in the elementary school. Hopefully, many children will develop a skill in estimating distance, weight, capacity, and time. Some teachers may wish to help children develop skill in drawing geometric figures. Some teachers set goals for upper-grade children which include developing skills in reasoning and even in "proof" of simple ideas. In elementary mathematics the most fundamental skill, by far, is that of computation with whole and rational numbers. It is these specific computational skills involving addition, subtraction, multiplication, and division and the processes related to these operations with which we are particularly concerned in the discussion that follows.

Two types of skills, power skills and speed skills, are available for completing each arithmetic process. A *power skill* is any effective way to find an answer. A *speed skill* is the most efficient way to find an answer. A power skill is a process through which a given problem is attacked by means of some technique which, though possibly quite inefficient, can produce a correct solution. This power skill may involve a long, tedious process, one which may be totally unrelated to the most efficient method for arriving at the solution. On the other hand, when a speed skill is employed, the problem is attacked with the most efficient technique available, and the problem is solved relatively quickly, usually in a mechanical fashion.

For example, suppose a child wants to find the sum of 27 and 48. If he simply starts at 48 and counts on 27 more, he is using a power skill. If, however, he finds the answer by using the usual algorithm for addition, then a speed skill is being employed.

Two additional points are worth noting about the previous example. First, in order to utilize the power

POWER SKILL B
– Bundles and Grouping



POWER SKILL C
– Expanded Notation

$$\begin{array}{r} 27 \\ +48 \\ \hline \end{array} \qquad \begin{array}{r} 20 + 7 \\ 40 + 8 \\ \hline 60 + 15 \\ 75 \end{array}$$

POWER SKILL D
– Addition Algorithm with Intermediate Step

$$\begin{array}{r} 27 \\ +48 \\ \hline 15 \\ 60 \\ \hline 75 \end{array}$$

Figure 10

skill, the child needed a clear concept of addition as it relates to counting. Thus, a power skill relies on a previously learned concept. As the child uses the concept in a power-skill situation, he gains new confidence in his ability to do something he "hasn't had yet." Secondly, the teacher should observe the evolution from power to speed. In finding the sum of 27 and 48, the initial power skill involved a basic concept of addition and the counting process. In practice, the child may continue the evolutionary trek from power to speed by next utilizing power skills B, C, and D as shown in figure 10.

Note that each of these power skills represents a small step toward the ultimate, more efficient speed skill. When considering this process of evolution, it should also be noted that the earlier stages in a power-skill sequence often involve manipulative materials with subsequent power skills exhibiting a transition from the concrete to the more abstract. This physical beginning, which utilizes bundles and grouping, is illustrated as Power Skill B in Figure 10.

The use of power skill is available to all children. The slower child may well attempt the problem by the only means he knows, one which may often be quite laborious. For example, in finding the quotient $5863 \div 72$, the slower child might subtract 1 seventy-two at a time until he has reduced the dividend to some number less than seventy-two. The more able and creative child might tire of this method and attempt to subtract some multiple of seventy-two, such as 10 seventy-twos. Since each child is working on his own for a period of time, the development of power skill is extremely helpful in working with individual differences.

One decision that the teacher must make in relation to each child is the extent to which he should be encouraged to develop an efficient speed skill for a given algorithm. Obviously, skills are important and should be taught in elementary mathematics, yet it is the good judgment of the teacher that plays the crucial role in guiding a given child from power to speed. For certain processes, children should probably never be forced to attain a speed skill, but should be allowed to operate at the power-skill level. Other children should be directed toward the speed skill as quickly as possible in order that they may proceed to more interesting aspects of mathematics. In rare instances, a child might profit from an initial consideration of a speed skill with no previous power-skill development of a given process. The emphasis on the role of conceptual power in the performance of a skill is a key feature of the so-called "new" mathematics. It is quite probable that we cannot predict the future mathematical needs of children in our classes today, but we can help them develop the confidence, even in the area of learning skills, to utilize concepts previously learned to discover some of the basic processes for themselves.

EXERCISE SET 6

1. Write *power* or *speed* depending on the type of skill you think is being employed.

	Specific Skill	Example
A	Using sets to find sums	 $3 + 2 = \square$
B	Using number line to find sums	 0 1 2 3 4 5 $3 + 2 = \square$
C	Counting fingers to find sums	 $3 + 2 = \square$
D	Memorizing that $3 + 2 = 5$	Think: 3, 2 $\rightarrow$ 5 $3 + 2 = 5$
E	Thinking about "take away" to find differences	 $5 - 2 = \square$
F	Using the inverse relation (missing addend) to find differences	Think: ? + 2 = 5 $5 - 2 = \square$
G	Memorizing that $5 - 2 = 3$	Think: 5, 2 $\rightarrow$ 3 $5 - 2 = 3$
H	Using sets to find products	 $3 \times 6 = \square$
I	Using the number line to find products	 0 5 10 15 $3 \times 5 = \square$
J	Using logic (basic principles) to find products	Since $5 \times 5 = 25$, $6 \times 5 = \square$ or Since $3 \times 5 = 15$, $6 \times 5 = \square$

2. Four different power skills are shown for finding $91 \div 7$. These skills would lead up to finding this quotient by "ordinary short division."

$$7\overline{)91}^{\,13}$$

In what order should these be presented?

A

$$\begin{array}{r|l} 7\overline{)91} & 10 \\ \underline{70} & \\ 21 & 3 \\ \underline{21} & \\ 0 & \end{array}$$

B Subtract 1 seven at a time.

$$\begin{array}{r} 91 \\ \underline{-\ 7} \\ 84 \\ \underline{-\ 7} \\ \vdots \end{array}$$

C

$$\begin{array}{r} 13 \\ 7\overline{)91} \\ \underline{70} \\ 21 \\ \underline{21} \end{array}$$

D Group 91 objects into sets of 7.

3. Complete the "Learning a Skill" lesson on pages I-19 and I-20; then do these exercises.
   - **A** Discuss the skill you learned and the way you learned in terms of power skills and speed skills.
   - **B** What part of the lesson helped you evolve a speed skill?
   - **C** What were your feelings about the lesson? How could it be improved?

Generalizations

Imagine that one of your students is engaged in an investigation in which he was asked to cut out a large quadrilateral and draw colored lines connecting the midpoints of each side of the quadrilateral. The question stimulating the investigation was, "Can you make an odd-shaped quadrilateral so that when you connect the midpoints you do not form a parallelogram?" As a result of this investigation and the subsequent discussion of his findings, the child was led to form a generalization: "The segments connecting the midpoints of any quadrilateral form a parallelogram."

In another lesson, a child might be responding to an investigation question which asked: "If you cut off the corners of a triangle and place the tips at the centre of a circle, what part of the circle can you cover? Can you find a triangle for which this is not true?"

As the child completes the investigation and engages in the discussion which follows, he forms this tentative, unproved *generalization:* "If a compass is used to draw arcs on the corners of any triangle and these corners are cut off along the arcs, then these corners will cover exactly one half of a circle drawn with the same compass opening." This tentative generalization, of course, is the forerunner of the familiar generalization that the sum of the degree measures of the three angles of any triangle is 180.

A generalization provides the economy of moving from consideration of isolated, specific cases to a general statement which holds true for a complete set of numbers or geometric figures. For example, the generalizations stated above deal with the set of all quadrilaterals and the set of all triangles. The regular occurrence of the word "any" in the generalization statements implies that the observation is true for *every* such geometric figure.

The key to teaching a generalization effectively is to provide children with appropriately chosen examples (or instances) which lead them to the generalization. An approach often used by teachers to help children learn generalizations is that of *guided discovery*. In this approach the teacher uses carefully sequenced questions and carefully chosen examples to focus the child's thought on the generalization to be discovered.

It is instructive for children in the upper elementary grades to have experiences in forming generalizations which seem obvious from a set of examples, but which, in fact, do not hold true. For example, consider the equations below.

$$\boxed{1} \times \boxed{1} - \boxed{1} + 11 = 11$$
$$\boxed{2} \times \boxed{2} - \boxed{2} + 11 = 13$$
$$\square \times \square - \square + 11 = ?$$

Figure 11

If 1 is written in the box and the operations are performed, the result is 11, which is a prime number. If 2 is written in the box, the result is 13, also a prime number. Upper-grade children are likely to conjecture that the sum is always a prime number. When they try 3, the sum is 17, also a prime. Similarly, the child finds that the numbers 4, 5, 6, 7, 8, 9, and 10, when written in the box produce a prime number. A child accustomed to forming generalizations from even fewer examples than this will likely conclude that this formula will always produce a prime number. It is instructive to note that when the next number, 11, is written in the box, the result is 121, which, being divisible by 11, is not a prime. This example illustrates the important idea that, even though the generalizations the child might make seem quite plausible and are most often true, it is only by means of a mathematical proof of a generalization that one can be completely sure that it is correct. These proofs, of course, are often not accessible to elementary school children. Thus, a healthy attitude might be characterized by references to generalizations which include phrases such as, "appears to be true," "is probably true," or "could most likely be proven."

Often a search for a generalization is initiated by a question such as, "Do you see any patterns?" For example, several simple generalizations might be formulated about the multiplication table in Figure 12. One child might observe that every number on the main diagonal of the table is a square number. Another student might observe that for every number on one side of this main diagonal, such as 10, there is a matching number symmetrically placed on the other side of the main diagonal.

×	0	1	2	3	4	5	6	7	8	9
0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9
2	0	2	4	6	8	10	12	14	16	18
3	0	3	6	9	12	15	18	21	24	27
4	0	4	8	12	16	20	24	28	32	36
5	0	5	10	15	20	25	30	35	40	45
6	0	6	12	18	24	30	36	42	48	54
7	0	7	14	21	28	35	42	49	56	63
8	0	8	16	24	32	40	48	56	64	72
9	0	9	18	27	36	45	54	63	72	81

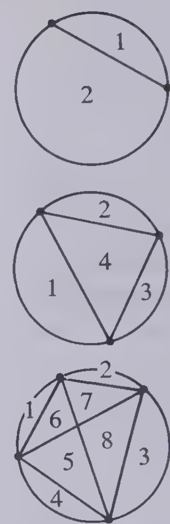
Figure 12

This last generalization is the table counterpart of the commutative principle for multiplication; that is, in the case of 12, 6 × 2 = 2 × 6 or 4 × 3 = 3 × 4. Another generalization that might be reached by careful consideration of the table is that the only primes in the table occur in the one-row or one-column of that table. Still another interesting generalization suggested by the table is that the sum of any number in the two-row and a number below it in the five-row will equal the number below these numbers in the seven-row. Of course, there are many other generalizations ranging from the very simple to the more complex that could be made about this multiplication table.

Perhaps the illustrations above will suggest that the mathematics available to the elementary school child is replete with possibilities for discovery of generalizations. The teacher's task is to create a learning environment in the classroom, not only in terms of physical materials and situations, but in terms of attitude toward learning and toward children, which provides opportunities for discoveries of generalizations and an atmosphere in which it is rewarding to make these discoveries. The teacher should be ever aware of the possibility that the habit of seeking generalizations may well be one of the most valuable things the child learns from his experiences in mathematics.

EXERCISE SET 7

1. Choose a text from the *Investigating School Mathematics* series and list some generalizations which the students who study this text might discover.
2. Investigate the Madison Project shoe boxes and complete the activities for at least two boxes.
3. The illustrations and the table which follow show that if you connect two points on a circle, you divide the interior of the circle into two regions; if you connect three points on a circle, you divide its interior into four regions; if you connect four points on a circle, you divide its interior into eight regions. Note that the points chosen should not be evenly spaced on the circle.



Number of points on a circle	Number of regions formed inside circle
2	2
3	4
4	8
5	
6	

   A Fill in the table to show how many regions are formed if five points on a circle are connected.
   B Form a generalization about the right-hand column of the table.
   C Test your generalization by finding out how many regions are formed inside when six points on a circle are connected.
4. Devise an investigation which might enable a student to discover this generalization: "The sum of the degree measures of the angles of a quadrilateral is 360."
5. Write some questions you would ask and show some examples you would use in guiding a child to discover one of the following generalizations.
   A The commutative principle for multiplication
   B The volume of a "box" is found by multiplying length times width times height.
   C In measuring length, the shorter the unit, the greater the measure.
   D Any angle inscribed in a semicircle is a right angle.
   E Every even number ends in 0, 2, 4, 6, or 8.
6. Complete the "Learning a Generalization" lesson on page I-20; then answer the following questions.
   A What generalization did you learn from the lesson?
   B How many specific examples did you consider before you understood the generalization?
   C In what way did you use the generalization after you discovered it?

Facts

In elementary mathematics, there are certain bits of information that are used so frequently that it is

beneficial for the child to be able to recall them quickly when they are needed. These items are ordinarily called *facts*. There are three main types of facts that are of major concern. The first type of fact is one which evolves from a concept. It might be an example of a specific concept ("Two is a prime number," "25 is a square number," "A parallelogram is a quadrilateral"), or it might be a characteristic of a specific concept, possibly even a part of the definition for the concept ("An isosceles triangle has two congruent sides," "An even number is a number divisible by two," "A pentagon has five sides"). Examples of, or characteristics of, concepts are not always considered as facts; only if such an example or characteristic is deemed important enough to be remembered for immediate recall, is it considered to be a fact and committed to memory.

A second type of fact is a fact derived from a generalization; that is, if a generalization is simple, or deemed important enough to remember for immediate recall, it might often be considered a fact. For example: "The sum of the squares of the lengths of the legs of a right triangle is equal to the square of the length of the hypotenuse of the right triangle"; or "The length of the segment joining the midpoints of two sides of a triangle is one half the length of the third side." Each of these statements might be considered facts since they are sometimes useful for immediate recall. A third type of fact—one that is given a great deal of attention in the elementary school mathematics program—is the type of fact derived from a power skill. For example, the child may have utilized a sequence of power skills for finding sums such as $4 + 3$. He may have used sets of counters, centimetre strips, jumps on the number line, or reasoning from facts such as $3 + 3 = 6$. These power skills, based on certain important concepts, provided the evolutionary progression toward the final speed skill used in finding sums. In this particular case, however, the speed skill used is simply that of memorizing the sum. Whenever the speed-skill stage involves memorization, the particular learning which was classified as a skill or a process during the power-skill stage is reclassified as a fact. The basic addition and multiplication facts fall into this category, and they are given major attention in the elementary school. It is these facts to which primary attention will be given in this section.

A first important point to be made in discussing the teaching of facts is that extensive power-skill work preceding the memorization stage can pay valuable dividends. The broad base of understanding provided by the power-skill work removes the aura of magic from this aspect of mathematics and not only makes the task of memorization of the facts easier, but helps the child view it as a "reasonable thing to do." Figure 13, for example, shows some of the power skills that might be utilized in the initial development of procedures for finding products. Careful development using some or all of these power skills can give the child a basic feeling for a procedure by which products may be found.

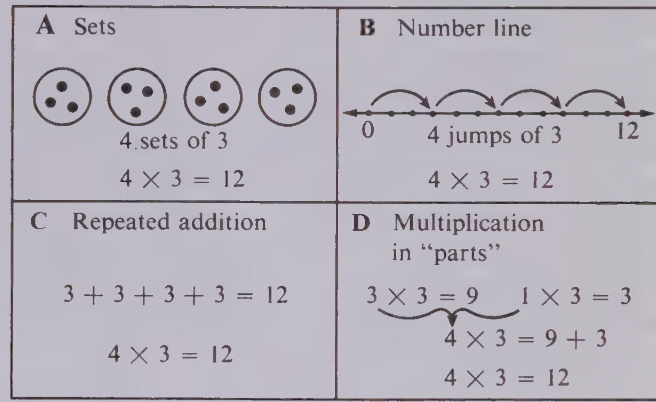


Figure 13

The teacher must use good judgment in deciding when a given child should move from this power-skill stage to memorization of the facts. The appropriate time could vary extensively depending upon the ability and experiences of the child. If the power-skill work is started early in the elementary grades, the child will have ample time to reap the benefits of this basic experience with materials and concepts before the transition to speed skill is made.

When the time has come to memorize the facts, it is important for the child to have a clear idea of the nature of this goal and the reasons it is appropriate. The teacher should even take time to help the child see the very clear difference between "figuring out the fact" and "memorizing the fact." Hopefully, he could help the child develop a feeling for situations in which the facts will be used and in which immediate recall would be quite valuable and time-saving for the child.

After the addition or multiplication facts to be memorized have been placed in perspective, the teacher should seek interesting situations and creative ways in which to practice recalling the facts. For example, the children might make their own flash cards and use a timer to see how long it takes them to give these facts. If desired, two children could work together and see which of them could go through the flash cards most quickly. Another game utilizes a pair of homemade colored dice and an empty multiplication table (Figure 14) for each child. As the game

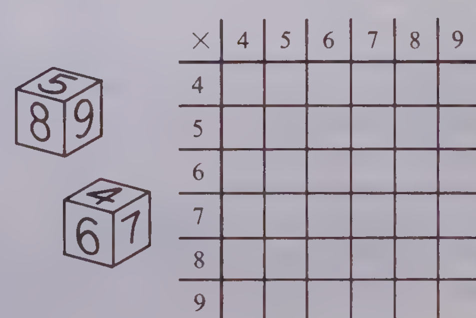


Figure 14

proceeds, a child rolls his dice and writes the product of the numbers on the dice in the appropriate space on his multiplication table. His partner then does the same thing when it is his turn. If a child arrives at an incorrect product or writes the product in the wrong space in the table, he is penalized by missing a turn. The object of the game is to see who can complete the table first. Various modifications of this game are possible, including one in which each child works independently and keeps a tally of the number of times he rolls the dice and also keeps track of the time it takes. The basic objective, of course, is to provide an interesting situation in which the child is motivated to recall multiplication facts rapidly.

Some children may need to spend considerable time in the power-skill stage before they begin to memorize. If there are children who have attempted to memorize the facts and find the job more difficult than anticipated, the teacher may want to consider allowing them to prepare a fact card on which they write the facts that they still do not know. Perhaps it would be realistic and beneficial to let some children use this fact card during the year whenever they desire, thus relieving the tension that could result from difficulties they encounter in memorizing the facts at one specific time. As the school year progresses, the teacher may want to suggest from time to time that a particular child concentrate on one of the troublesome facts and attempt to memorize it so that he can remove it from his fact card. The accomplishment of this goal, of course, would merit recognition and reward. After one fact is removed, the child might start working on removing another fact. The ultimate goal would be to remove all of the facts by the end of the year. Teachers who are interested in helping children learn mathematics in a comfortable way may find that a more realistic, less pressured approach to learning facts may enable the child to find greater enjoyment and success in his mathematical experience.

EXERCISE SET 8

1. Invent a game that could be used to help children practice recalling addition or multiplication facts.
2. Find a commercially produced game that is designed to help children practice recalling facts.
3. Complete the "Learning Some Facts" lesson on pages I-20 and I-21 of this text; then answer the following questions.
   - **A** How many of the facts did you know?
   - **B** What techniques did you use to help you memorize the remaining facts? Did you find this lesson difficult?
   - **C** Can you imagine some of the difficulties your children might have in learning facts?
   - **D** Did you find any mnemonic devices which were helpful in remembering the facts?

Attitudes

In his poem "Arithmetic," Carl Sandburg wrote:

Arithmetic is numbers you squeeze from your head
  to your hand to your pencil to your paper
  till you get the answer.
Arithmetic is where the answer is right
  and everything is nice
  and you can look out of the window
  and see the blue sky—
  or the answer is wrong
  and you have to start all over and try again
  and see how it comes out this time.

. . . . .

Arithmetic is where you have to multiply—
  and you carry the multiplication table in your head
  and hope you won't lose it.*

The attitude toward mathematics, school, one's ability, and learning in general that one senses on reading this part of Sandburg's poem is surely typical of many children in classrooms today. Perhaps, it was a feeling similar to this that caused Huckleberry Finn to say:

> I had been to school 'most all the time and could spell and read and write just a little and could say the multiplication table up to six times seven is thirty-five, and I don't reckon I could get any further if I was to live forever. I don't take no stock in math, anyway.

There are many different kinds of attitudes exhibited by children who have been exposed to classroom mathematical experiences in different parts of the world. There are, of course, the more general attitudes that a child has toward his teacher, toward his school, toward his fellow students, and toward the process of education. All too often the child's attitude toward education in general is that suggested by Charles Schulz in this *Peanuts* cartoon.

© 1971 United Feature Syndicate, Inc.

---

*From *Complete Poems*, copyright, 1950, by Carl Sandburg. Reprinted by permission of Harcourt Brace Jovanovich, Inc.

Two of the attitudes to be considered here, however, are the child's attitude toward mathematics and the child's attitude toward himself as he relates to mathematics. It has been said that the mathematical experiences of a child before the age of 11, and the responses he has been encouraged to make to those experiences, largely determine his potential mathematical development. If this is so, then a child's attitude toward mathematics and his feelings about how he relates to mathematics are extremely important considerations for the classroom teacher.

A moment's reflection on the number of people who are willing to say that they hate mathematics and on the multitude of others who seem to harbor a fear regarding their inability to cope with ideas of mathematics leads the teacher to realize that he does indeed teach attitudes, whether he tries to or not. Clearly, the teacher who conducts a classroom in which children's achievements are evaluated almost exclusively on the basis of how many right answers they can come up with must surely engender attitudes in children which differ greatly from those engendered by the sensitive teacher who recognizes the child's need to think his own thoughts and to become involved in an exciting exploration of ideas that interest him. Or, consider the difference between the teacher who teaches only speed skills and facts and the teacher who recognizes the central importance of concepts and generalizations, as well as the facts and skills. The child exposed to the first teacher must surely have a feeling toward mathematics, and his ability to interact with it, that is far different from that of the child who learned with the second teacher.

If what happens in the classroom is of such importance in developing attitudes within the child, then the teacher may want to reevaluate his approaches to instruction by reconsidering certain fundamental questions. What subject matter and methods most effectively instill within the child the feeling that mathematics is interesting, fun, and a source of adventure? Will these means provide an opportunity for the child to exercise his freedom of choice and to make decisions about what he does with mathematics? Aldous Huxley said: "A child is a genius until the age of ten." Could it be that our classroom approaches squelch this genius? Can we select mathematical experiences and materials that enable the children to experience success and thus maintain that sense of worthiness and prestige with peers that is of such importance? Can we structure these experiences in such a way that the child maintains within this atmosphere of freedom a sense of security and safety, thus avoiding the fear that can erode his ability to approach mathematical situations with confidence? Can we help children see the usefulness and importance of mathematics without boring them?

Clearly, the questions just raised are difficult to answer and specific techniques for developing healthy attitudes are hard to come by. But even though prescriptions for developing attitudes are scarce, many of the ideas about teaching suggested in earlier sections of this text can provide assistance for the teacher. The investigation, for example, provides the child with an opportunity to make independent decisions and to interact with mathematics and materials and encourages him to take responsibility for his own learning. As difficult as it may seem at times, a child's acceptance of responsibility for his own learning inculcates an attitude that is ultimately invaluable. Also, the manipulative materials or activities that are made available to the child in the investigation situation provide an interaction with the physical world that is often extremely valuable in making mathematics real to a student. Unless a child is ready for more abstract thinking, he cannot be induced to sense the adventure in mathematics without a physical environment to explore. Opportunities for attitude development are implicit not only in the investigation phase of a lesson but in the discussion as well. If a teacher can convince the child that his ideas are important, then the child finds himself in a situation, albeit a mathematical one, in which *he* feels important. His prestige with his peers increases and he feels successful. Exercises in the utilization phase of a lesson that begin simply and gradually increase in difficulty can also help the child feel that he can do mathematics on his own; and, of course, carefully selected extension activities can provide the child with a variety of opportunities to experience the fun of mathematics.

Not only do the phases of the learning experience provide unique opportunities of attitude development, but the particular types of learnings involved within these phases also have their effect. The teaching of concepts and generalizations provides the child with a feeling of power regarding mathematics, for when he experiences the thrill of discovering a concept or a generalization, or when he uses these to solve a problem, he is also developing a useful and wholesome attitude toward mathematics learning. He is developing a habit of reacting to a mathematical situation which will be invaluable when he later encounters mathematical situations possibly undreamed of today. Also, careful teaching of skills and facts can provide the child with that basic sense of security that comes simply from being able to do something or to remember something.

Figure 16

Regarding the child's level of confidence in his ability to cope with mathematical problems, one of the child's paramount needs is to experience success, and as mentioned previously, having entertaining experiences with mathematics might decrease the fear that can erode his confidence. To provide these experiences, the teacher might create in the classroom a "Fun with Mathematics" centre (see Figure 16) that contains mazes, puzzles, design materials, and so on. This centre represents an extra effort to encourage the child to successfully play with mathematics. Some of the materials that might be in such a centre are as follows: the soma cube, the tangram pieces, 2-cm cubes, materials for curve stitching, a kaleidoscope, pattern blocks, Cuisenaire rods, multi-base arithmetic blocks, geoboards, a wide variety of counters, attribute blocks, scales and balances, timers, calendars, measuring tapes and rulers, yarn and string, an assortment of boxes and cans, magazines and catalogues, mirrors, dice, play money, graph paper, assorted plane and solid shapes, abacus, pegboard, compass, mathematical balance, etc.

Perhaps, as you consider the attitudes more carefully and reevaluate the effects of your approaches to instruction, you will find other ways to help children develop a healthy attitude toward mathematics and an enthusiasm for the enjoyment it can offer. Each day as the teacher enters the classroom with plans for a learning experience, he might well ask himself: "What effect will *this* have on the attitudes of the students in my classes?"

EXERCISE SET 9

1. Select a text from the *Investigating School Mathematics* series and find at least five activities which could contribute to the child's development of a positive attitude toward mathematics.
2. Explain how you think some of the other types of learning might also contribute to better child attitude toward learning in general and mathematics specifically.
3. Complete the "Learning an Attitude" lesson on page I-21 of this text; then answer the following questions.
   - **A** Was the lesson fun?
   - **B** How did you feel when you had finished the lesson?
   - **C** Did the lesson change any of your ideas about mathematics?

IV. Some Learning Experiences for the Teacher

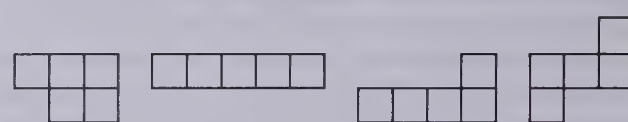
In Section II you were introduced to an outline for a learning experience which involved preparation, investigation, discussion, utilization, and extension. In Section III the types of things children learn—concepts, skills, generalizations, facts, and attitudes—were categorized. In this section, we combine these ideas and use them in presenting five learning experiences designed especially for the teacher. That is, in order to gain a first-hand view of lessons which develop these types

Lesson 1. Learning a Concept

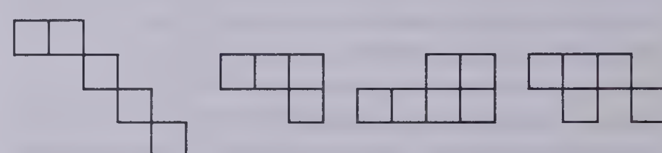
What is a pentominoe?

Investigating the Ideas

Each of these is a **pentominoe.**



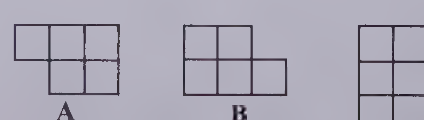
None of these is a **pentominoe.**



How many more pentominoes can you find and show on graph paper?

Discussing the Ideas

1. How many pentominoes did you find?
2. Can you give some characteristics of a pentominoe?
3. How would you "broadly classify" a pentominoe?
4. Can you define a pentominoe?
5. Are the pentominoes in Figures A, B, and C the same?



A **B** **C**

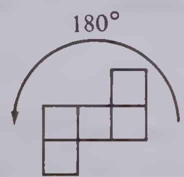
6. How could you convince someone that you have found all possible pentominoes?

of learning, the teacher will have experiences with each of these in the five lessons; and, in order to become more familiar with the suggested structure for a learning experience, each of these five lessons will involve an investigation, a discussion, a utilization, and an extension of the ideas.

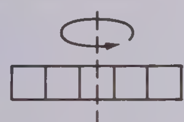
It might be valuable for the teacher, after he has become involved in each of these lessons and has completed the activities, to rethink and discuss his reactions to the various phases of the lesson structure and to the various types of learnings involved. In this way, he might gain a new insight into the way the children in his classes might react to these kinds of situations.

Using the Ideas

1. Which of the pentominoes can be folded to form a box with the "lid missing"?
2. Some pentominoes can be rotated about a point 180° and returned to their starting position. These pentominoes are said to have 180° rotational symmetry. Which pentominoes have 180° rotational symmetry?



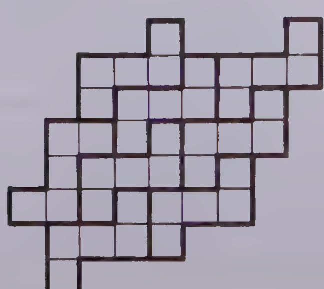
3. Some pentominoes can be flipped about a line and returned to their starting position. Such pentominoes are said to have reflectional symmetry. Which pentominoes have reflectional symmetry?



4. What do you think a hexominoe would be? How many hexominoes can you find?

Extension

Some pentominoes can be used to tessellate (fill without overlapping) the plane, as shown below. Can you find at least two more pentominoes that can be used to tessellate the plane? Show the tessellations on graph paper.



Lesson 2. Learning a Skill

Can you find the product of two 2-digit numbers "in your head"?

Investigating the Ideas

Follow these steps for writing the *answer only* for 74×36.

Step 1	Step 2	Step 3
Think $4 \times 6 = 24$	*Think* $4 \times 3 = 12$; $7 \times 6 = 42$; 54; Add 2: 2; 56	*Think* $7 \times 3 = 21$; Add 5: 5; 26
Write 4, Remember 2	Write 6, Remember 5	Write 26
$\begin{array}{r} 36 \\ \times 74 \\ \hline 4 \end{array}$	$\begin{array}{r} 36 \\ \times 74 \\ \hline 64 \end{array}$	$\begin{array}{r} 36 \\ \times 74 \\ \hline 2664 \end{array}$

Can you use this method to write answers only for the products below? Check your answer using the "long" method.

$\begin{array}{r} 53 \\ \times 48 \\ \hline \end{array}$ $\begin{array}{r} 37 \\ \times 62 \\ \hline \end{array}$ $\begin{array}{r} 45 \\ \times 23 \\ \hline \end{array}$ $\begin{array}{r} 67 \\ \times 32 \\ \hline \end{array}$

Discussing the Ideas

1. Explain this statement: In Step 1 you are finding the number of ones.
2. In Step 2 you are finding the number of _?_.
3. The 2 you remembered is really 2 _?_.
4. Explain what you are finding in Step 3.

Using the Ideas

Write answers only for each product.

1. $\begin{array}{r} 28 \\ \times 42 \\ \hline \end{array}$ 2. $\begin{array}{r} 46 \\ \times 33 \\ \hline \end{array}$ 3. $\begin{array}{r} 37 \\ \times 42 \\ \hline \end{array}$ 4. $\begin{array}{r} 82 \\ \times 56 \\ \hline \end{array}$ 5. $\begin{array}{r} 53 \\ \times 34 \\ \hline \end{array}$

6. $\begin{array}{r} 64 \\ \times 27 \\ \hline \end{array}$ 7. $\begin{array}{r} 29 \\ \times 63 \\ \hline \end{array}$ 8. $\begin{array}{r} 48 \\ \times 35 \\ \hline \end{array}$ 9. $\begin{array}{r} 53 \\ \times 53 \\ \hline \end{array}$ 10. $\begin{array}{r} 27 \\ \times 64 \\ \hline \end{array}$

Extension

1. Study the figures below for finding the product of two 3-digit numbers.

$\begin{array}{r} 352 \\ \times 436 \\ \hline 2 \end{array}$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 72 \end{array}$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 472 \end{array}$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 3472 \end{array}$ $\begin{array}{r} 352 \\ \times 436 \\ \hline 153472 \end{array}$

2. Use the method shown in exercise 1 to find each product.

125	757	841	525
$\times 365$	$\times 426$	$\times 215$	$\times 525$

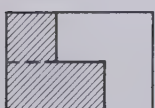
*3. Devise a rule for multiplying two 4-digit numbers.

Lesson 3. Learning a Generalization

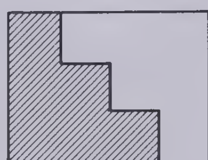
Can you find a pattern?

Investigating the Ideas

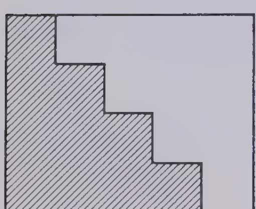
Use the small square in Figure A as the unit. Can you find the area of each shaded part in two different ways? For each part, write an equation to show that the two ways of calculating the area give the same result.



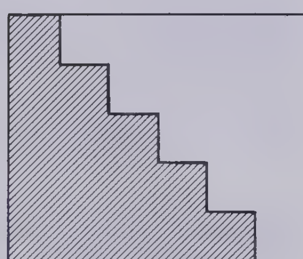
A



B



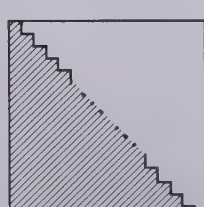
C



D

Discussing the Ideas

1. A Describe one way you found for finding area in the figures above.
   B Describe another way you found.
   C Did you find any other way?
2. Can you write an equation to show that these two methods give the same area?
3. A Suppose there are 50 vertical segments in the "stairsteps" of Figure E. What is the area of the shaded part?
   B Which of the two methods for finding the area would you use?
   C Can you write an equation about this?



4. Can you find the area of the shaded portion of Figure E if there are 100 vertical segments?
5. Can you use what you have learned so far to explain this generalization?

$$1 + 2 + 3 + 4 + 5 + \ldots + n = \frac{n\cdot(n+1)}{2}$$

Using the Ideas

1. Without adding each number, find the sum of the whole numbers through 25.
2. Find the sum of the first 75 whole numbers.
3. Find the sum of the first 200 whole numbers.
4. What is the sum of the first 1000 whole numbers?

Extension

1. What is this sum? $50 + 51 + 52 + 53 + \ldots + 99 + 100$
2. Can you find a short way to find the sum of
   A these even numbers? $0 + 2 + 4 + 6 + 8 + \ldots + 100$
   B these odd numbers? $1 + 3 + 5 + 7 + 9 + \ldots + 99$

*3. Can you state a rule for what you found in exercise 2 by using a variable?

Lesson 4. Learning Some Facts

Can you learn some "new" facts?

Investigating the Ideas

Many rapid "human Calculators" consider these products to be facts.

$\times$	10	11	12	13	14	15
10						
11						
12						
13						
14						
15						

How many of these "facts" can you give without calculating?

(Record the facts you know and shade that portion of the table with a red pencil. Then fill in the remainder of the table by figuring out the remaining facts.)

Discussing the Ideas

1. Which facts in the table need not be memorized provided you know the others and also know the commutative principle? Shade these facts blue.
2. A How many facts altogether are in the table?
   B How many facts remain to be memorized?
3. A What is the "largest" fact?
   B Which facts are over 200?
   C Which facts are in the 190's?
   D Do you notice other patterns in the table that might help you remember certain facts?

Using the Ideas

1. Give these products as quickly as possible.

A 15×15	**E** 13×13	**I** 11×13
B 15×14	**F** 14×12	**J** 11×14
C 14×14	**G** 11×11	**K** 11×15
D 13×15	**H** 11×12	**L** 12×13

2. Make flash cards for the "facts" in exercise 1 that you do not know. Practice with a friend.
3. In exercise 1, start with part L and, following reverse order, give each of the products as quickly as possible.

*4. Make a large multiplication table with all numbers up to 20. Mark out the "facts" you know. How many of these "facts" are left to memorize?

*5. A person who knew the distributive principle and the facts in the table referred to in exercise 4 looked at the multiplication 143×15 and wrote 2145. How did he do it so quickly?

Extension

Study the facts for these powers of 2.

$$
\begin{aligned}
2^2 &= 2 \times 2 &&= 4\\
2^3 &= 2 \times 2 \times 2 &&= 8\\
2^4 &= 2 \times 2 \times 2 \times 2 &&= 16\\
2^5 &= 2 \times 2 \times 2 \times 2 \times 2 &&= 32
\end{aligned}
$$

1. Give the next six powers of 2.

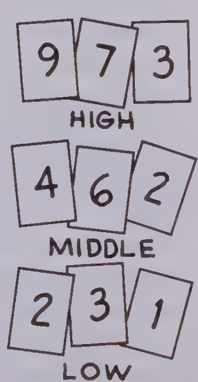
*2. Can you find some mnemonic aids to help you memorize the first ten powers of 2?

Lesson 5. Learning an Attitude

Let's try a place-value game.

Investigating the Ideas

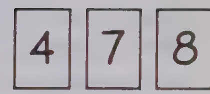
Use 3 sets of 9 cards, each with the digits 1 through 9. Shuffle the 27 cards and deal 3 to each player. Each player then forms a 3-digit numeral, places his cards face down in order, and declares (starting to dealer's left and rotating clockwise) whether his number is high, middle, or low. Play the game in groups of three players.



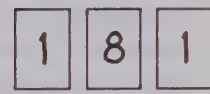
Discussing the Ideas

1. One player arranged his cards like this and declared that he would try for the low hand. What was wrong with his strategy?

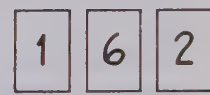
2. What is wrong with this arrangement for a middle hand?



3. If you were dealt these cards, would you try for a high or low? Why?



4. Suppose you are last to declare. Everyone else has declared either low or middle. What would you do with these cards?



Using the Ideas

1. Try playing this game with 2 or more other people.
2. Try the game with the rule that you can declare only high or low.
3. Make up rules for a game in which you turn up the cards one at a time starting with the ones' digit card.

Extension

1. Invent a place-value game in which 4 or 5 cards are dealt to each player.

*2. Find or invent another game or activity that strengthens understanding of the concept of place value.

V. Some Thoughts About Evaluation

The strategy of preparation, investigation, discussion, utilization, and extension is a flexible organizational plan that allows each teacher an opportunity to make a modest beginning toward an activity-oriented mathematics program. The lesson categorization of concept, skill, generalization, fact, and attitude provides a framework that allows each teacher an opportunity to apply the teaching strategy to various types of learning situations. Since there are different types of learning, it is reasonable to assume that there should be different types of evaluation used to measure these learnings.

When considering the facts and skills, for example, emphasis should be placed on child accountability. The teacher should determine the learning outcomes, consider performance objectives for these outcomes, and help the child attain these objectives. The evaluation of this attainment is most easily completed by use of fact and skill tests which determine the child's level of achievement. Since the child needs considerable practice in remembering facts and performing skills, the procedure for helping them is reasonably straightforward.

When evaluating concepts, generalizations, and attitudes, however, the desired performance objectives are often quite difficult to verbalize. We have mentioned earlier that concept learning often takes place

over a relatively long time span, that concepts are extended and broadened, and that concepts mature with each subsequent set of related experiences. Clearly, it is difficult to write a performance objective which specifies the exact level of concept maturity appropriate for a given child at a given time. Whenever possible, objectives for simple concepts should be written, and an attempt should be made to write test items which will show whether children understand these concepts. These items should involve requests for children to give examples of concepts, characteristics of a concept, and even, in some cases, a definition of the concept. For more difficult concepts, the evaluation of children's progress might be made through observation and recorded by means of a check-list which specifies certain levels of development for the given concept. The teacher should be alert for situations in which the child actually uses the concept correctly and should recognize also that understandings which are only partially developed indicate positive achievement. The teacher should also search for instances where the child has shown an ability to form concepts, for this is one of the desired learnings.

When evaluating the child's understanding of generalizations, the teacher should specify the simple generalizations which should be learned by all children. Specific performance objectives and the subsequent test items should be written to evaluate these generalizations. Beyond this, the teacher should again evaluate in greater depth through personal observations or interviews with the children. In the area of generalizations, the teacher should be ever aware that a child who is in the habit of looking for patterns or generalizations has learned a great deal. The teacher should also recognize that a child who can form a generalization from a sequence of specific examples has developed an understanding of a process that is extremely important. We would be remiss if we evaluated only the factual part of the learning of generalizations. As noted earlier, however, although these are important goals of mathematics learning, it is very difficult to write performance objectives for these goals. Whenever possible, objectives should be written which go deeper than facts and skills, but in the absence of objectives, the teacher should feel free to use other means of evaluation, including interviews to evaluate student learning.

While attitudes are not easy to measure in a conventional way, it is suggested that teachers frequently observe children and talk to them about their feelings about mathematics. It is important to realize that one's philosophy toward testing can also have a marked influence on the child's attitude toward mathematics. Testing should be reasonable and realistic, and the child should understand its purpose. The spirit of evaluation should be one of helpful assessment, rather than of critical evaluation. If children participate with teachers in understanding (if not in developing) the goals of instruction, the testing procedure can be a positive influence on the child's attitude and ability to improve.

It is hoped that the teacher will constantly take a broad view toward evaluating mathematics learning among his children. In the long run, evaluation of a child's learning should depend upon the interaction of that child and his teacher. For this interaction to be successful, it may be necessary for the teacher to reexamine his own beliefs about how children learn mathematics. As each teacher makes modest beginnings toward an activity-oriented approach to mathematics learning, he might ask himself the following questions:

1. Do I respect each child as an individual with unique interests, abilities, sensitivities, and significant thoughts?
2. Does the learning environment of my classroom provide a natural, free atmosphere in which children can explore, make decisions, be independent, and encounter exciting new experiences?
3. Does the learning experience also include a supportive, non-judgmental atmosphere in which children have enough routine activities to provide a comfortable threshold of security?
4. Is the child's need for earned success recognized in my classroom?
5. Do I recognize and treat mathematics as a dynamic, ever-growing discipline which offers limitless new vistas to be explored and an inexhaustible variety of new problems to be investigated and solved?
6. Do I view mathematics as a subject of beauty and a source of pleasurable fulfillment of intellectual curiosity?
7. Do I appreciate the significance of my role as a fellow-learner rather than merely a source of information?
8. Is my overall attitude toward mathematics one that encourages a basic freedom to learn through use of manipulative materials in an investigative environment, and through free discussion and exchange of ideas?

As a teacher evaluates the children in his class, he should also reevaluate his approach to mathematical learning. The goal of this short text has been to help in that reevaluation by encouraging the teacher to read, study, observe, experience, experiment, and reconsider. If that goal has been achieved, perhaps his resulting basic beliefs about children, mathematics, and evaluation methods will help him create a new climate of interaction that will spark more effective learning experiences in his classroom.

EXERCISE SET 10

1. Give a set of performance objectives for each lesson completed in Section IV.
2. Create an evaluation tool for each set of behavioral objectives given in exercise 1.

General Bibliography

Biggs, Edith, and James MacLean, *Freedom to Learn: An Active Learning Approach to Mathematics* (Don Mills, Ontario: Addison-Wesley, 1969).

Boyer, Carl B., *A History of Mathematics* (New York: John Wiley & Sons, Inc., 1968).

Copeland, Richard, *How Children Learn Mathematics: Teaching Implications of Piaget's Research* (New York: The Macmillan Co., 1970).

Dienes, Z. P., *Building Up Mathematics* (London: Hutchinson Educational Ltd., 1960).

Elliott, H. A., James MacLean, and Janet Jorden, *Geometry in the Classroom: New Concepts and Methods* (Toronto, Ontario: Holt, Rinehart and Winston of Canada, Ltd., 1968).

Forbes, Jack, and Robert Eicholz, *Mathematics for Elementary Teachers* (Menlo Park, Cal.: Addison-Wesley, 1971).

Members of the Association of Teachers of Mathematics, *Notes on Mathematics in Primary Schools* (New York: Cambridge University Press, 1967).

National Council of Teachers of Mathematics, *Insights into Modern Mathematics* (23rd Yearbook, 1957); *The Growth of Mathematical Ideas*, Grades K–12 (24th Yearbook, 1959); *Enrichment Mathematics for the Grades* (27th Yearbook, 1963); *Topics in Mathematics for Elementary School Teachers* (29th Yearbook, 1964); *More Topics in Mathematics for Elementary School Teachers* (30th Yearbook, 1969), Washington, D.C.: National Council of Teachers of Mathematics).

Newman, J. R., *The World of Mathematics* (New York: Simon and Schuster, 1956).

Nuffield Mathematics Project, *I Do, and I understand* (New York: John Wiley & Sons, Inc., 1967).

School Mathematics Study Group, *Studies in Mathematics, Vol. IX, A Brief Course in Mathematics for Elementary School Teachers,* Revised Edition (Stanford University, 1963).

The Schools Council, *Mathematics in Primary Schools* (Curriculum Bulletin No. 1. Available from Selective Educational Equipment, Newton, Mass., 1964).

Williams, E. M., and Hilary Shuard, *Elementary Mathematics Today, Grades 1–8* (Menlo Park, Cal.: Addison-Wesley, 1972).

Bibliography of Resources for Active Learning

Abbott, Janet, et al., Franklin Mathematics Series: *From Fingers to Computers; Learning About Measurement; Learn to Fold—Fold to Learn; Making and Using Graphs and Nomographs; Mirror Magic; Patterns and Puzzles in Mathematics; Pencil and Paper Geometry* (Chicago: Lyons and Carnahan, 1970).

Bates, John, Donald Irwin, and Garry Hamilton, *Developmental Math Cards* (Don Mills, Ontario: Addison-Wesley, 1970).

Clarkson, Dave, *Math Activity Cards* (New York: Macmillan Co., 1969).

Cohen, Donald, *Inquiry in Mathematics via the Geo-Board* (New York: Walker, 1967).

Cohen, Donald, *Maths Mini Lab* (Newton, Mass.: Selective Educational Equipment, 1971).

Davis, Robert (Madison Project), *Discovery in Mathematics: A Text for Teachers; Student Discussion Guide* (Menlo Park, Cal.: Addison-Wesley, 1964).

Dienes, Z. P., *Multibase Arithmetic Blocks,* Tasks and Manual (New York: Herder and Herder, 1961).

Elementary Science Study: *Attribute Games and Problems; Mirror Cards; Tangrams* (St. Louis: Webster Division, McGraw-Hill Book Co., 1968).

Fletcher, Harold, Arnold Howell, and Ruth Walker, *Mathematics in Modules* (Menlo Park, Cal.: Addison-Wesley, 1973).

Galton, Grace, Arlene Fair, and Patricia Davidson, *Chip Trading Activities,* Set I (Fort Collins, Colorado: Sigma, Scott Scientific, 1970).

Huff, M. Elizabeth, Donald Irwin, *Activities in Geometry* (Don Mills, Ontario: Addison-Wesley, 1973).

Knaupp, John, and Gary Bitter, *Mathematics Activities Kit* (Menlo Park, Cal.: Addison-Wesley, 1972).

Mathex: Matching and Graphing No. 1; Numeration No. 2; Operations No. 3; Geometry No. 4; Measurement and Estimation No. 5; Graphing and Probability No. 6; Numeration No. 7; Operations and Problem Solving No. 8; Geometry No. 9; Measurement No. 10 (Toronto, Ontario: Encyclopaedia Britannica Publications Ltd., 1970).

Nuffield Mathematics Project: *Environmental Geometry; Mathematics Begins* 1; *Computation and Structure* 2, 3, 4; *Beginnings* 1; *Shape and Size* 2, 3; *Pictorial Representation* 1; *Graphs Leading to Pictorial Representation* 1; *Graphs Leading to Algebra* 2 (New York: John Wiley, 1967–1969).

Turner, Ethel, *Teaching Aids for Elementary Mathematics* (New York; Holt, Rinehart and Winston, 1966).

Wirtz, Robert, Morton Botel, and Max Beberman, *Toward Improving Computation* (Washington, D.C.: Curriculum Development Associates, 1970).

Wirtz, Robert, et al., *Math Workshop:* Games and Enrichment Activities (Chicago: Encyclopaedia Britannica Educational Corp., 1964).

Wirtz, Robert, Morton Botel, and B. G. Nunley, *Discovery in Elementary School Mathematics* (Chicago: Encyclopaedia Britannica Educational Corp., 1963).

INTRODUCING THE METRIC SYSTEM

Canada is committed to the metric system of measurement. You may be aware of this but may not have a clear idea of exactly what the metric decision means to you as a *teacher*. It is hoped that this section will serve three purposes—

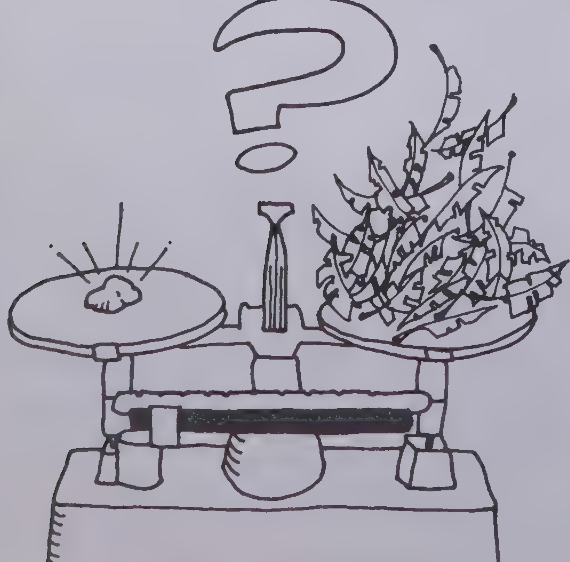
1. give you an idea of how the metric decision will affect you,
2. help you understand the metric system of measurement, and
3. give you some hints for teaching the metric system of measurement to your students.

History and Rationale

The English system of measurement developed from man's need to measure size and distances using units from the most readily available object—himself. He utilized his palm, span, finger, an ell, and a fathom for length; his foot, step, pace, an arrow's flight, and a day's journey for distance; and a handful, shellful, hornful, or gourdful for capacity.

There was little need for standardization until man began to travel and trade with other men. When "standard units" were developed, a new problem arose. Different countries used different definitions for the same unit. The foot was, at first, the length of any man's foot. In some countries, it was the length of the king's foot (since he was the "ruler") and this foot could change as the "rulers" changed. Later an effort was made to standardize some units; for example, England and Scotland decreed the foot to be 12 inches. Unfortunately, England and Scotland didn't use the same definition for the inch.

Today, in the age of technology, one still finds different units in those countries which are not yet metric. Canada and the United States are neighbouring countries, yet they use two different definitions for the gallon. A question at which people in metric countries must laugh is "Which is heavier, a pound of gold or a pound of feathers?" A pound of feathers is heavier since feathers are weighed by the avoirdupois pound (1 avoirdupois pound—7 000 grains) and gold is weighed by the troy pound (1 troy pound—5 760 grains). Which is heavier, an ounce of gold or an ounce of feathers? An ounce of gold is heavier. There are 12 ounces in the troy pound, so one ounce of gold weighs 480 grains; there are 16 ounces in the avoirdupois pound, so an ounce of feathers weighs 437.5 grains.



Out of such confusion there developed a need for a simple, standardized system of measurement. In 1670 Gabriel Mouton, a French abbé, developed a system of measurement organized according to the decimal system of numeration. It took over a hundred years for a system of measurement like the one Mouton put forth to get official sanction. In 1790 the French National Assembly appointed a committee to study the measurement situation and see if a rational system of measurement was possible. In 1795 France adopted a decimal system of measurement, defining the base unit of length to be the *metre* (from the Greek word *metron,* "a measure").

The metric system did not use parts of the human body as units. The metric system did not develop haphazardly adding more and more units as the need arose. The metre was defined as one ten-millionth of the distance from the North Pole to the equator, along the meridian passing near Dunkirk, Paris, and Barcelona. One can see that such a definition would be difficult to replicate in any one country. Also, the length of the metre changes as the position of the North Pole changes; at the time that the metre was defined, scientists were unaware that the position of the North Pole changed.

In 1870, because of the problem of replicating and comparing metric units from country to country, France called a meeting of the metric countries to develop a "unified metric system of measurement". In 1875, the *Treaty of the Metre* was signed to establish the General Conference on Weights and Measures which meets to determine the official definitions for the units used in the metric countries. In 1960 the Conference adopted the Système International des Unités (SI). It is this SI metric system that is most used throughout the world.

A Popular System

The popularity of the metric system stems from two characteristics—the high degree of standardization and its simplicity.

In the entire metric system there are only seven base units! They are **metre** (length), **kilogram** (mass), **second** (time), **ampere** (electric current), **degree kelvin** (thermodynamic temperature), **candela** (luminous intensity), and **mole** (amount of substance).

All units used in the metric system are related to these seven base units. The units you will be most concerned with (because they are the ones used in everyday living) appear in Table 1:

Table 1: Metric Units to be Studied

Quantity	**Unit**	**Symbol**
Length	metre	m
Mass	kilogram	kg
Capacity	litre	ℓ*
Temperature	degree Celsius	°C

*As a rule of thumb, the cursive letter (ℓ) is used as a symbol for the litre to avoid confusion with the numeral (1), however, in symbols such as ml (millilitre), kl (kilolitre) the cursive form is not used.

All other units to be discussed can be represented by the product of one of the units and a power of 10. For example, every possible unit of length can be developed by multiplying the number of metres by the appropriate power of 10.

Table 2: Metric Units of Length

Name (Symbol)	**Metres**
*kilometre (km)	10^3m or 1000 m
hectometre (hm)	10^2m or 100 m
decametre (dam)	10^1m or 10 m
*metre (m)	10^0m or 1 m
decimetre (dm)	10^{-1}m or $\frac{1}{10}$ m
*centimetre (cm)	10^{-2}m or $\frac{1}{100}$ m
*millimetre (mm)	10^{-3}m or $\frac{1}{1000}$ m

*preferred units

To make the system simpler the same prefixes are used with all units. For example, a millimetre (mm) is $\frac{1}{1000}$ of a metre, a millilitre (ml) is $\frac{1}{1000}$ of a litre, a milligram (mg) is $\frac{1}{1000}$ of a gram, etc.

According to the class, you may want to introduce the symbol "m" for metre, "cm" for centimetre, etc. The plurals, metres and centimetres, are also symbolized "m" and "cm", not "ms" or "cms." Remember, these are symbols and not abbreviations and no period is used after a symbol.

Countries which have been completely metric for several years find that some terms such as "decimetre" are not used in everyday living. People will talk of a book being 28 centimetres long rather than 2.8 decimetres long. You may wish to explain the term "decimetre," but it is not necessary.

Most people who feel that the metric system is complex are those who convert back and forth between the metric and English systems of measurement. When teaching the metric system, conversion to the English system is not necessary and should be avoided!

The metre is defined world-wide to be 1 650 763.73 wave lengths in a vacuum of the orange-red line of the spectrum of krypton 86. This is quite a definition! There are two reasons why such a complex definition was adopted—

1. the length never varies and

2. this measurement can be replicated in laboratories throughout the world.

From this brief history of the metric system it is hoped you will take three main thoughts—

1. The metric system resulted from concentrated effort to develop a rational system of measurement. It did not develop haphazardly.

2. The problem of standardization has been solved in the metric system.

3. The metric system is both popular and useful because of its simplicity.

Activities

Experience and activity are key words in the teaching of measurement. Measure things! The success of this material will depend upon the amount of experience each participant has with the activities. The limited number of activities that are presented should stimulate possibilities for many more. Although the content is approached through activities and measuring experiences, there is a need for exercises to further these experiences and to structure metric thinking. Two points should be emphasized—

1. It is *important* that *you* as well as your class do the activities in this section.

2. The activities will be more fun if done in a group situation.

Looking at Table 1 in the *History and Rationale* section, you will notice that you have to be concerned with only four base units. So, let's use the frontal attack, start right in on length, and begin inching our way down the metric road.

Length, Area, and Volume

In the groups where the metric system has been argued for years, there were two camps. One group wanted to use the centimetre, gram, and second for the core of the system and the other the metre, kilogram, and second. The latter group has prevailed.

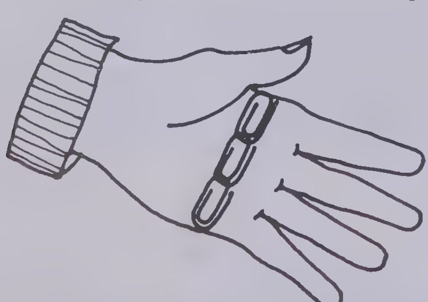
It is strongly urged that first grade teachers **not** start with the metre. It is very difficult for first graders to handle a metre ruler. The same argument may be advanced for the kilogram and litre. Length will be approached as it should be covered with students, i.e., first measure with arbitrary units, then use the centimetre, next use the 10-centimetre (decimetre), and finally the metre. All measurement should be approached as a three step process—

1. Select a unit.
2. Partition the object to be measured into units.
3. Count the number of units used. That number is the measure of the object.

ACTIVITY 1

Measuring objects with an arbitrary unit. Students should do several activities of this type using arbitrary units such as their thumb, a paper clip, pencil, crayon, cutout of their shoe, width of their hand (a unit in the English system used for measuring the height of horses), cubit (another "English" unit, the length of the forearm from the elbow to the tip of the middle finger), or other selected units. For your experience measure the chalk eraser, the width of your hand, the width of this book, and the length of a pencil using a paper clip as the unit.

In the illustration, a "paper clip train" is being used to measure the width of a hand. Follow the three steps mentioned previously in the measurement process.



Record all answers. Then measure the object again using pieces of paper the length of a thumbnail. Repeat the process measuring other objects.

In class emphasize four points—

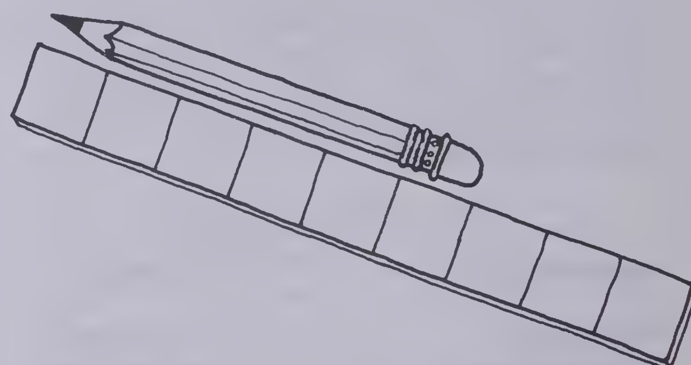
1. The first unit should be lined up with the "starting point" of the object.
2. The units should touch, but not overlap.
3. The "train" should be straight.
4. The units should be "rounded off" to the unit that has its right end nearest to the "finishing point" of the object.

In doing activities where arbitrary units are used, the need for standardized units becomes obvious. Ask several children to measure the same object, each with his own pencil. On the chalkboard, place their statements such as "The table (or whatever object you pick) is 5 pencils wide." "The table is 7 pencils wide." "The table is 8 pencils wide." Children will soon see that when pencils of differing lengths are used, different answers will result.

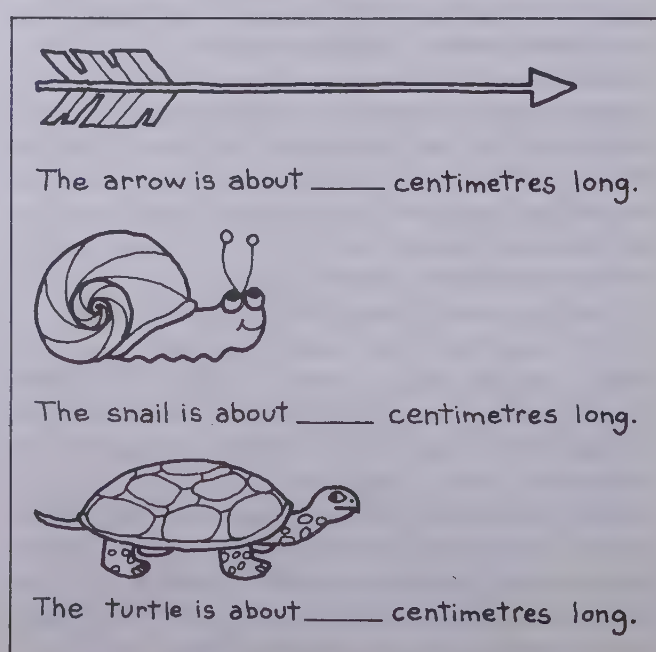
ACTIVITY 2

Developing the concept of a centimetre. Probably the first metric unit the children will make use of is the centimetre. You will need (and each student in the class will need) 9 centimetre strips—9 pieces of paper or cardboard 1 cm by 1 cm square.

The children, especially the younger ones, should have the experience of measuring many objects using centimetre strips. (If at the time you present this activity your students have studied two-digit numbers, have them measure objects longer than 9 cm.)



Using the centimetre strips, measure the length of a paper clip, a piece of chalk, the Cuisenaire 6-rod, the width of a hand, and the width of a thumb to the nearest centimetre. In this initial activity, actually use centimetre strips and not a ruler marked in centimetres. An exercise the children can do at their desks is to measure the pictures of objects drawn on a duplicator master. The pictures can be of predetermined length. Measure the pictures below.



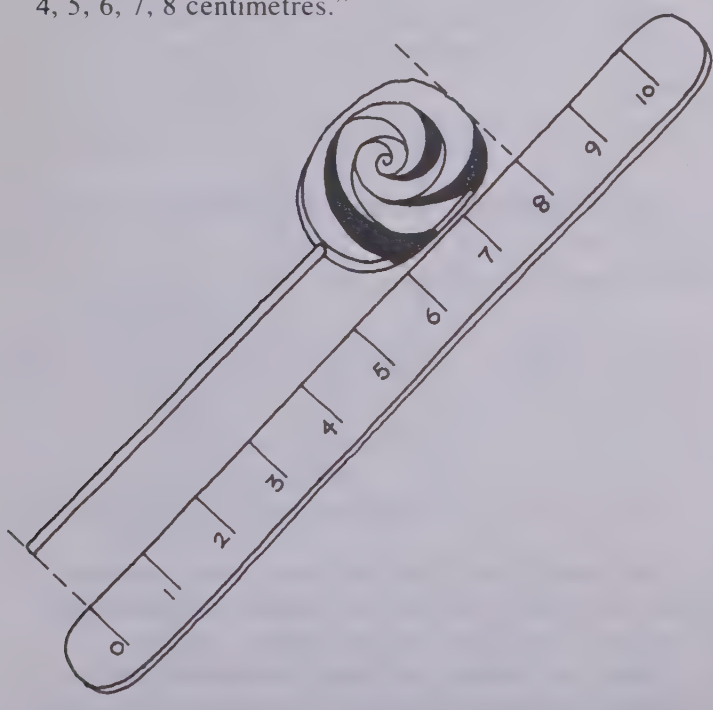
In exercises like these, the length can be controlled. Some answers should require "rounding up," and some "rounding down." The word "about" is important in the sentence since a measurement is an approximation. As the children progress you can have them write not only the number but also the name of the unit.

ACTIVITY 3

Measuring with centimetre rulers. When the children have learned to use the centimetre strips in the measurement process, a ruler marked off in centimetres (not millimetres) should be introduced. It is strongly urged that the child construct his own 10-cm ruler during his first introduction to metric measure. He can do this by constructing a 10-cm train on a 10-cm long piece of paper, pasting the train on the paper, then numbering the cars from 1 to 10. Another approach is to construct a 10-cm ruler in front of the class. Then hand out 10-cm long pieces of paper already marked off in centimetres and have the children number the centimetres from 1 to 10.

The next few activities should involve the measuring of an object with a centimetre train, a 10-cm ruler, and finally with only a 10-cm ruler. When measuring an object with a 10-cm ruler work toward getting your students to "read the ruler" rather than counting the centimetres as they did with the trains.

In the example illustrated the child should learn to round off to the nearest centimetre and then read the ruler, "8 centimetres," instead of counting "1, 2, 3, 4, 5, 6, 7, 8 centimetres."

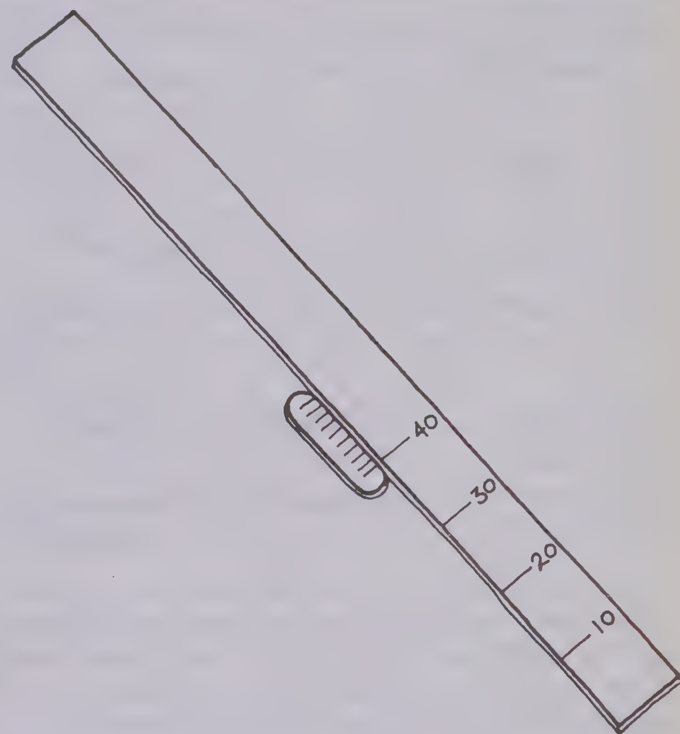


After the children have become skilled in using a 10-cm ruler, they should be given activities requiring them to measure objects which are longer than 10 cm. When working with 5-and-6-year olds, be careful that the measure of the object is not a number the children haven't studied. In the activities concerning measurement it is the process that should be emphasized; the numbers themselves should never be a source of difficulty.

Now, using your 10-cm ruler, measure the length and width of this book and length of your forearm, the length of your foot, and length of your span (what is your span?).

ACTIVITY 4

The metre and notation. Initially, you may want to have your students measure objects with metre-long strips of unmarked cardboard. Then ask them to number the centimetres on the metre strip in groups of 10 using their 10-cm strips. Before proceeding further, have the class subdivide these cardboard metre rulers into centimetres. It is important that you do the activities with the same type of ruler your students will use. If you have a classroom set of wooden metre rulers, use one of them. Ideally, the rulers used should be marked off in centimetres, but if the ruler is marked off in centimetres (cm) and millimetres (mm) no harm is done. Measure the length, width, and height of your desk rounding off to the nearest metre.



The measurements for a desk, accurate to the nearest metre, might be 2 m long, 1 m wide, and 1 m high. Such measurements would not be helpful. The metre is used for much longer measurements, such as the length and width of the classroom, the playground, the school, the block, etc. To measure the dimensions of objects such as desks, tables, bookshelves, and people, a metre ruler may be used and the results recorded in centimetres. For example, a desk may be 152 cm long, 76 cm wide, and 74 cm high.

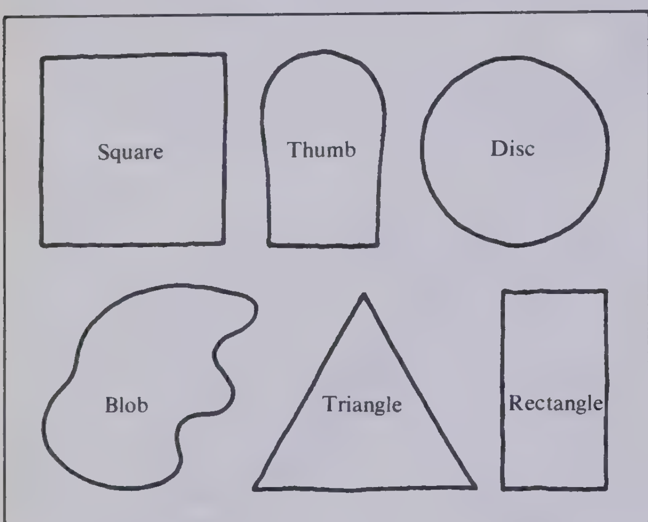
You might say: I am 178 cm tall; what is your height (in centimetres)?

Just as 153 cents is written as \$1.53, 153 centimetres is written as 1.53 metres. This can be interpreted as 1 metre and 53 centimetres which is read as "one point five three" metres. Do not dwell on the mathematical use of the notation – it is not necessary!

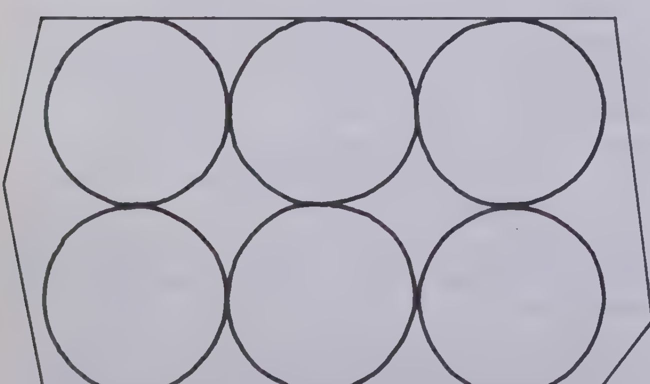
With your class, record the dimensions of your classroom, your desk, their desks, your height, and their heights in terms of centimetres, then in terms of metres using the decimal notation.

ACTIVITY 5

Area using arbitrary units. Here are some examples of area units:



Let the children give names to the units. Then follow the measurement process: select one of these units, match it against the area of some object, and count the number of units used. For example, the irregular figure below has an area of about 6 discs



(if disc is the name given to the unit used). Emphasize that you are trying to "cover" the object. The units should be "even with the edge" of the object, the units should touch, but not overlap, each other. Direct the children's attention to the parts of the object that are not "covered."

Make a cutout of some irregular area such as your thumb and make copies of it out of paper. Use your "thumb" to find the area of the top of a chalk eraser, of the irregular figure measured with the discs, of a cutout of your shoe, and of figure X.

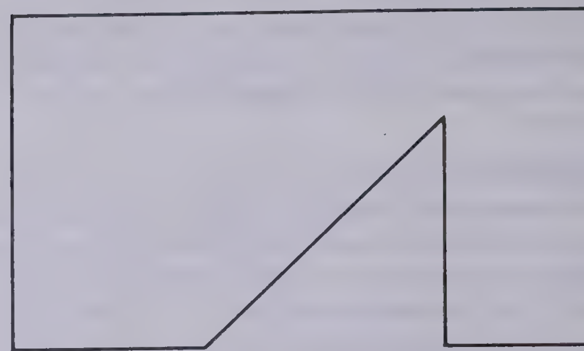


Figure X

Record the answers on the chalkboard in sentence form –

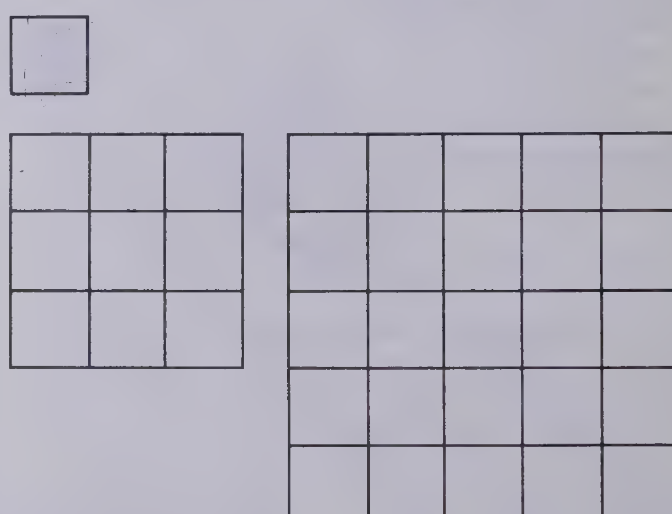
"The figure has an area of about ______ thumbs."

Have your class perform similar activities.

ACTIVITY 6

Area using the centimetre square (cm^2). Have the children make centimetre squares (or have them available for use). The children should have the experience of finding the area of many objects.

Make duplicator masters for some areas that the class can measure with their centimetre squares. The figures below are $1\,cm^2$, $9\,cm^2$, $25\,cm^2$, respectively.



You might point out that the square containing the $9\,cm^2$ has a side of 3 cm and the square containing the $25\,cm^2$ has a side of 5 cm.

Have the children use their centimetre squares to find the *area* of a stamp, a 10-cm ruler, the cutout of their thumb, the irregular figure which had an area of 6 discs, and figure X.

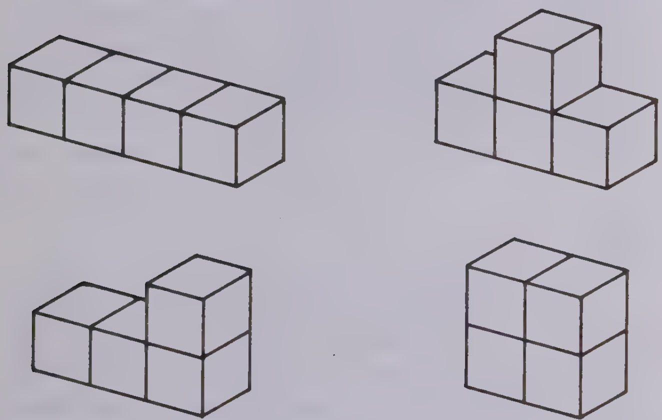
ACTIVITY 7

Volume, using the centimetre cube. In the initial development of the concept of volume, it is important

that children have the opportunity to construct several differently shaped objects each having the same number of volume units.

As with length and area, the study of volume should be introduced with activities making use of arbitrary units of volume, such as blocks, Cuisenaire rods, pencils, erasers, or even marbles.

Use 10 or 12 centimetre cubes in this activity. At first, let the children work on their own, constructing any objects they like. Encourage them to see that an object built of a specific number of cubes has a volume of the same number of cubes regardless of its shape. For example, the illustration shows 4 different constructions, each having a volume of 4 centimetre cubes (4 cm^3).



How many differently shaped objects can be constructed with a volume of 8 centimetre cubes? When those possibilities have been exhausted, try the activity with 10 cubes.

REVIEW: LENGTH, AREA, AND VOLUME

1. Have your class compare the length of their feet, spans, and cubits. Why are these units useless as standard units?
2. Complete these statements.
   a. 128 cm = ____m **e.** 1.06 m = ____cm
   b. 108 cm = ____m **f.** 10.01 m = ____cm
   c. 15 cm = ____m **g.** 23.86 m = ____cm
   d. 1010 cm = ____m **h.** 0.09 m = ____cm
3. What would be the length of the sides in a square containing:
   a. 36 cm^2 → ____ cm?
   b. 25 cm^2 → ____ cm?
   c. 4 cm^2 → ____ cm?
   d. 16 cm^2 → ____ cm?
4. How many different-shaped objects can you form with 6 centimetre cubes?

Capacity

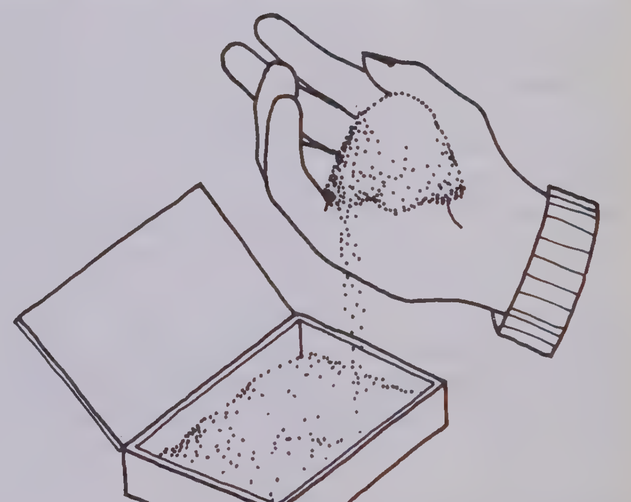
Capacity can be thought of as the amount of material a container will hold. Capacity is usually linked to liquid measure though you may have already had your classes measure capacity by using sand to avoid using liquids.

In the metric system of measurement, volume and capacity are directly related. A container with a volume of 1 cubic centimetre (1 cm^3) will hold 1 millilitre of water. One millilitre (1 ml) is one thousandth of a litre (0.001 ℓ).

The need for fractional names such as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$ etc. will diminish. The parts of the whole which need emphasis are—0.1, 0.2, 0.3, . . . , 0.9. Of course, in measurement, fractions could disappear completely, since $\frac{3}{8}$ of a meter is 0.375 m or 375 mm. However, when working with the litre (the unit of capacity in the metric system) don't worry now about using $\frac{3}{4}\ \ell$, $\frac{2}{3}\ \ell$, etc. if it is the amount you want the children to see or work with. Since the metric system is based on 10 and since 1, 2, 5 and 10 are the only divisors of 10, we will probably talk about halves, fifths, and tenths of metric units. The decimal notation ($\frac{1}{2}$ is 0.5) will prevail eventually, even at the primary level.

ACTIVITY 8

Capacity and arbitrary units. The most obvious capacity units are handfuls. Give each child a container to fill with water or sand or other material you prefer to use. Have the children fill the container



(milk carton, ice cream carton, cigar box, etc.) with "handfuls" of material. Have them record their results on a piece of paper: "My carton holds ______ handfuls of ______." Compare the wide range of results. Re-emphasize the need for a standard unit to measure capacity. If further experience is necessary, you may want to repeat the project with cups brought from home (since there are so many different sized and shaped cups). Try the activity yourself or get several containers such as an ice cream carton, a milk carton, a wastebasket, a big cooking pan, and a litre container.

On a piece of paper write a pair of sentences for each container:

"The (name of container) holds about (guess) litres.

The (name of the container) actually holds (result) litres.

In the first blank "guestimate" the number of litres the container will hold. In the second, write in the results of measuring the object.

Don't forget the three step measuring process—

1. Select the unit—the litre.
2. Match the unit against the object—fill the object using the litre.
3. Count the number of units (litres) used.

When the container is full (it is best to have a "fill line" just below the top of the container) round off to the nearest whole litre according to whether more or less than half of the last litre was used.

ACTIVITY 9

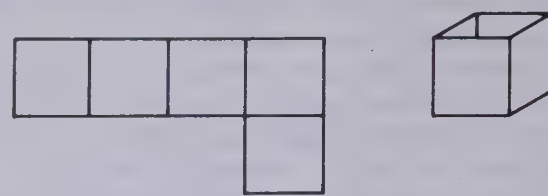
Working with the litre. Get a container that holds a litre of water(and, ideally, has submarkings for each 100 ml). When you are collecting containers for your classroom, try to get as many different shapes as you can. It is important, especially in early experiences, that the children see that litre containers can come in many different shapes. It is the quantity the container will hold, not its shape that determines a capacity of 1 litre.

Once you get a litre container you can make many more. Pour a litre of water into a container and mark a "fill line" for 1 litre on the outside with tape, or, if possible, cut the container so that it holds just 1 litre. Suggested existing containers which can be cut are quart, half-gallon, and gallon milk cartons, round quart, half-gallon, and gallon ice cream cartons. Containers that can be marked might be various shaped pans, cooking bowls, large tin cans, and bottles or jugs. Most activities for introducing the metric units should be accompanied by some estimation exercises. Have the students estimate and record how many litres a container will hold, then measure the container to see about how many litres it does hold. Compare records.

ACTIVITY 10

Introducing the millilitre. The litre is a unit for capacity that is used for milk, gasoline, paint, and other quantities of considerable size. The litre is not used to measure small quantities, such as toothpaste, soda pop, medicines, frozen orange juice, etc. The unit used for the smaller measures is the millilitre (ml). If your school is going to get a set of metric capacity containers, try to get them in these sizes—1 ℓ, 500 ml, 200 ml, 100 ml, 50 ml, 20 ml, and 10 ml. With such a set (whether bought, given, or constructed) one can do all the activities that are necessary.

Construct a container with a volume of 1 cubic centimetre (1 cm^3) to demonstrate the size of the millilitre (ml). Trace the figure below, then cut it out and tape it together along the edges. If you avoid spillage your cube will hold 1 ml of water.



The children need several activities measuring the capacity of objects and recording the results in millilitres. Have them first guess and then measure the capacity of a thimble, a match box, a tablespoon, and a teaspoon. Record the results in sentences like—

"I estimate that the thimble holds about ______ ml.
It actually holds about _____ ml."

Mass

As the metric system becomes the predominant system of measurement you may hear talk about the difference between mass and weight. A lunar example may be the best way to show the difference. Now that we are in the space age, practically everyone knows that a man weighs less on the moon than he does on the earth. For example, a 300-kg man on earth would weigh about 50 kg on the moon, but he would have the same mass on the moon as he does on earth. Weight is dependent upon gravity, mass is not. Begin to stress the use of the correct metric term, mass.

The base unit of mass in the metric system is the kilogram (kg). For example, we say "I have a mass of 78 kg.

ACTIVITY 11

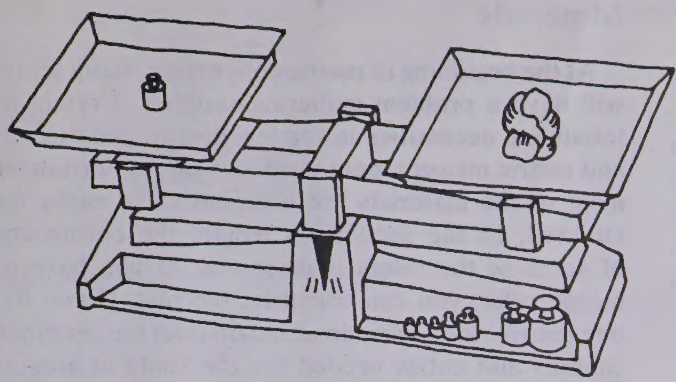
Arbitrary units of mass. To find the mass of an object you will need a balance and some arbitrary units such as paper clips, pencils, Cuisenaire rods, pennies, or other objects. Put a pencil on one side of the beam and then "balance the pencil" with pennies (or multiples of any other small unit). Record the results on paper in a sentence like:

"The pencil has a mass of about _____ pennies."

Repeat the activity with at least three other objects.

ACTIVITY 12

The unit used for small masses is the gram (g). This activity is very similar to the last. You will need gram masses. If you have a classroom set, that's great! If you don't, you can make one.



Put a gram mass on one side of the balance and balance it with a lump of clay or plasticine. Label your clay "1 g." In a similar manner make a set of clay or plasticine "masses" in multiples such as: 5 g, 10 g, 20 g, and 50 g. Use several small objects as test objects (a paper clip, a nickel, a penny, and a pencil). However, before you have the children put one of the test objects on the balance, ask them to estimate its mass in grams. Then find the mass of the object. Record both the guess and the result.

The quarter has a mass of about (guess) grams.
It actually has a mass of (result) grams.

Repeat the activity using other objects. Do you and the class get better at estimating mass?

ACTIVITY 13

Measuring mass using the kilogram. Hopefully, all schools will have metric scales available for finding the mass of children and other large objects using kilograms. For this activity, have each child find his own mass and then make and label a cutout of himself (perhaps using his projected shadow). Have him record his height and mass in metric units on the cutout.

Then you and your class might measure the mass of other objects, such as your own chairs, the textbooks used in the course of one day, litre of water (don't count the container—first find its mass when empty), a dictionary, and even the principal of the school (if he agrees). As mentioned earlier, there is a direct relationship between volume and capacity in the metric system of measurement. In fact, there is a direct relationship between volume, capacity, and mass. A container whose volume is 1 cubic cm (cm^3) holds 1 ml of water and the 1 ml of water has a mass of 1 g. A container whose volume is 1000 cubic cm (or 1 cubic decimetre) holds 1000 ml of water (or 1 litre), and the water has a mass of 1000 g (or 1 kilogram). What did you get for the mass of one litre of water?

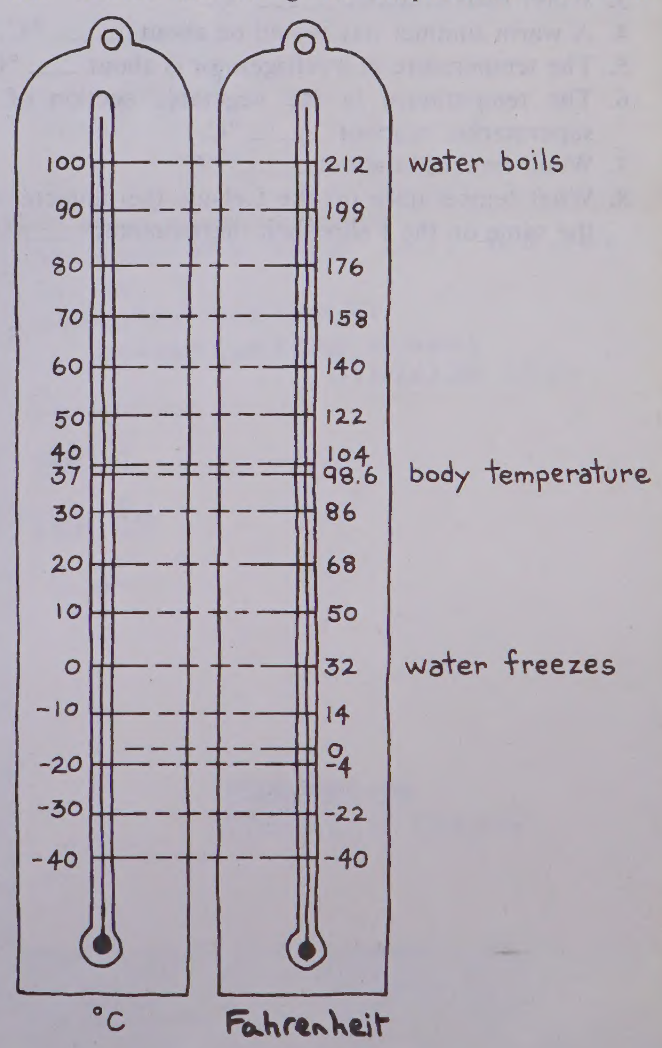
N.B. It is a good idea to label some of the objects in the room as you introduce each metric unit. For example, the aquarium may be 70 cm long, 40 cm wide, 35 cm high; have a water surface area of 2 800 cm^2, volume of 98 000 cm^3; a capacity of 98 ℓ of water and a mass of 12 kg. If the children label the objects as they study particular units, they will begin to think metric.

REVIEW: CAPACITY AND MASS

1. When finding the mass of something using a balance beam, how do you decide which unit to round off to?
2. Fill in the answers:
   a. 28 ml of water has a mass of about ______ grams.
   b. 170 ℓ is ______ ml.
   c. 3.12 kg is ______ g and 438 g or ______ kg.
   d. It would take ______ ml of water to balance 1 kg.
3. a. Will a car get a higher or a lower number of miles per litre than miles per gallon? (Is the litre larger or smaller than the gallon?)
   b. Will a car get a higher or a lower number of kilometres per gallon than miles per gallon? (Is the kilometre longer or shorter than the mile?)
   ★ c. Gasoline consumption rates will be given in kilometres per litre. Will a car get a higher or a lower number of kilometres per litre than miles per gallon?

Temperature

This last section covers the introduction of a metric unit, the degree Celsius (°C), for which there is no physical model. On the Celsius scale for temperature, water boils at 100° C and freezes at 0° C. The unit is named after the Swedish scientist, Anders Celsius, who created the centigrade temperature scale. The



Celsius and centigrade scales are the same, but centigrade is no longer the proper term since the centigrade is a unit used to measure angles in the metric system.

The best way to get used to the Celsius temperature scale is to use it! It is almost a necessity that you have a Celsius thermometer. However, if you have a demonstration model of the Fahrenheit thermometer, you can rescale it using the nomograph shown here.

ACTIVITY 14

Graphing temperatures. Be sure to give the children lots of opportunities to read the temperature and record it in degrees Celsius (° C). Perhaps you could institute a morning weather report given by a different child each day to get the class to use Celsius thermometers and to give them a feeling for what the temperature is when expressed in degrees Celsius (° C). The previous day's high and low temperatures (taken from a newspaper account) could be recorded on a wall graph.

REVIEW: TEMPERATURE

1. My body temperature is about ______ °C.
2. Normal room temperature is about ______ °C.
3. Water boils at about ______ °C.
4. A warm summer day would be about ______ °C.
5. The temperature in a refrigerator is about ____ °C.
6. The temperature in the vegetable section of a supermarket is about ______ °C.
7. Water freezes at about ______ °C.
8. What temperature on the Celsius thermometer is the same on the Fahrenheit thermometer? ____ °C

Materials

At the beginning of metric conversion, many schools will have a problem gathering supplies. Certain materials are necessities in the teaching of measurement and metric measurement is no exception. Fortunately, most of the materials are inexpensive or easily constructed. In the section on length, the construction of some of the rulers is discussed. If you have one metric ruler, you can construct the rest. If you have one metric ruler, you can also construct the centimetre squares and cubes needed for the study of area and volume.

The construction of units of capacity and mass have also been discussed. When it comes to temperature you should have a thermometer available for classroom use. If it is a Fahrenheit thermometer, then you should rescale it to degree Celsius using the nomograph given earlier.

Following is a list of companies and government agencies that are currently producing materials or can give some assistance with this problem of teaching the metric system of measurement.

Addison-Wesley (Canada) Ltd. – Don Mills, Ontario
Buntin Gillies & Co. Ltd. – Ottawa, Ontario
Cameron Products – Bramalea, Ontario
Canadian Metric Association – (P.O. Box 35) – Fonthill, Ontario
Contrasts 20 – Calgary, Edmonton, Vancouver, Winnipeg, Regina (Nearest Barber-Ellis Office)
Kruger Pulp and Paper Ltd. – Moncton, Toronto, Hull, Montreal (Nearest Office)
Information Canada (Under Government of Canada) (Nearest Office)
Jack Hood School Supplies Co. Ltd. – Stratford, Ontario
Lufkin Rule Co. of Canada Ltd. – Don Mills, Ontario
Lily Cups Ltd. – Scarborough, Ontario
MacLean-Hunter Learning Materials Co. – Toronto 101, Ontario
Metric-Aids Ltd. – Toronto, Ontario
Moyer-Vico Ltd. – Moncton Weston, Winnipeg, Saskatoon, Edmonton, Vancouver and the Longueuil Co. in Chambly (Nearest Office)
The National Council of Teachers of Mathematics – 1906 Association Drive, Reston, Virginia 22091
Sargent-Welch Scientific Co. of Canada Ltd. – Weston, Ontario
Spectrum Education Ltd. – Toronto, Ontario
Spicars International Ltd. – Scarborough, Ontario
Toronto Dominion Bank (Nearest Office)

Book Three

ons and sets: 23
numbers, and nu
ting sequences: 3
aring sets: 58
t's My Rule": 68
ion Machine: 70,
plication and sets
counting sequenc
plication patterns
uct sets: 162
on and sets: 170,
and intersection
and even numbers
nal logic: 16, 24, 6
, 189, 296–297

ping by tens to 4-digit
merals: 30–45, 48
alities: 42–43, 97, 120–121, 282
value in computation: 92, 98, 106,
, 242, 248, 294, 300
oning: 251
numbers: 222–223
tive numbers: 272–273

tion and subtraction
ated: 50–57, 66–71
parison subtraction: 58–59
principles: 60–65
rences and missing addends: 66–67
ng and subtracting 2- and 3-digit
mbers: 90–121

cepts of multiplication: 122, 162–163
principles: 138–145, 236–239
iplication facts: 146–155
ng and multiplication: 162–163
cepts of division: 170–181, 186–189,
6–281, 284–291
iples and factors: 218–221
iplying with multiples of 10: 226–231
iplication algorithm: 240–249
sion algorithm: 294–303
ainders in division: 300–303

es, fourths, eighths: 14–15, 18–23

cience: 59, 110, 232–233, 243, 247, 249
cial Studies: 47, 102–103, 111, 159,
61, 165, 191, 291, 308
surement: 27, 46, 108, 115, 130–131,
34, 235, 245, 283, 293, 299
ure Stories: 134, 156, 189, 190, 296–297
rt Stories: 23, 67, 117, 134, 157, 173,
93, 241, 253, 290, 298, 307
cellaneous: 119, 164, 177, 305

surement (not exact): 12–15
mating area: 18
mating weights: 108–109
mating products: 250–251

and even numbers: 214–217
iples and factors: 218–221
e numbers: 222–223

ncept of measurement: 4–29
ntimetres: 8–15, 20–21
a (square centimetres):
6–19, 20–21
ume: 24–25
es: 26–27
e: 245

iple geometric figures: 74–79
les and triangles: 80–87
allel lines: 196–199
les: 80, 84, 198–199
ygons: 200–207
nmetry: 210–211, 264–265
phing: 256–263, 268–269, 272–273
nslations: 266–267
cles: 208
ading a chart: 108–111
ps as scale drawings: 165, 283
graphs: 270–271

uitive introduction to
unctions: 70–71, 150, 185
aphing: 256–274
gative numbers: 272–273
thematical Activities: 309–317

ontinued from inside front cover]

Book Four

inequalities: 44–45, 81, 95, 225, 228–229,
231, 235–236, 240–243, 330–333
Base-ten machine: 52–53
Reading large numbers to 9 places: 48–51
Negative numbers: 312–313

Inverse relation (+ and −): 58–59
Basic principles: 66–67
Regrouping in addition: 68–75
"Adding" and "subtracting"
money: 76–77, 92–93
Regrouping in subtraction: 82–84, 88–90
Estimating sums and differences: 172–173

Inverse relation (× and ÷): 116–119
Basic principles: 122–125, 182–183
Multiplication facts: 126–131, 134–137
Concept of division: 138–139
Multiples of 10, 100, and 1000 used as
factors: 156–167
Multiplication algorithm: 184–187,
190–193, 198–199
Division algorithm: 220–231,
234–236, 240–243
Factors and products: 252–255

Fractional numbers in measurement: 10–13,
24–25, 316–317, 328–329
Formal introduction to fractional
numbers: 262–267
Equivalent fractions: 268–279,
284–287, 318–325
Fractions used to compare objects: 282–283
Lowest terms fractions: 288–289, 328–329
Concept of fractional numbers: 316–323
Inequalities: 330–333
Addition of fractional
numbers: 334–335, 340–341
Fractional names for whole
numbers: 336–337
Mixed numerals: 338–339

Science: 46–47, 85, 86, 91, 146–147,
203, 237, 245, 247
Social Studies: 54–55, 87, 151, 189, 201
205, 239, 293, 327, 346
Measurement: 18–19, 33, 74–75, 77, 93, 143,
197, 249, 281, 295
Picture Stories: 120, 140, 188, 196, 200, 243
Short Stories: 61, 121, 132, 141, 178,
188, 196, 200, 244, 345
Miscellaneous: 63, 153, 155, 179, 341

Estimating length: 8–9
Estimating perimeter: 14
Estimating 1000 things: 40–41
Meaning of estimation: 170–171
Estimating sums and differences: 172–173
Estimating products and quotients: 174–177
Estimation in story problems: 178–179
Using estimation to find quotients: 224–225,
228–229, 234–235

Even and odd numbers: 250–251
Factors and products: 252–253
Greatest common factor: 254–255
Prime numbers: 256–259

Linear measure: 6–13, 30, 339
Perimeter: 14–17, 349
Area: 20–25, 194
Volume: 26–27, 195
Liquid measure: 28–29

Space figures: 98–103, 106–107, 206–207
Points, angles, lines, line segments: 102-105
Simple closed curves: 108-109
Polygons: 110–111
Symmetry: 112-113, 216–217
Circles: 208–213
Congruent figures: 214–217
Co-ordinate geometry: 296–313
Transformations: 304–307
Graphing negative numbers: 312–313
Maps: 18–19, 293, 317
Bar graphs: 91, 189, 308–309
Charts: 346

Intuitive work with functions: 64,
144–145, 258
Intuitive work with number pairs: 262–267
Distance on maps: 18–19, 156
Mathematical Activities: 347–355

Book Five

Sets: 4–7
Logic and sets: 8–11
Patterns: 12–17
Number line sequences: 58–61
Union and intersection of sets: 176–179
Sets of equivalent fractions: 190–195
Rational number sequences: 212–213, 232
Sets of ordered pairs: 302–319

Symbols for numbers: 20–21
Place value concepts: 22–27
Inequalities: 28–31, 214–215
Numeration systems other than
base ten: 32–37
Fraction numerals: 186–189
Decimal numerals: 252–257
Integers: 316–319

Solving equations: 40–43, 46–49, 54–57
Operations related: 44–45
Inverse relation (+ and −): 50–51
Basic facts: 52–53
Basic principles: 62–65
Estimating sums and differences: 104–109
Review of addition and subtraction: 114–121

Solving equations: 40–43, 46–49, 54–57
Operations related: 44–45
Inverse relation (× and ÷): 50–51
Basic facts: 52–53
Basic principles: 62–65, 122
Estimating products and quotients: 98,
104–109, 155
Special products and quotients: 100–103
Multiplying 2- and 3-digit factors: 124–127
Dividing: 132–147, 156–163
Money problems: 164–165

Meaning of fractions: 186–189, 198–199
Equivalent fractions: 190–197
Lowest terms fractions: 200–201
Concept of fractional numbers: 208–213
Inequalities: 214–217
Ratio: 218–219
Addition and subtraction using
fractions: 226–245
Fractional numbers in measurement: 246–247
Decimal numerals: 252–257
Addition and subtraction using
decimals: 258–261
Decimals and the metric system: 262–263
Multiplication using fractions: 288–291
Basic principles: 240–241, 292–293
Division using fractions: 294–296

Science: 130,
265, 267,
Social Stud
151, 158
Measurem
Picture Sto
Short Stori
245, 297
Miscellane
239, 261

In measure
Story pr
Roundin
Estimati
quoti
Estimati
144–1

Factors
Factor t
Greates
Least c
Least c
Modular

Length:
Angle m
Areas a
Decima
Volume
figure

Number
226–2
Points,
Congru
Angles:
Symme
Perimet
polyg
Space f
Graphi
figure
Relatio
enlar
Tessell
Graphi
Charts:
Bar and
Ratio a

Functio
Numera
Averag
Time, rate, and distance: 152–154
Maps: 220–221
Tessellations: 314–315
Negative numbers: 316–319
Modular (clock) arithmetic: 322–329
Mathematical Activities: 333–341

Book Six

Naming sets: 4–5
Classifying attribute pieces: 6–11
Logical reasoning: 10–11
Patterns: 12–15, 216–217
Union and intersection of sets: 130–135
Fractions and sets: 142–145
Sets of equivalent fractions: 144–145
Repeating decimal sequences: 248–249
Sets of ordered pairs: 282, 332

Symbols for numbers: 18–19
Inequalities: 20–21, 150–153, 214–215,
228–229, 349
Billions: 22
Rounding numbers: 24–25, 234–235
Exponents and scientific notation: 26–29,
34, 96–97, 252–253
Number bases other than base ten: 30–33, 35
Decimals and scientific notation: 252–253
Integers: 340–352

Review of computation: 36–37, 40–45, 84–89
Basic principles: 38–39
Writing and solving equations: 52–53

Review of multiplication skills: 36–37,
40–45, 90–97
Basic principles: 38–39, 50–51
Special products and quotients: 46–49
Writing and solving equations: 52–53
Review of dividing skills: 100–115
Money problems: 116–117

Review of fractions: 142–143
Equivalent fractions: 144–145, 150–151
Lowest terms fractions: 146–147
Concept of fractional numbers: 148–149
Inequalities: 152–153, 214–215, 228–229
Addition and subtraction of fractional
numbers: 156–175
Basic principles: 168–169, 186–187, 192–193
Mixed numerals and improper
fractions: 166–177
Multiplication of fractional numbers: 182–200
Reciprocals: 194–195
Division of fractional numbers: 202–211
Decimal names for fractional
numbers: 224–227, 248–251
Basic operations with decimals: 230–251
Decimals and scientific notation: 252–253
Ratio and percent: 304–327

Science: 28–29, 94–95, 99, 108–109, 174,
198–199, 200, 210, 213, 233, 240, 247,
255, 257, 281, 309, 329

Tessellations: 76–77, 294–295
Clock arithmetic: 136–137
Special functions: 216–217
Probability (ch 14): 330–338
Integers (ch 15): 340–352
Maps and scale drawings: 212–213, 310–311

ISM

AFFER • FLEENOR

ADDISON-WESLEY (CANADA) LIMITE